Constitutive models for practice

in ZSoil v2014

Rafal OBRZUD

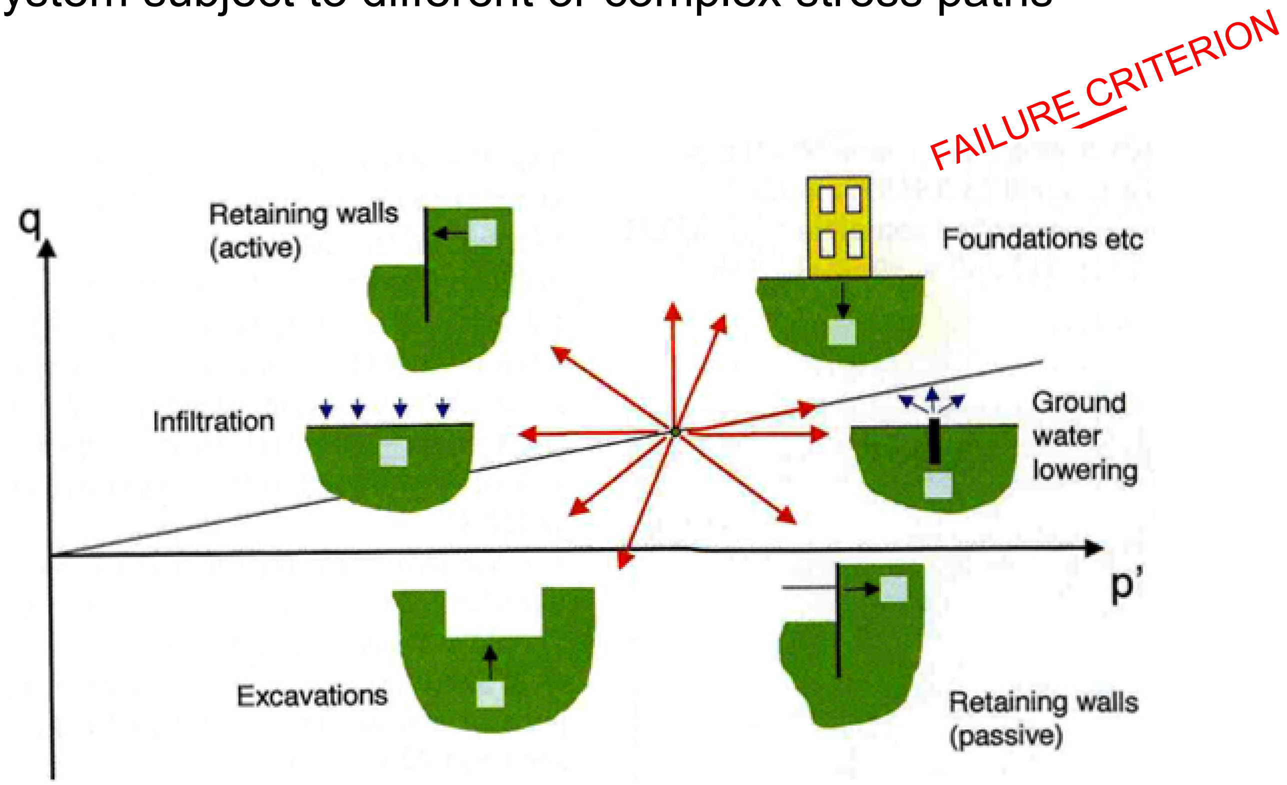
ZSoil® August 2015

Contents

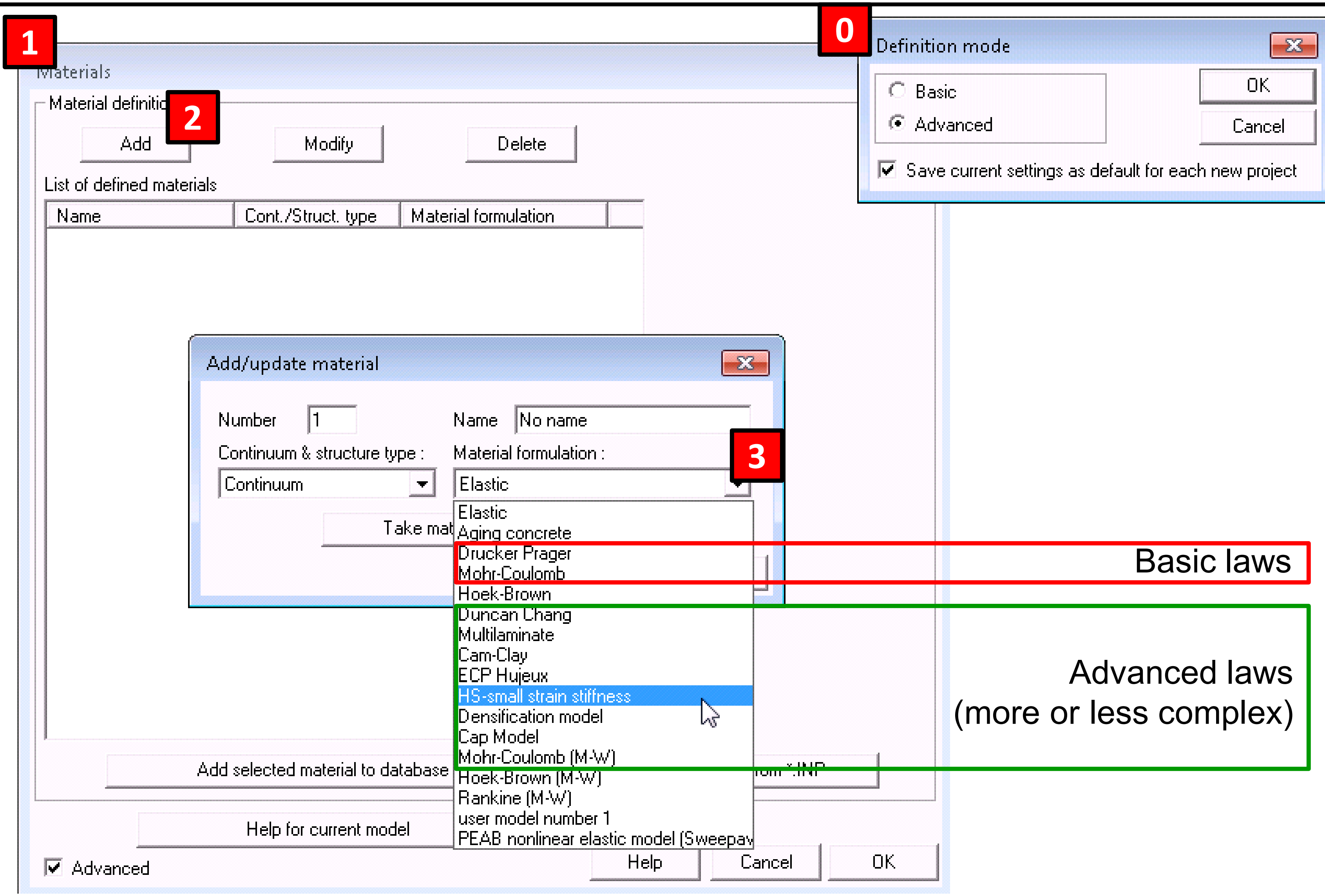
Constitutive laws for finite element analysis

Goal of FE analysis:

reproduce as precisely as possible stress-strain response of a system subject to different or complex stress paths

