Seismic Evaluation of Constructions with Z_SOIL.PC
Nonlinear Structural Dynamics and Static Pushover Procedures

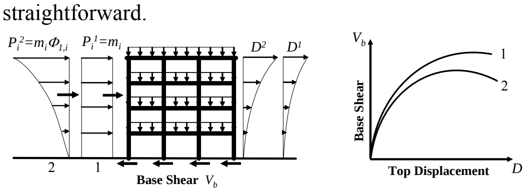
Introduction

Eurocode 8 proposes linear simplified static analysis, linear modal analysis, nonlinear pushover analysis and nonlinear time-history analysis.

This note gives an overview of the steps followed to implement pushover and dynamic analyses in Z_Soil.PC and presents a test-bed application to a reinforced concrete building.

Nonlinear static pushover procedure in Eurocode 8

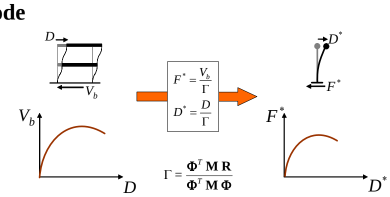
The Nonlinear Static Pushover Procedure in EC8 follows the N2 method developed by Fajfar . The method consists of applying constant load shapes to the building model. The load shapes represent the lateral loads applied by the ground motion. The load intensity is increased in a pseudo-static manner. The structure model can be planar (2D) or spatial (3D), depending on the regularity characteristics of the building. The N2 method was developed using a shear building model, i.e. a frame model with floors rigid in their planes. Furthermore, vertical displacement are typically neglected in the method and only the two horizontal ground motion components, x and y, are considered. Extension to the general case of a fully deformable frame is straightforward.



Load distribution for pushover analysis according to EC8 and pushover response curves

The N2 procedure transforms the response of the MDOF system into the response of an equivalent SDOF system. This is necessary in order to compare the building capacity curve with the demand, expressed in the design codes by the design spectra, which refer to SDOF systems.

Equivalent SDOF model and capacity diagram in Eurocode 8



Capacity curve: transformation from response of MDOF to equivalent SDOF

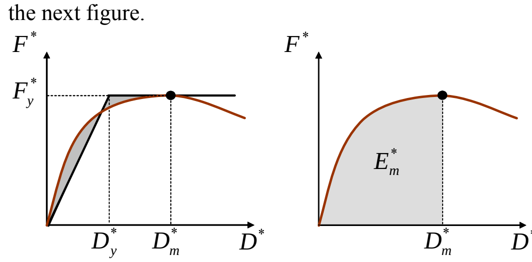
Derivation is straightforward from the equation of motion of a MDOF building subjected to base ground motion,i.e:

$$\mathbf{M\ddot{U}} + \mathbf{F}(\mathbf{U}) = -\mathbf{M}\,\mathbf{R}\,a$$

where damping is neglected, M is the mass matrix , U and F are vectors representing displacements and internal forces, respectively, R is the influence vector and a is the ground acceleration as function of time, i.e. a = a(t). a is given in one direction only. In the linear elastic case $\mathbf{F} = \mathbf{KU}$ (where $\mathbf{K}$ is the structure stiffness matrix), in the nonlinear case $\mathbf{F}$ depends on the displacement history.

Linearization of the capacity curve and comparison to demand spectrum

In order to compare the capacity curve to the demand curve given by the design spectrum, the nonlinear pushover curves of the SDOF are approximated by elastic-perfectly plastic (or bilinear) curves. According to EC8 this transformation can be based on the equal energy principle. A target displacement D_m^* is assumed, and equal energy is assumed between bilinear and nonlinear pushover curves. This simple procedure is illustrated in the next figure.

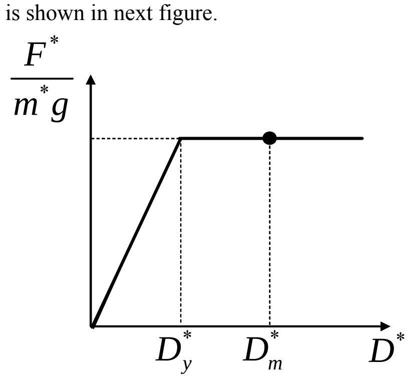


Bilinearization of the capacity curve of SDOF

The bilinearization gives the yield force and the yield displacement $D_y^* = 2\left(D_m^* - \frac{E_m^*}{F_y^*}\right)$, which allow the initial elastic period to be computed as:

$$T^* = 2\pi\sqrt{\frac{m^* D_y^*}{F_y^*}}$$

Also, the capacity curve is transformed into capacity spectrum by normalizing the force F^* with respect to the SDOF weight $m^* g$. The resulting capacity spectrum is shown in next figure.