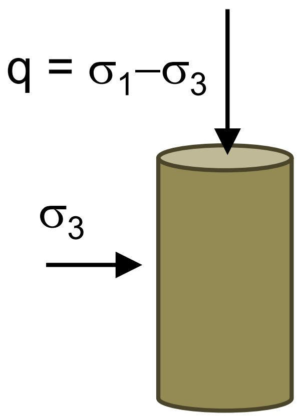


Continuum models available in ZSoil v2014

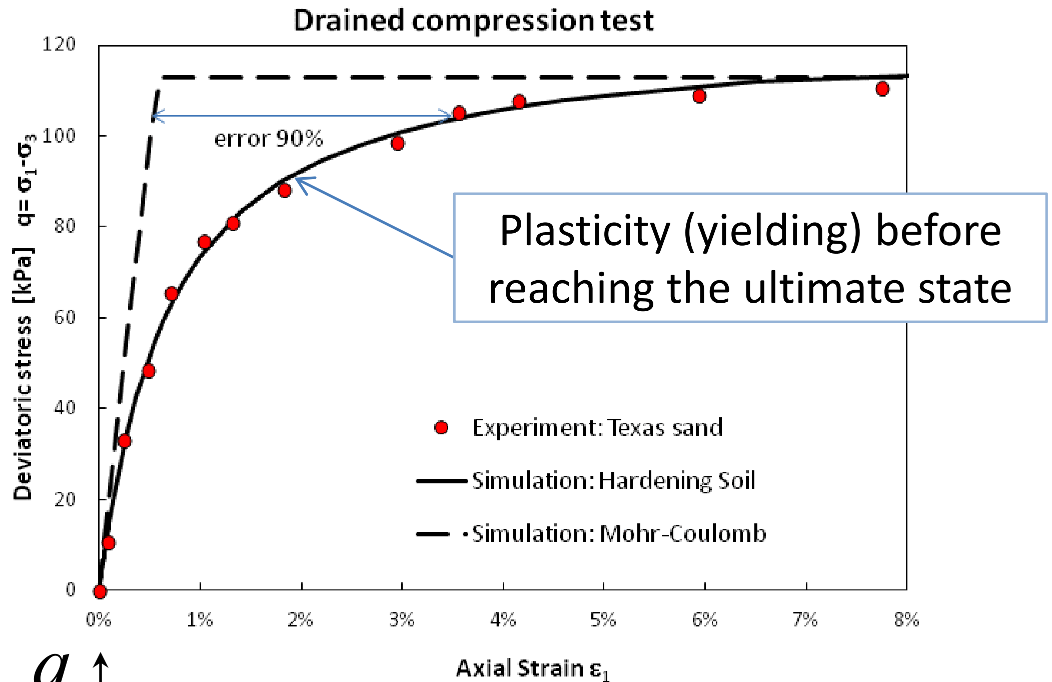


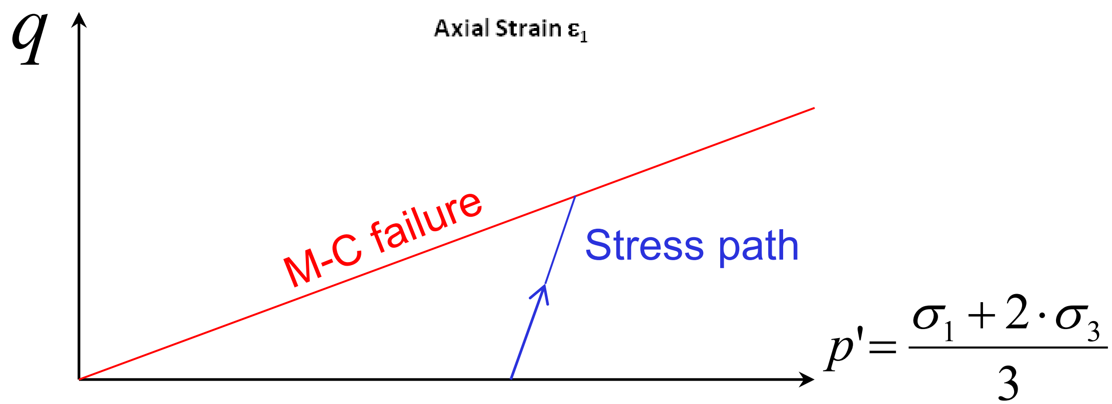
Why do we need advanced constitutive models?

Better approximation of soil behavior for simulations of practical cases



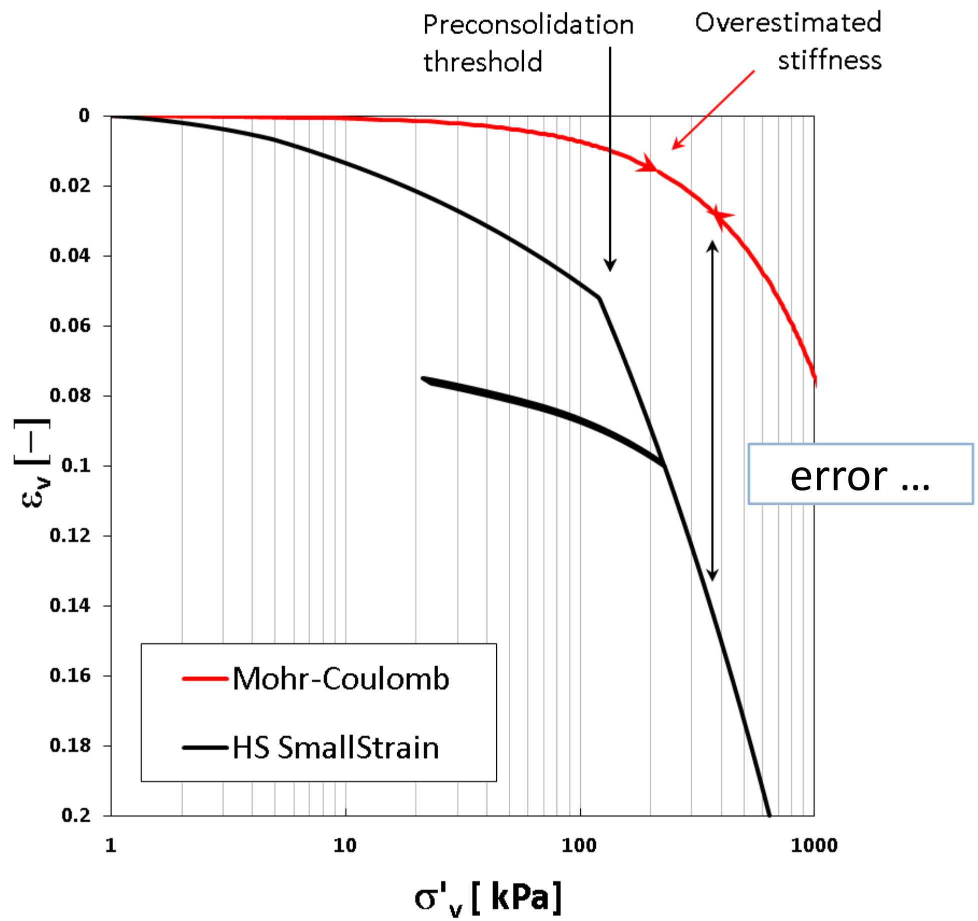
Triaxial stress conditions
(e.g. under footing)

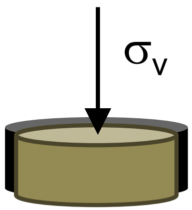




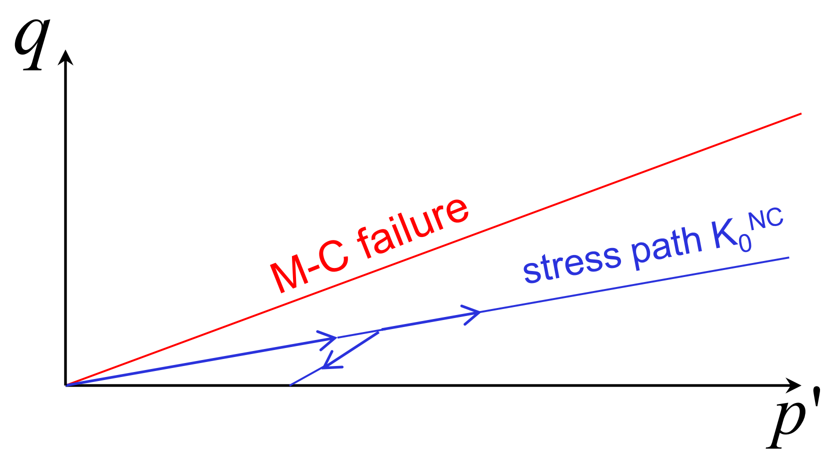
Why do we need advanced constitutive models?

Accounting for stress history and volumetric plastic straining





Odometer test conditions (e.g. very wide foundation)



Why do we need advanced constitutive models?

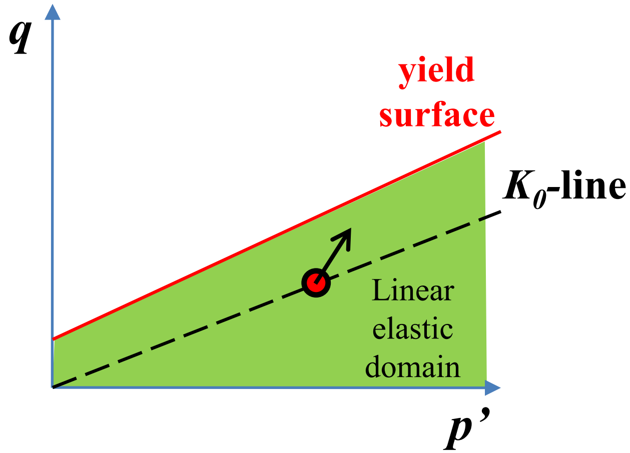
- ENGINEERING CALCULATIONS
  - ULTIMATE STATE ANALYSIS
    - Bearing capacity
    - Slope stability
  - DEFORMATION ANALYSIS
    - Pile, retaining wall deflection
    - Supported deep excavations
    - Tunnel excavations
    - Consolidation problems

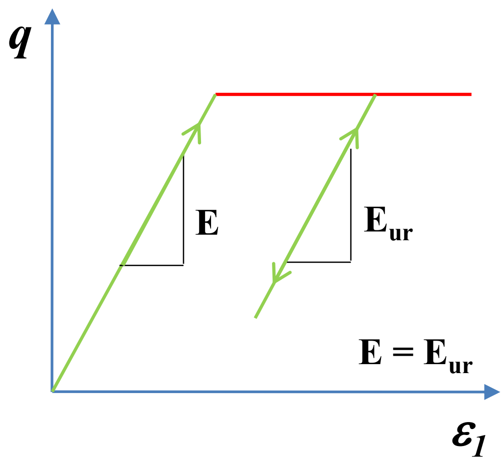
Basic models e.g. Mohr-Coulomb

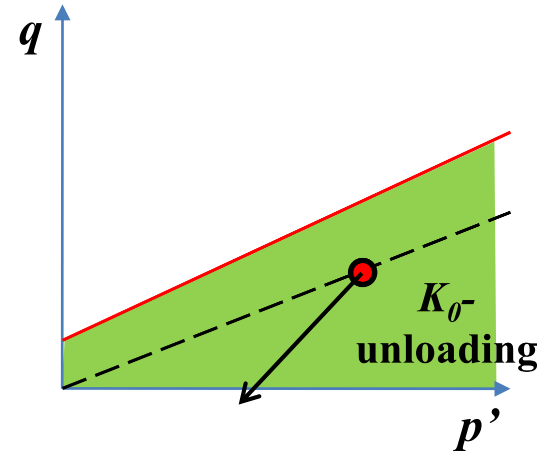
Advanced soil models

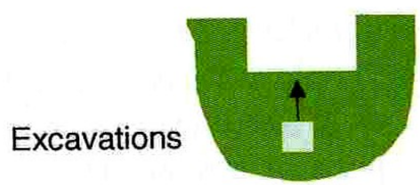
Basic differences between implemented soil models

Mohr-Coulomb



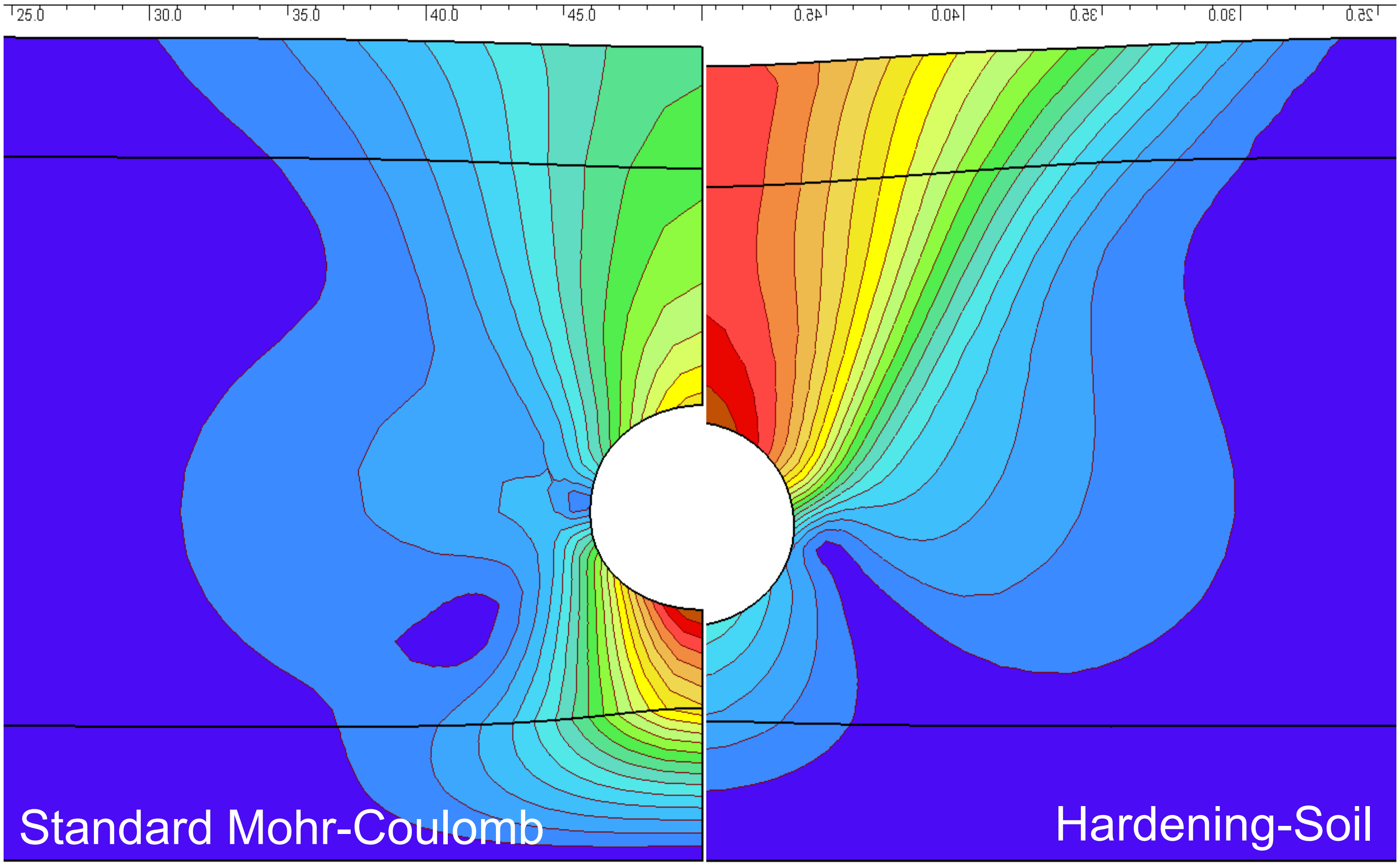






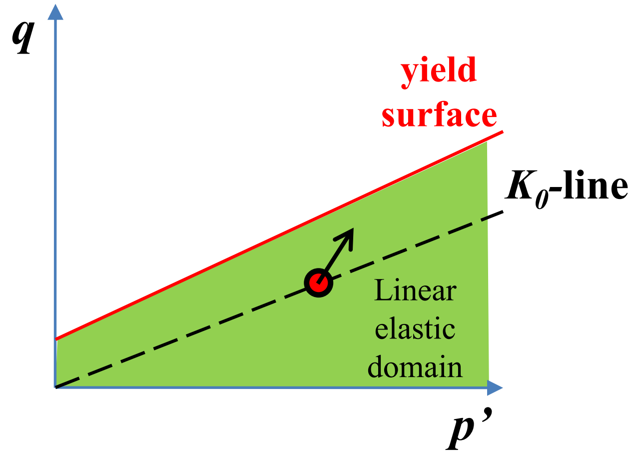
Practical applications of the Hardening Soil model

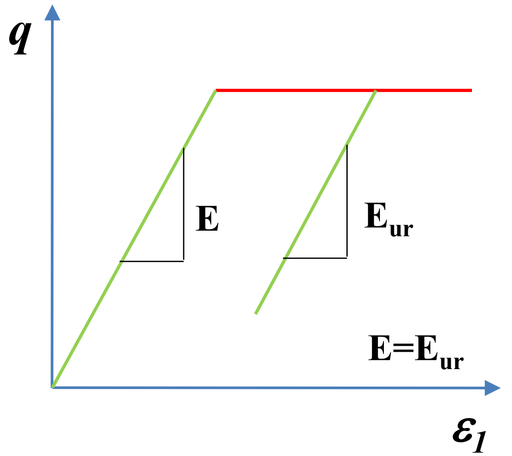
Tunneling



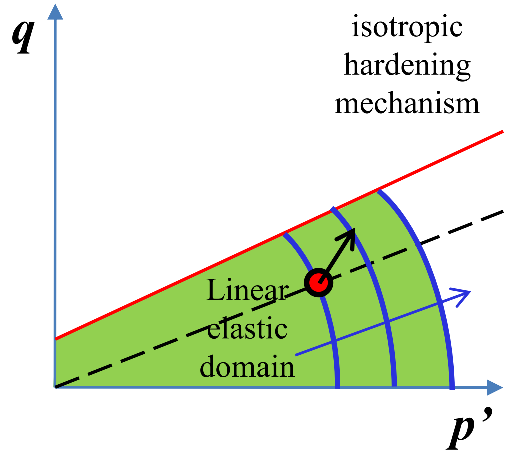
Basic differences between implemented soil models