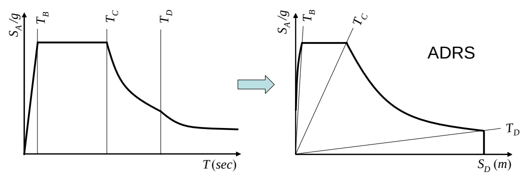


SDOF capacity spectrum

The demand on the building is given by the design spectrum provided by the design codes. In order to compare capacity and demand, the first step is to transform the format of the design spectrum from the classical Acceleration A vs Period T format to the ADRS format, i.e. Acceleration A vs. Displacement D. Acceleration and Displacement are related by

$$S_D = \left(\frac{T}{2\pi}\right)^2 S_A$$

The transformation to the ADRS spectrum is shown in next figure. Lines from the origin represent constant period.



Transformation to ADRS linear spectrum

The capacity spectrum is now compared to the ADRS demand spectrum. The comparison is not immediate, because the capacity spectrum is nonlinear, while the ADRS spectrum given by design codes is linear.

For a SDOF system with an bilinear plastic behavior, the acceleration spectrum SA and the displacement spectrum SD can be determined as

$$S_A = \frac{S_{Ae}}{R_\mu}$$

$$S_D = \frac{\mu}{R_\mu} S_{De} = \frac{\mu}{R_\mu} \frac{T^2}{4\pi^2} S_{Ae} = \mu \frac{T^2}{4\pi^2} S_{Ae}$$

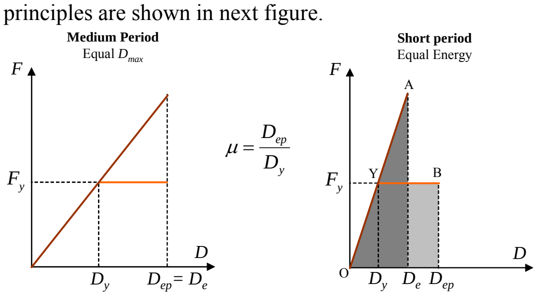
where subscript e indicates elastic, μ is the ductility factor = maximum inelastic displacement/yield displacement, and R μ is the reduction factor due to ductility.

The reduction factor R μ can be found in different ways, some analytical, other approximated. In the simple version of the N2 method, the following approximated expressions are given:

$$R_\mu = (\mu - 1)\frac{T}{T_C} + 1 \qquad T < T_C$$

$$R_\mu = \mu \qquad T < T_C$$

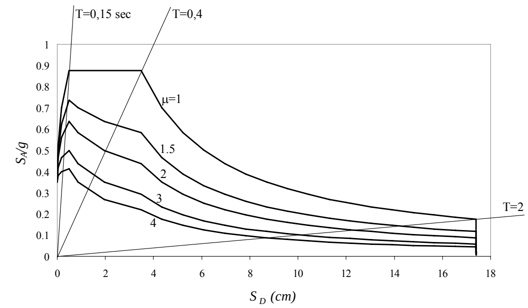
where T_C is a characteristic period of the ground motion that depends on the soil type and is given by EC8. It typically corresponds to the transition from the constant acceleration range (short-period range) to the constant velocity range (medium-period range) in the response spectrum. The above equations suggest that in the short-period range the equal displacement principle is applied (elastic and inelastic SDOFs have the same maximum displacement), while in the medium- and long-period range the equal energy principle is applied. These principles are shown in next figure.



Transformation elastic response – bilinear response: equal maximum displacement and equal energy assumptions

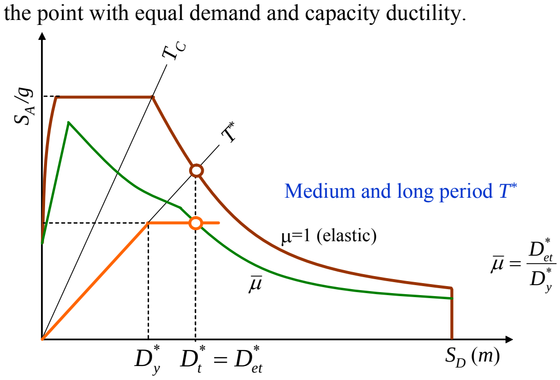
Above equations are used to obtain inelastic demand spectra of constant ductility, as shown below. The inelastic demand spectra can also be determined by a rigorous procedure by using nonlinear dynamic analysis.