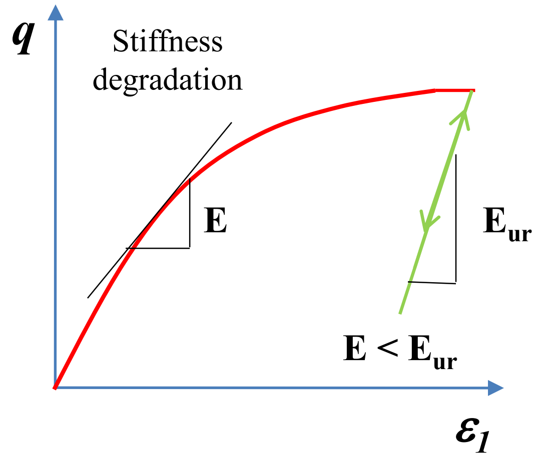
Mohr-Coulomb

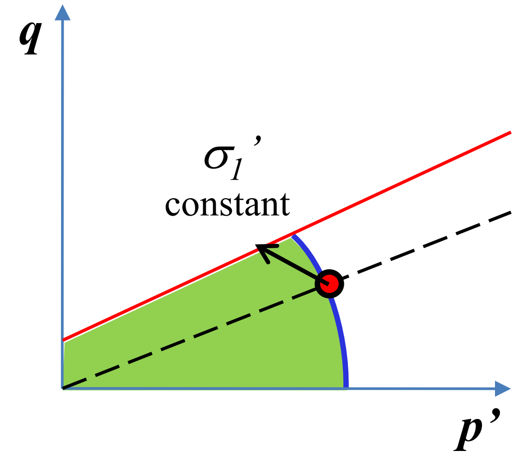


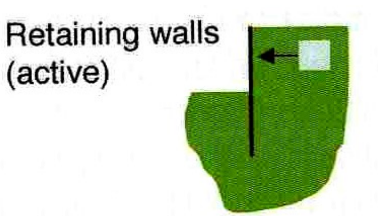


Volumetric cap models



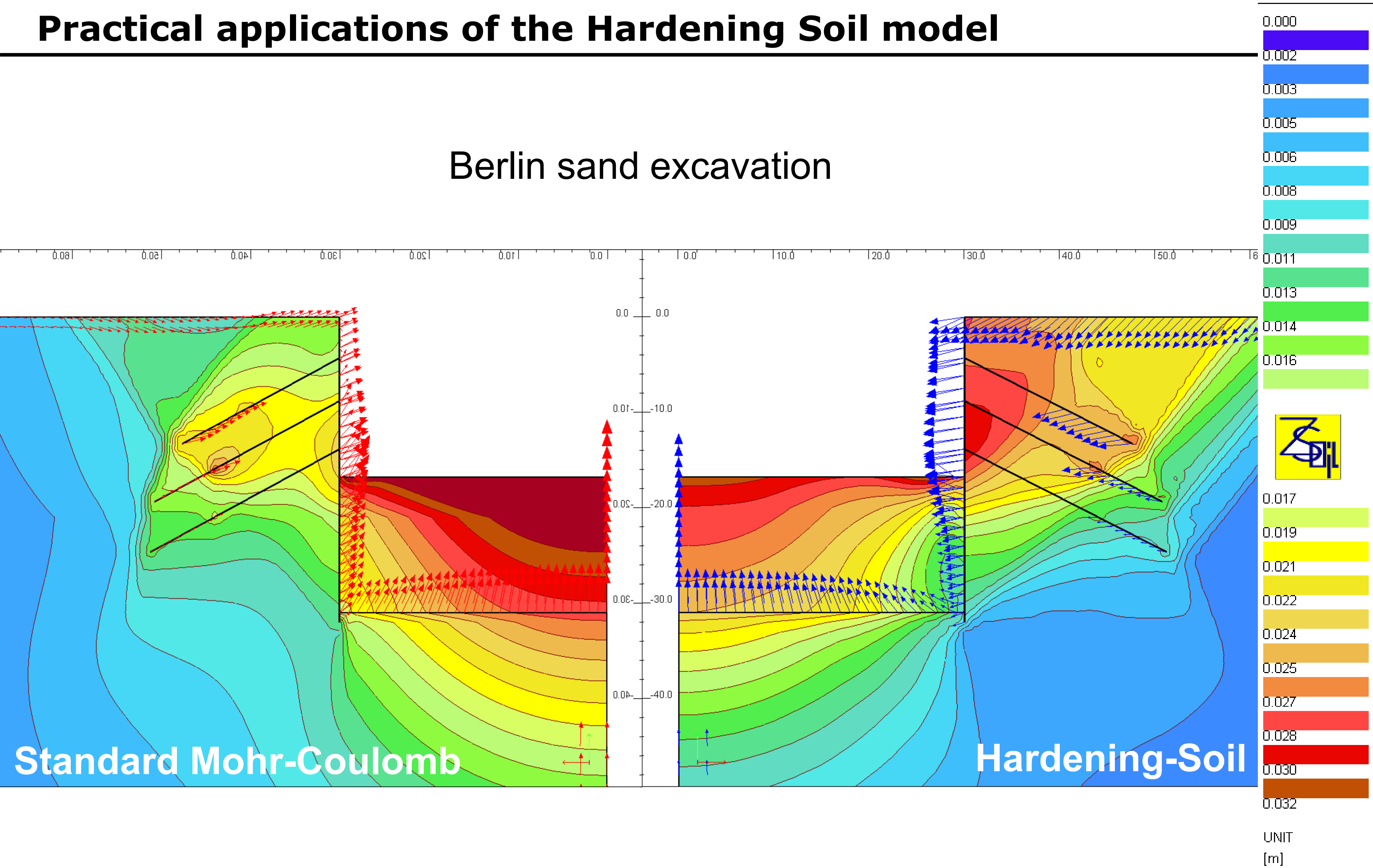






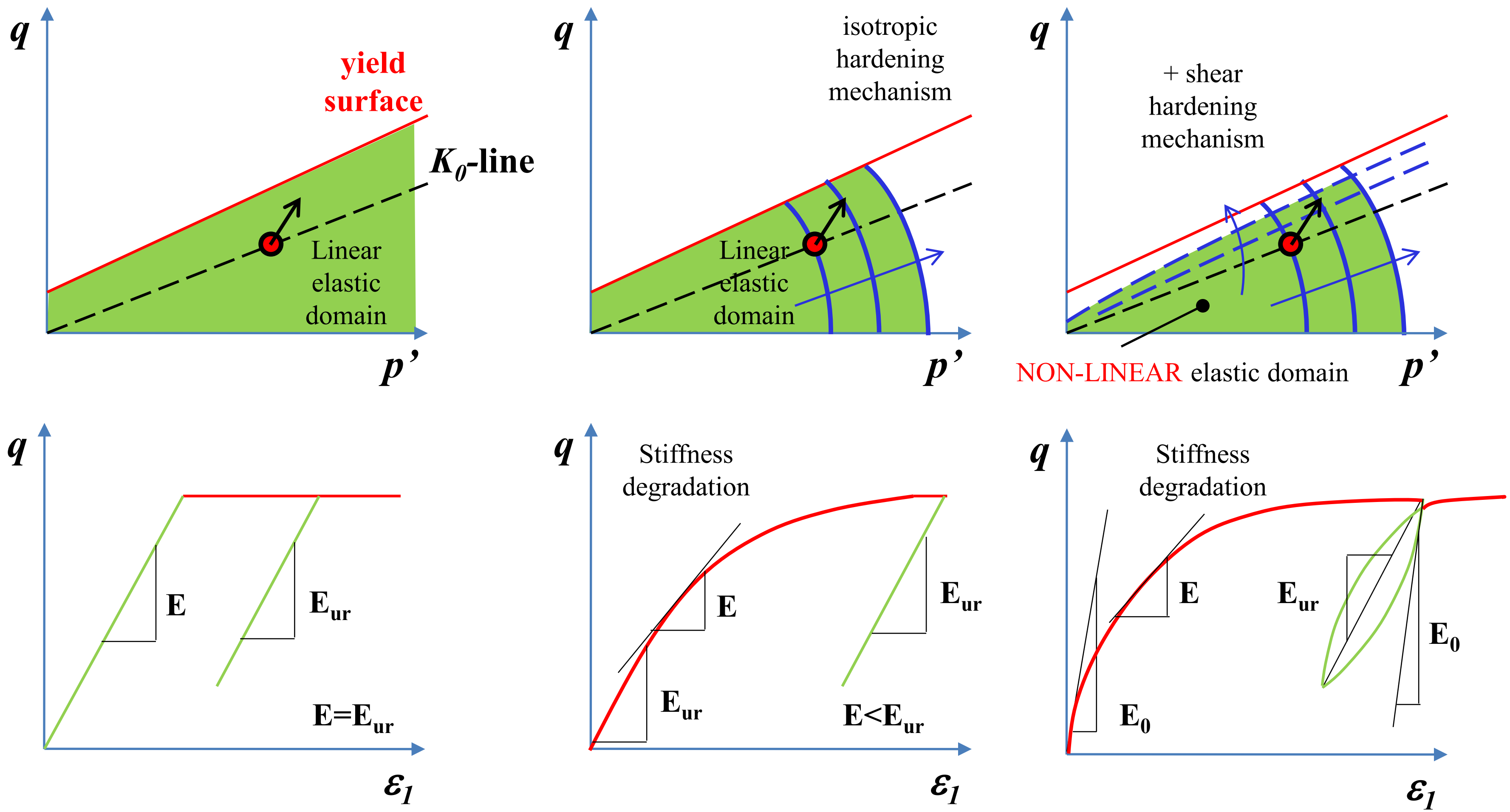
Practical applications of the Hardening Soil model