Using the procedures illustrated in the section, the seismic demand on the equivalent SDOF can be determined. The steps are schematically illustrated for



Demand spectra for constant ductilities in AD format(based on EC8 spectrum for Zone 1, Soil type A)

a bilinear oscillator with medium or long elastic period T*. Given the elastic demand spectrum and the bilinear capacity spectrum, from a theoretical point of view the target displacement is determined by finding the inelastic demand spectrum of ductility $\bar{\mu}$ that intersects the capacity spectrum in a point corresponding to a capacity ductility $\bar{\mu}$. In other words, the design point is given by the point with equal demand and capacity ductility.



Capacity and Demand spectra for medium and long period T*:determination of the target displacement for SDOF

Dynamics in Z_Soil.PC

This section briefly reviews the most important features of the procedures used in program Z_Soil for dynamic problems. No special procedures are needed at the computational levels for earthquake engineering analysis.

Frames

Recent versions of Z_Soil include advanced models for frame analysis. In particular, nonlinear beams with fiber section models are available. The cross section is divided into fibers and the constitutive laws of each fiber is assigned from the consititutive law library available in Z_Soil. Both displacement-based and force-based formulations are available. Force-based elements **Error! Reference source not found.** are exact within the classical Euler-Bernoulli beam theory. As for geometric nonlinearities, these are considered in the general framework of the program and thus follow a corotational approach.

Equations of motion

The following equations of motion are used for nonlinear dynamics:

$$\mathbf{Ma} + \mathbf{Cv} + \mathbf{N}(\mathbf{d}) = \mathbf{F}$$

where a, v and d represent approximations of $\ddot{\mathbf{U}}$, $\dot{\mathbf{U}}$ and $\mathbf{U}$, respectively. In order to be consistent with the notation of the Z_Soil user manual, $\mathbf{N}$ indicates the internal forces and $\mathbf{F}$ the time dependent externally applied forces, and Rayleigh damping is used :

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$$

where α and β are obtained by imposing damping rates at 2 different modal frequencies (i,j):

$$\begin{Bmatrix} \xi_i \\ \xi_j \end{Bmatrix} = 0,5 \begin{bmatrix} 1/\omega_i & \omega_j \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}$$

C takes into account minor nonlinearities ignored in **N**;K is typically approximed by the initial stiffness.

Seismic input

Seismic input is given as imposed displacements or accelerations of the ground. Indipendently of the input format, the solving algorithms can find the solution in terms of accelerations, displacements, or velocities.

Nonlinear algorithms for arbitrary dynamic input

Newmark and HHT algorithms are available in Z_Soil to solve equilibrium at any time $t_{n+1} = (n+1)\cdot\Delta t$.

Newmark's algorithm consists of the following equations:

$$\mathbf{M}\,\mathbf{a}_{n+1} + \mathbf{C}\,\mathbf{v}_{n+1} + \mathbf{N}(\mathbf{d}_{n+1}) = \mathbf{F}_{n+1}$$

with

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t\,\mathbf{v}_n + \frac{\Delta t^2}{2}\,[(1-2\beta)\,\mathbf{a}_n + 2\beta\ \mathbf{a}_{n+1}]$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + \Delta t[(1-\gamma)\,\mathbf{a}_n + \gamma\ \mathbf{a}_{n+1}]$$

where subscript n+1 indicate values at instants t_{n+1}. The last 2 equations are used to eliminate accelerations and velocities from the 1st and an equivalent static problem results, in terms of displacements only.

Let's consider the STATIC PROCEDURE first. If superscript i+1 indicates iterations, in the vicinity of the displacement $\mathbf{d}_{n+1}^{i+1}$ we have :

$$\mathbf{N}\left(\mathbf{d}_{n+1}^{i+1}\right) \cong \mathbf{N}\left(\mathbf{d}_{n+1}^{i}\right) + \frac{d\mathbf{N}}{d\mathbf{d}}\ \Delta\mathbf{d} = \mathbf{N}\left(\mathbf{d}_{n+1}^{i}\right) + \mathbf{K}_T\ \Delta\mathbf{d}$$

hence:

$$\begin{cases} \mathbf{K}_T \cdot \Delta\mathbf{d} = \mathbf{F}_{n+1} - \mathbf{N}\left(\mathbf{d}_{n+1}^{i}\right) & \left(\text{with } \mathbf{d}_{n+1}^{0} = \mathbf{d}_n\right) \\ \mathbf{d}_{n+1}^{i+1} = \mathbf{d}_{n+1}^{i} + \Delta\mathbf{d} & \end{cases}$$