Berlin sand excavation



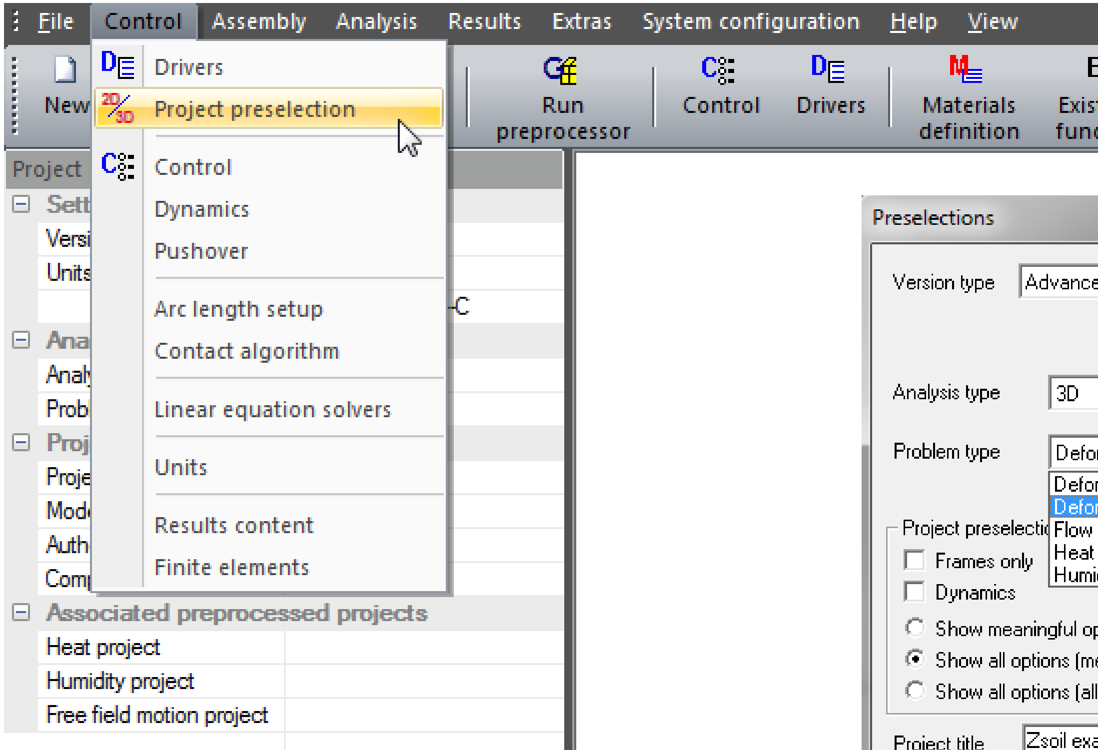
Basic differences between implemented soil models

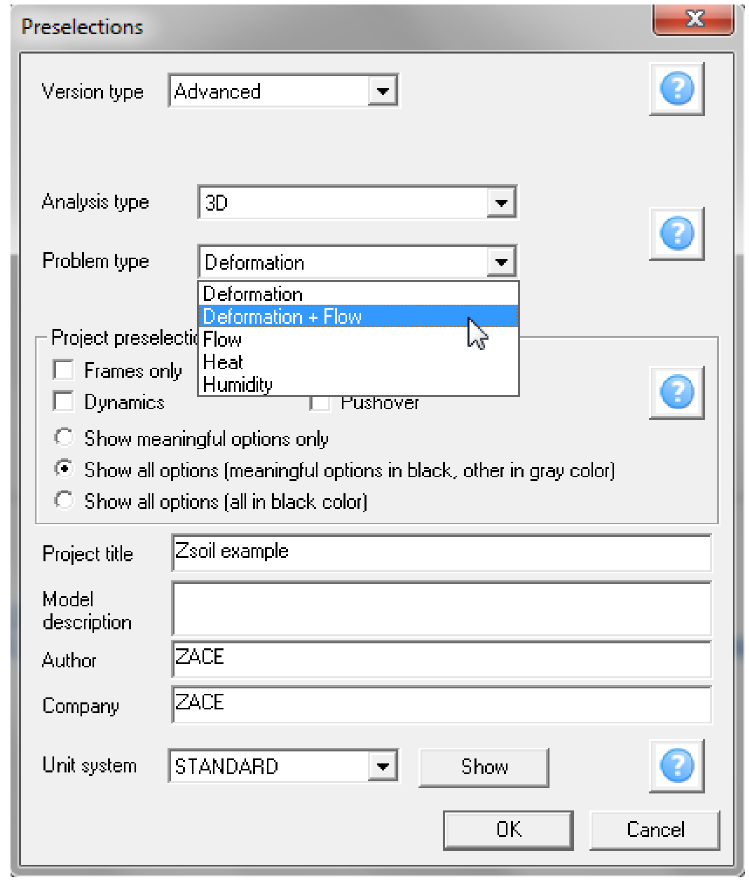
Mohr-Coulomb | Volumetric cap models | Hardening Soil models