Convergence is reached when:

$$\left\|\mathbf{F}_{n+1} - \mathbf{N}\left(\mathbf{d}_{n+1}^{i+1}\right)\right\| < TOLERANCE$$

Let's now consider the DYAMICS PROCEDURE in terms of incremental displacements $\Delta\mathbf{d}$

The equivalent static problem simply introduces:

$$\mathbf{N}^*(\mathbf{d}) = \mathbf{Ma} + \mathbf{Cv} + \mathbf{N}(\mathbf{d})$$

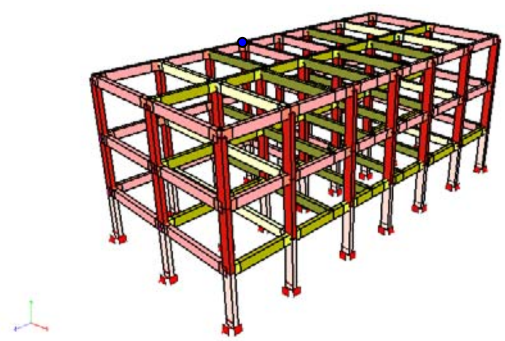
$$\mathbf{K}^* = \left(\frac{d\mathbf{N}^*}{d\mathbf{d}}\right) = \frac{\mathbf{M}}{\Delta t^2\beta} + \frac{\gamma}{\Delta t\beta}\,\mathbf{C} + \mathbf{K}$$

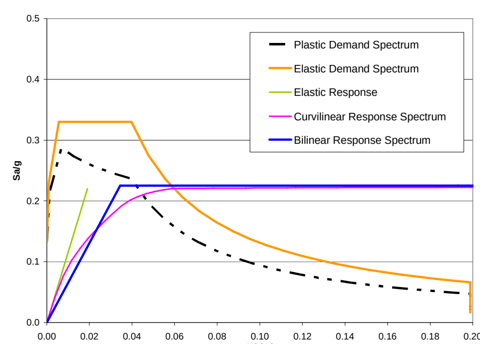
Application: Study of Bonefro buidling

An existing three-storey reinforced concrete building is studied using the nonlinear frame analysis capabilities of Z_Soil. The building is in Bonefro, Italy, and is a good example of residential buildings of the 70's and 80's in Italy, prior to the introduction of the seismic code in the early 80's .

Nonlinear pushover on 3D frame with Z_Soil (small displacements)

The next figure shows the building model and the top floor point at which displacements are measured. Force-based elements are used. Modal and uniform load distributions are considered.



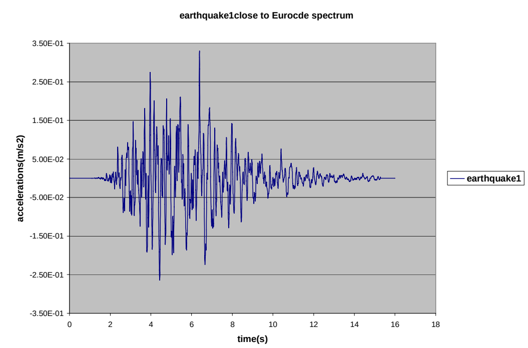


Determination of target displacement for modal loading in x direction load distribution

	Target displacement under Uniform load (SDOF)	Target displacement under Modal load (SDOF)	Target displacement Uniform load (MDOF)	Target displacement under Modal load (MDOF)
2 D	0.051m	0.042m	0.0612m	0.0504m
3 D	0.052m	0.06m	0.067m	0.077m

Dynamics with simulated accelerograms

Three accelerograms compatible with the prescribed linear response spectrum are applied. The three accelerograms are simulated accelerograms generated with Sabetta's program. Input parameters are epicentral distance, earthquake magnitude and type of soil. Sabetta's program generates nonstationary artificial earthquakes according to an empirical method based on the regression of the relations of attenuation of a collection of earthquakes measured in Italy (95 accelerograms of 17 earthquakes, with magnitude 4.6 to 6.8). A significant number of earthquakes are generated and the average is taken. Input parameters adopted here are: magnitude = 6.02, epicentral distance = 22.8 km, type soil = shallow. A sample and the relevant mean spectrum are shown below. The three earthquakes were scaled to 0,33g PGA.