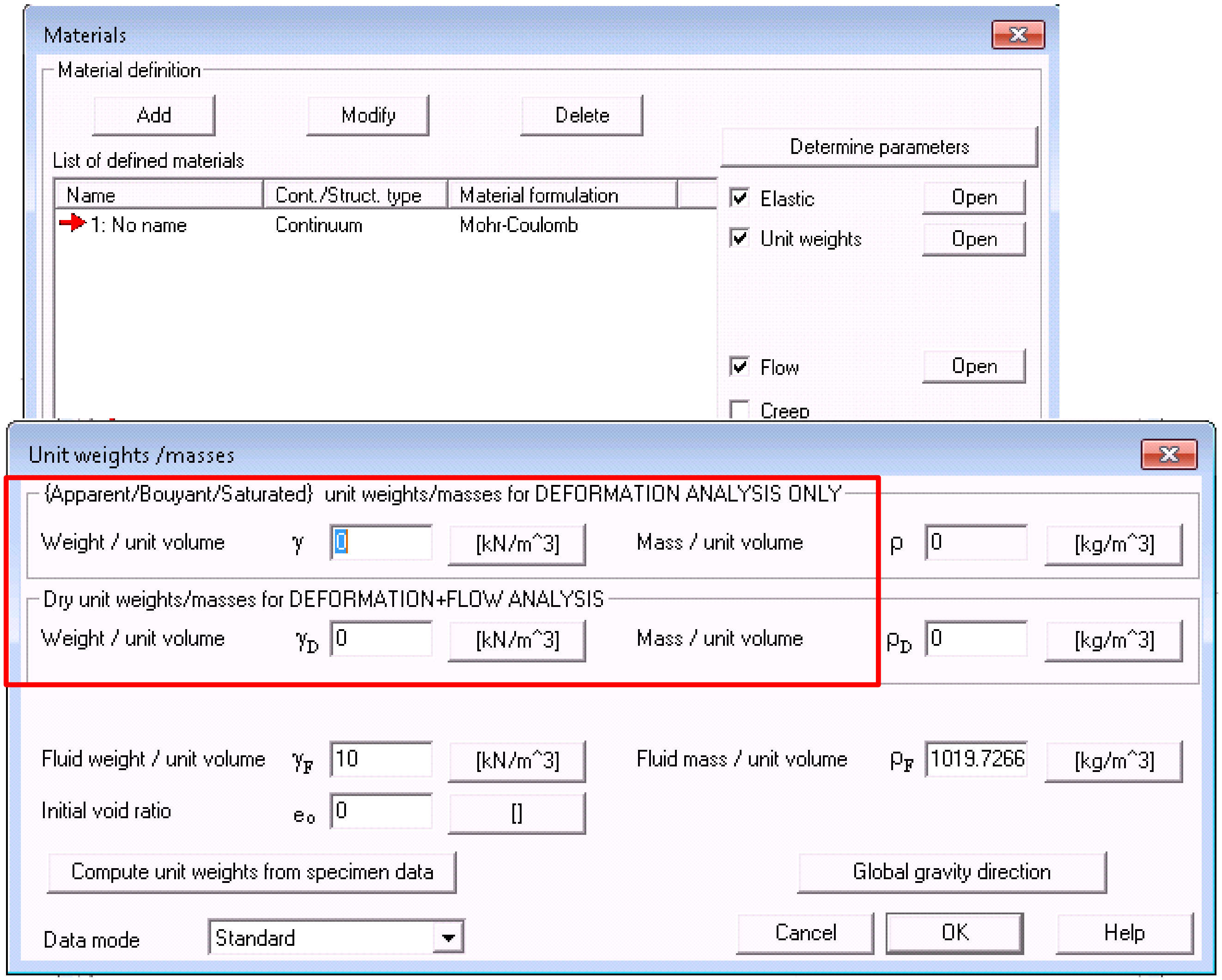
Contents

Defining type of analysis





Setting parameters: Initial state setup



Setting unit weight

DEFORMATION ANALYSIS

{Apparent/Bouyant/Saturated} unit weights/masses for DEFORMATION ANALYSIS ONLY

Weight / unit volume γ [0] [kN/m^3] Mass / unit volume

Total stress analysis | **Effective stress analysis**

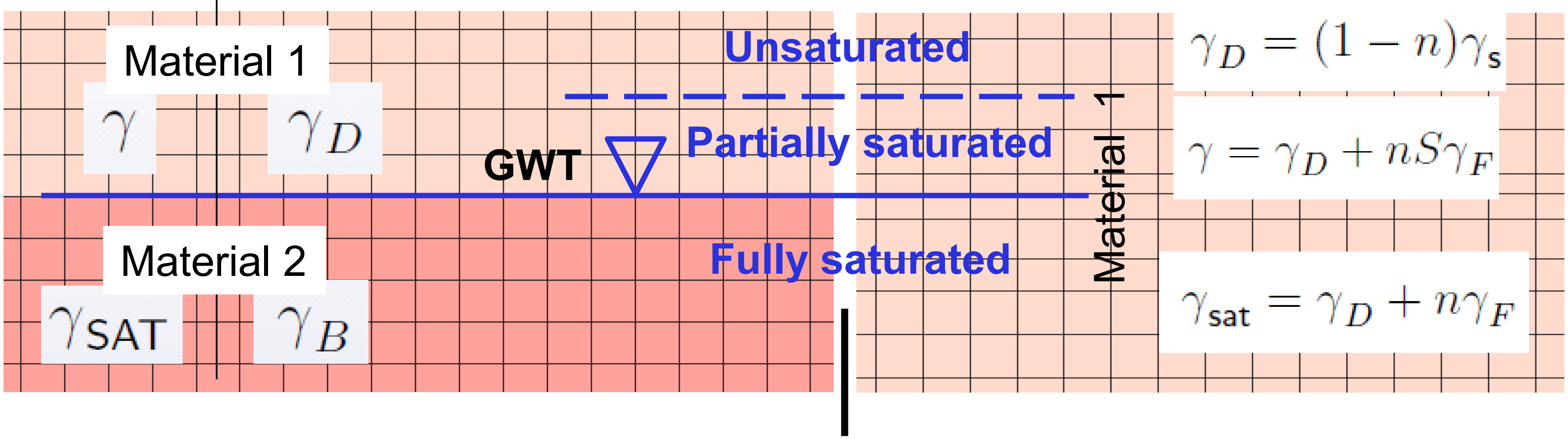
DEFORMATION + FLOW ANALYSIS

Dry unit weights/masses for DEFORMATION+FLOW ANALYSIS

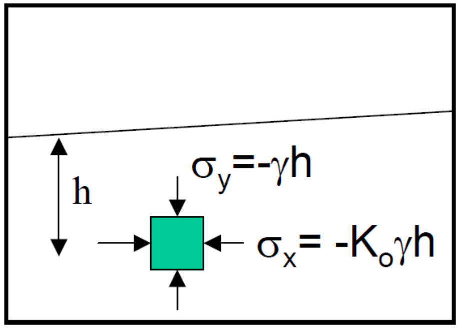
Weight / unit volume γ_D [0] [kN/m^3]

$$\gamma = \gamma_D + nS\gamma_F$$

Always effective stress analysis



Imposing initial stresses using superelements



Below GWT

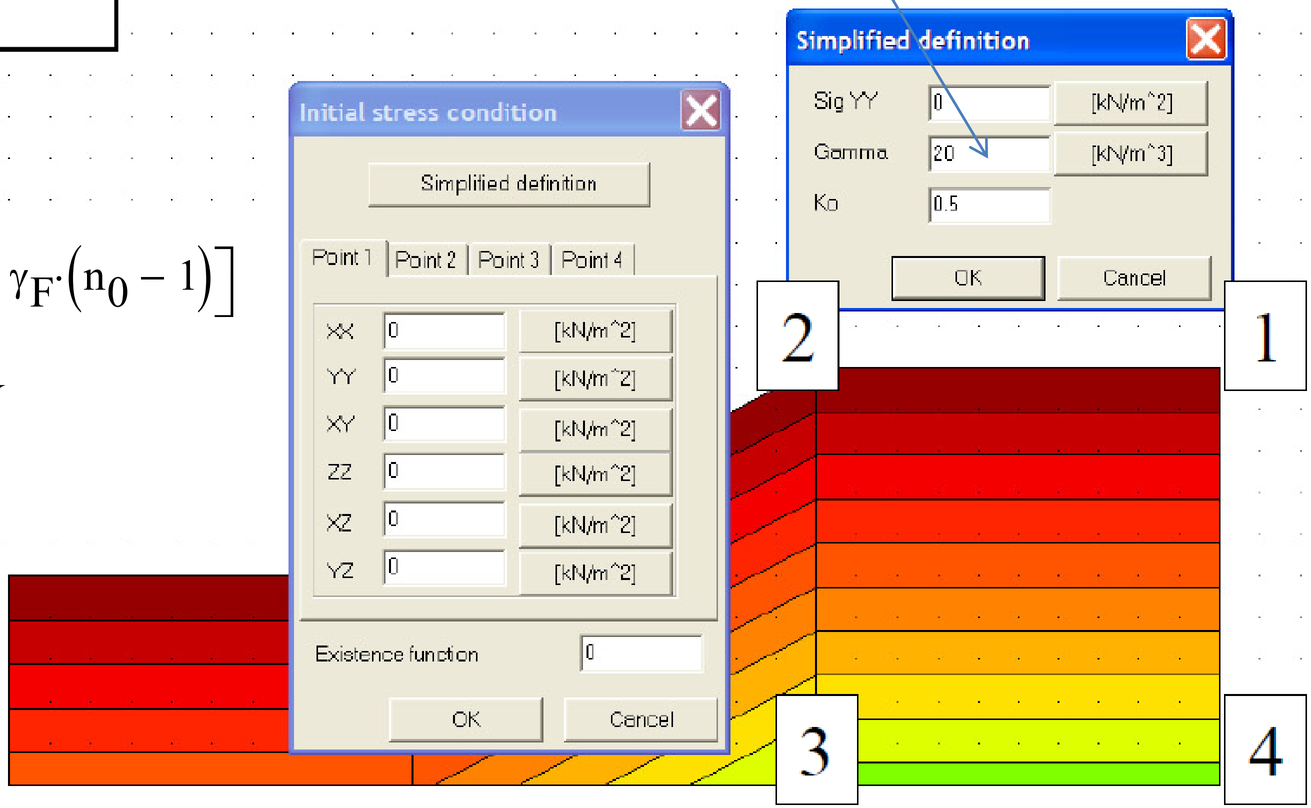
$$\gamma' := \gamma_{sat} - \gamma_F$$

Below GWT

$$\sigma'_{YY} := h \cdot \left[\gamma_D + \gamma_F \cdot \left(n_0 - 1 \right) \right]$$

$$\sigma'_{XX} := K_0 \cdot \sigma'_{YY}$$

$$\sigma'_{ZZ} := \sigma'_{XX}$$



NB. Effective initial stresses setup is mandatory for Modified Cam-clay

Contents

- Introduction
- Initial stress state and definition of effective stresses
- **Saturated and partially-saturated two-phase continuum**
- Introduction to the Hardening Soil model (HSM)
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From saturated to unsaturated soil

S - saturation degree

$$S = \frac{V_w}{V_v}$$

p_w – pressure in liquid phase

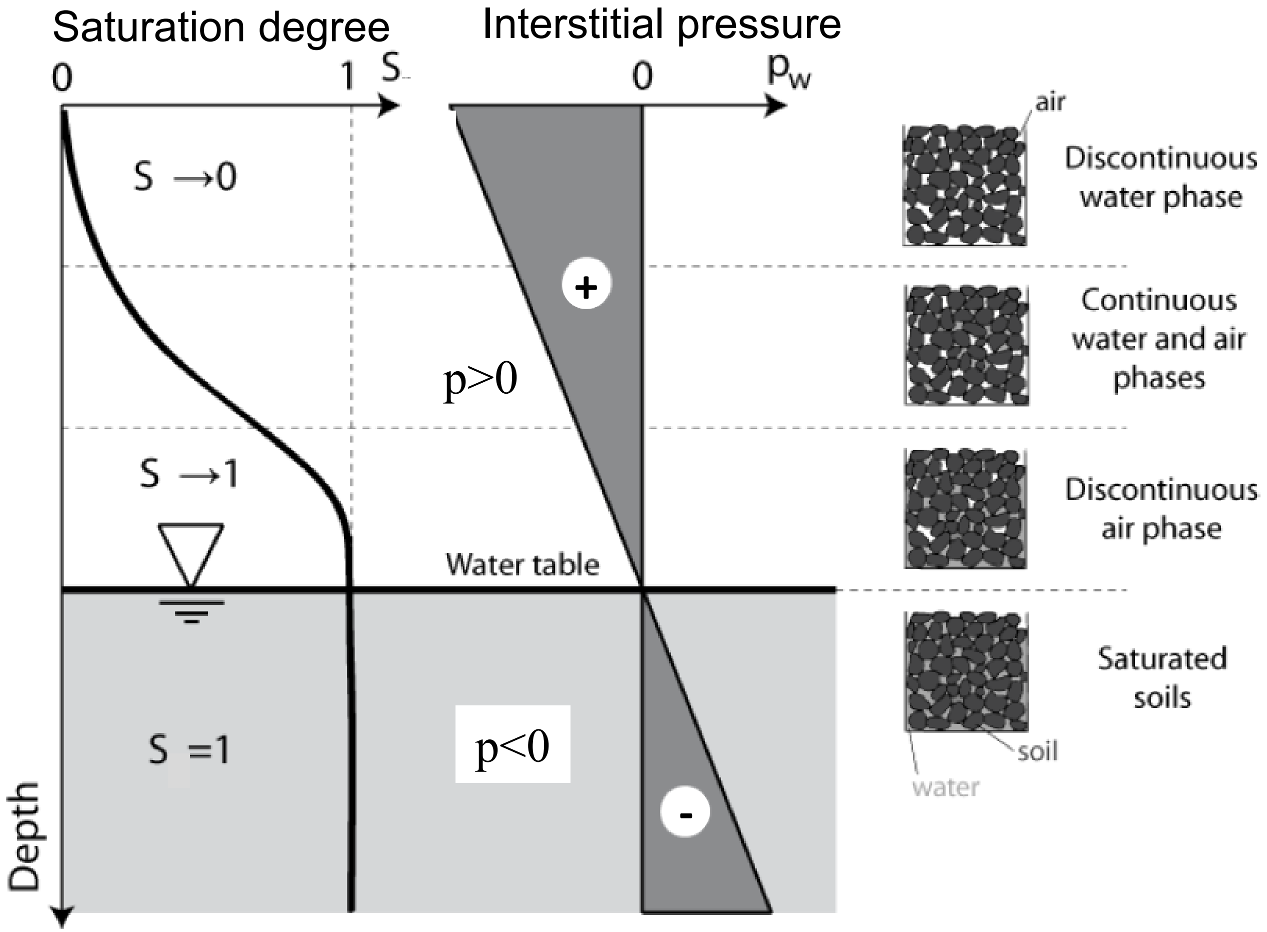


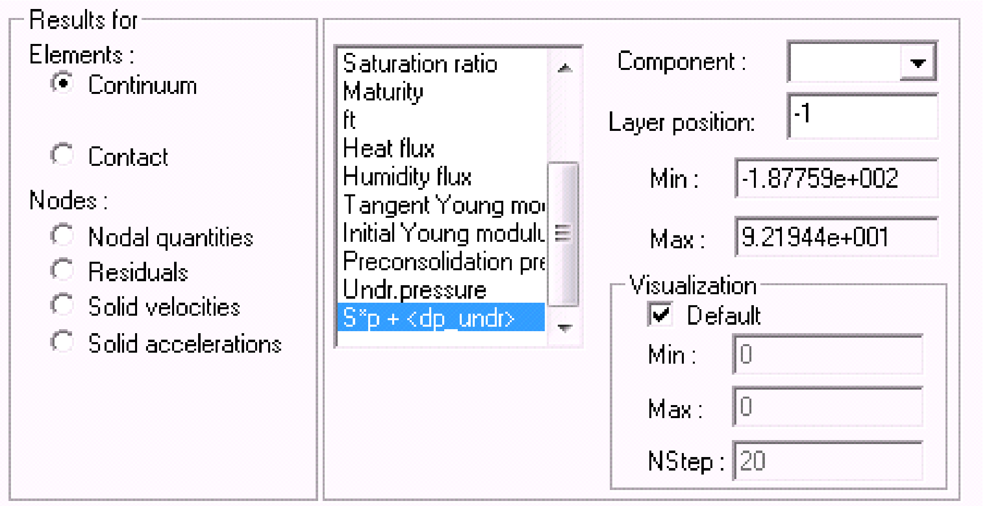
Figure 2.1 Schematic representation of the saturation phases in a soil profile with a ground water table beyond the surface, after Wroth and Houlsby (1985).

NB. fluid = liquid + gas

from Nuth (2009)

Effective stress in ZSoil

Fluid is considered as a **mixture of air and water** (single-phase model of fluid)



Bishop's effective stress in single-phase model for fluid

$$\sigma_{ij}^{tot} = \sigma_{ij}^{'} + \underbrace{S\ p}_{\substack{\text{suction stress} \\ \text{if } p > 0 \text{ (above GWT)}}}\delta_{ij}$$

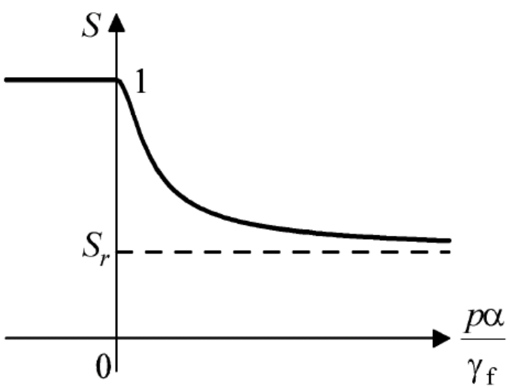
p – pore water pressure

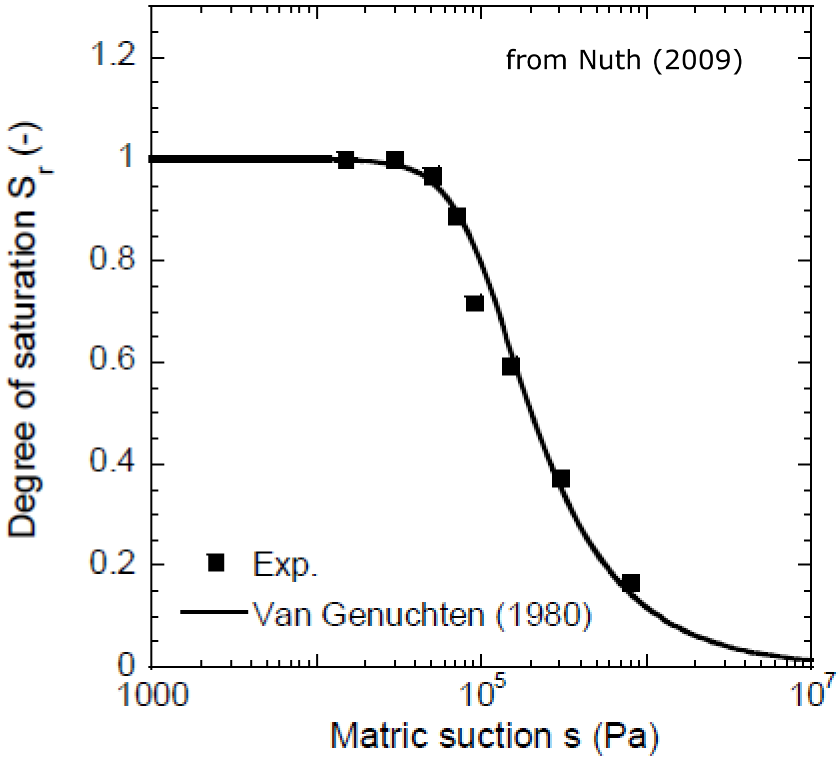
$S = S(p)$ – degree of saturation depending on pore water pressure

Deformation + flow analysis (soil water retention curve)

- Effect of an apparent cohesion can be easily reproduced by coupling:
  - constitutive model described by effective parameters
  - Darcy's flow in consolidation analysis
  - Soil water retention curve model (van Genuchten's model)

$$S = S(p) = \begin{cases} S_r + \dfrac{1 - S_r}{\left[1 + \left(\alpha \dfrac{p}{\gamma_F}\right)^2\right]^{1/2}} & \text{if} \quad p > 0 \\ 1 & \text{if} \quad p \leq 0 \end{cases}$$





Effective stress principle

- Bishop stress

$$\sigma_{ij}^{tot} = \sigma'_{ij} + S\, p\, \delta_{ij}$$

- All constitutive models are formulated in terms of effective stresses

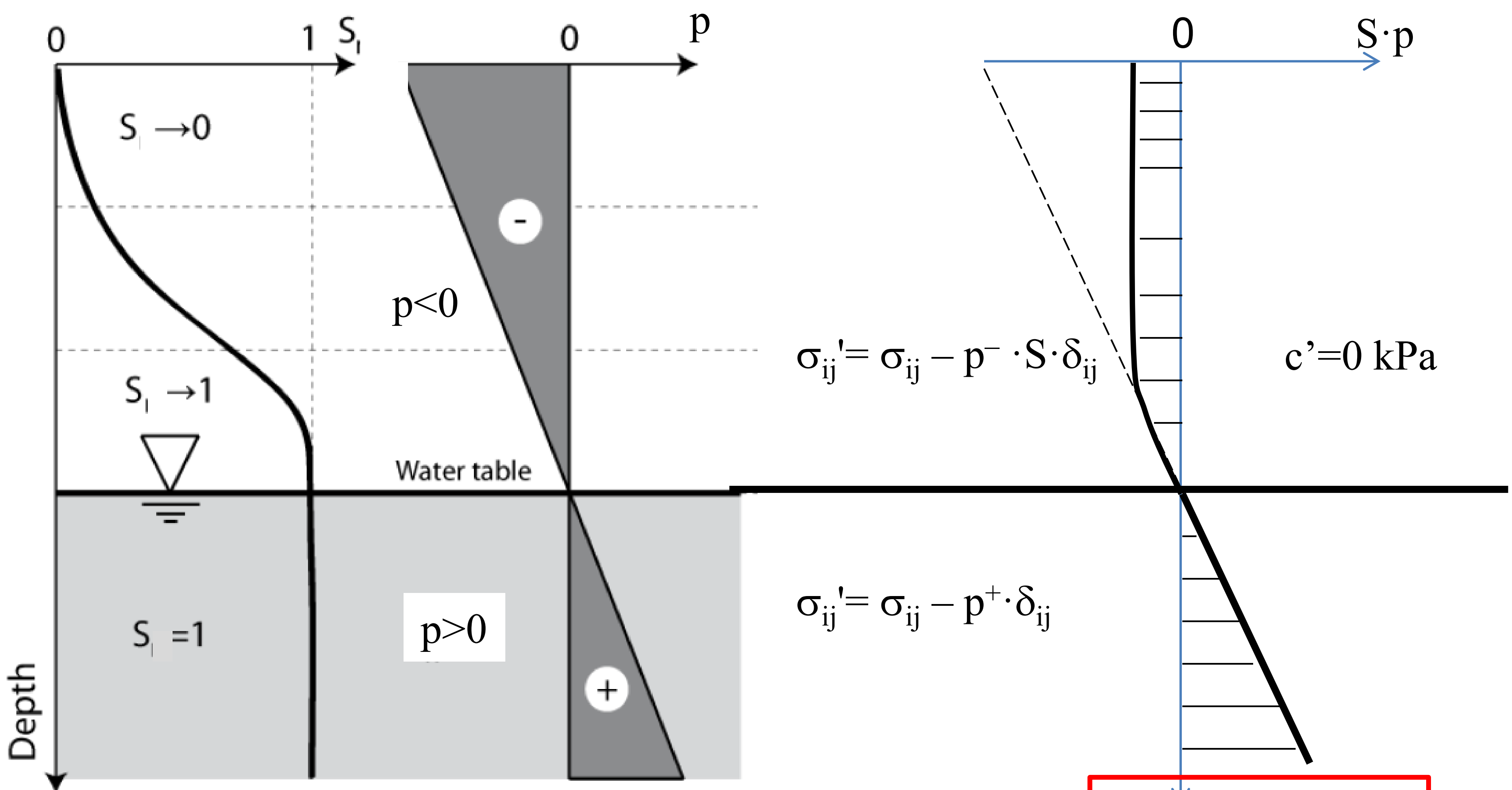
$$\Delta\sigma' = \mathbf{D}^{ep}\Delta\varepsilon$$