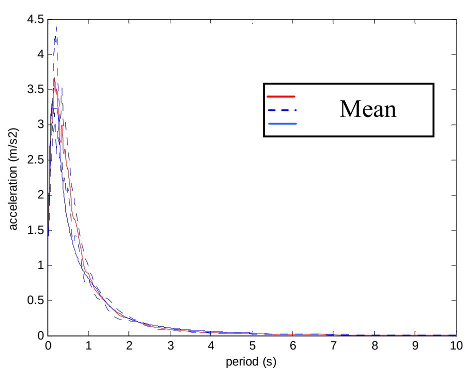


Earthquake 1

Nonlinear time history analysis of 3D model

A time-history analysis of the 3D model is performed with three accelerograms. The results for ground motion input applied in one direction only, x, are compared for pushover and time-history analysis. Stiffness proportional Rayleigh damping is prescribed, with 5% damping at 2 Hertz. The resulting values for Rayleigh damping are $\alpha = 0$ and $\beta = 0{,}008$.

When 3 accelerograms are used, EC8 prescribes that the maximum of the three maximum actions must be used for verifications. The maximum top floor displacement for the 3 earthquakes is 0.072m. The target displacement for the pushovere analysis is 0,077m .



Mean spectrum

Sensitivity to seismic parameters

A sensitivity of the time history analyses to the seismic parameters used in generating the accelerograms with the program by Sabetta is done next. The three parameters: magnitude, epicenter distance and soil type are changed. The reference values are magnitude = 6.02, epicenter distance = 22.8 km, shallow soil type. Results are briefly discussed below.

Observations and concluding remarks

The comparison between pushover and dynamics gives difference of 7% (dmax modal from pushover/ dmax dynamics) =0.077/0.072.

The sensitivity to Magnitude, in dynamics a (+-5%) variation in EQ magnitude yields a variation in max displacement of(+47% and –17%).

The sensitivity to epicenter distance, in dynamics (via Sabetta program) a (+100%, -50%) variation yields a variation in max displacement of (+31%, -10%).

The sensitivity to soil, in dynamics (via Sabetta program): shallow/deep = 0.072/0.065=1.1 the difference is about 10%.

Large displacements induce global softening and increase target displacements in 3D pushover +11% for uniform loading 0.058/0.052, +3% for modal loading 0.062/0.06.

No influence was observed in dynamics.

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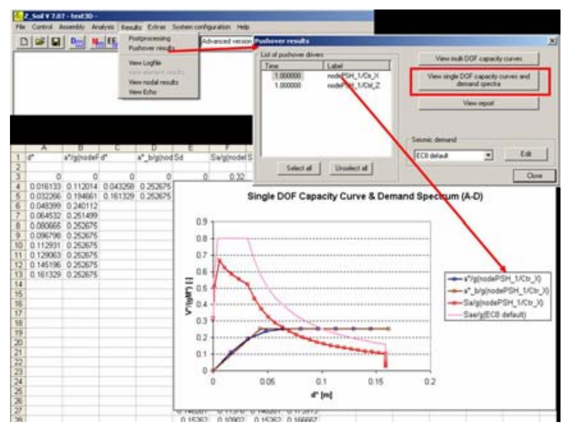
& J.-L.Sarf, BET J.-L.Sarf

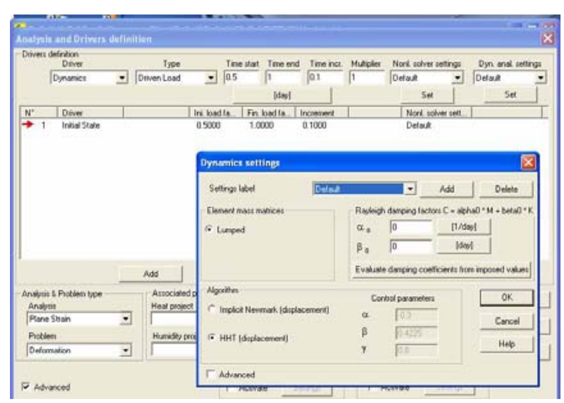
References

M.Belgasmia, E.Spacone, A.Urbanski, Th.Zimmermann. Seismic Evaluation of Constructions with Z_SOIL.PC. Nonlinear Structural Dynamics and Static Pushover procedures . LSC-EPFL Rep. 2006/1, and references therein.

Th.Zimmermann, A.Truty, A.Urbanski, K.Podles. Z-Soil user manual, Zace Services Ltd, 1985-2006.



Output screen for pushover analysis in Z_Soil.PC



Input screen for dynamic time-integration analysis in Z_Soil.PC