- Therefore effective stress parameters should be used
  - **Effective stiffness** parameters $E', E'_0, E'_{ur}, E'_{50}, \nu', \nu'_{ur}$
  - **Effective strength** parameters ϕ', c'
- Example:
  - for tertiary soils some codes may suggest $\phi = 15°$ and $c = 40$ kPa but these are NOT EFFECTIVE PARAMETERS
  - one would obtain from a drained triaxial test $\phi' = 27°$ and $c' = 7$ kPa
  - possible reason: geotechnical experience is often based on past triaxial test data, such as parameters interpreted from undrained triaxial tests but carried out on PARTIALLY SATURATED SAMPLES (no sufficient back-pressure was applied)

Suction pressure effect in ZSoil

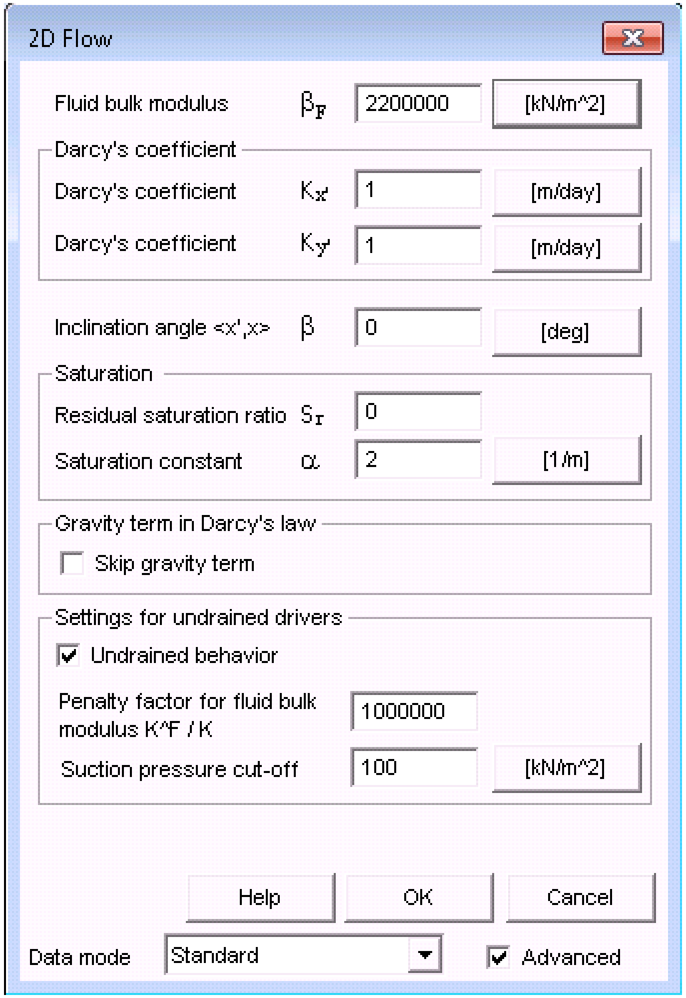
Let's consider stress sign convention from classical soil mechanics

Apparent cohesion effect in unsaturated, unstructured clay



Soil resistance: $q_f = \dfrac{2\sin\phi'}{1-\sin\phi'}\boxed{(\sigma'_1 + c'\cdot\cot\phi')}$

Setting flow parameters

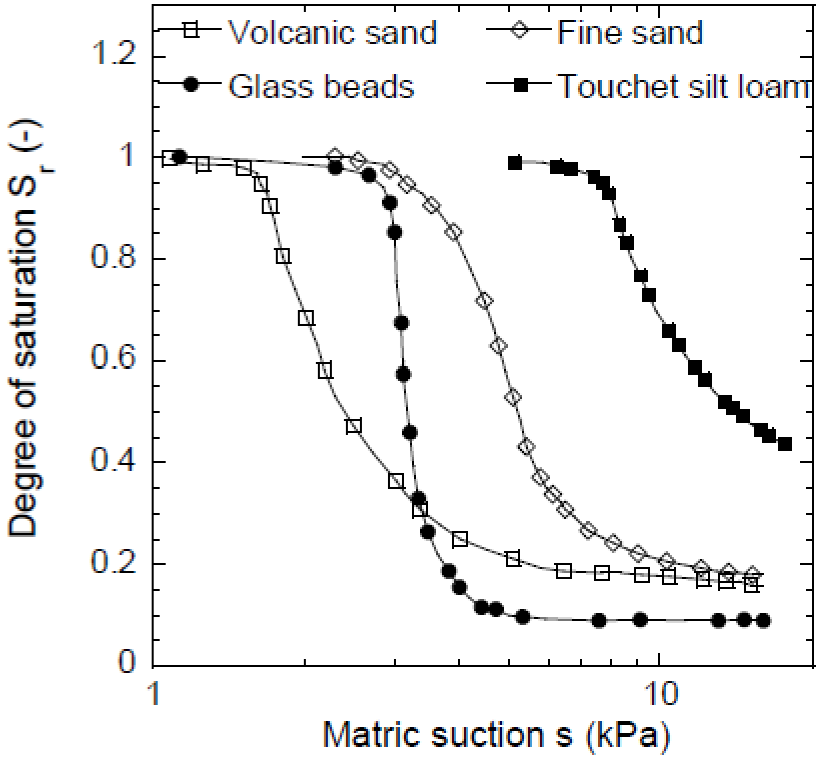


Darcy's law coefficients

Soil water retention curve constants

Setup for *Driven Load (Undrained)* analysis type

Setting flow parameters



from Nuth (2009)

Flow parameters from Yang & al. (2004)

Soil type	α [1/m]	S_r [-]
Gravely Sand	100	0
Medium Sand	10	0
Fine Sand	8	0
Clayey Sand	1-1.7	0.09-0.23

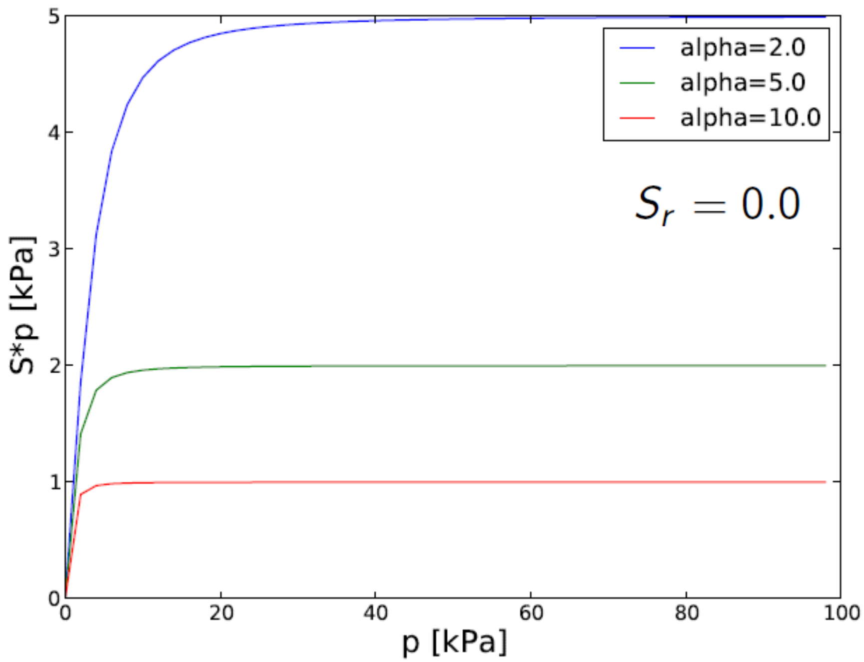
Fines content ⬀, constant α ⬂

α can be taken as an inverse of height of the capillary rise

Effect of SWRC constants on soil strength

- Reproducing an apparent cohesion
  - Effective stress principle (Bishop) $$\sigma_{ij}^{tot} = \sigma_{ij}^{'} + S\, p\, \delta_{ij}$$
  - Find a limit for $S{\cdot}p$ (for p $\rightarrow \infty$) using van Genuchten's model
  - For $S_r = 0.0$ and p $\rightarrow \infty$ $$S \cdot p \rightarrow \frac{\gamma_F}{\alpha}$$

Suction (apparent cohesion)



Positive pore pressure above GWT

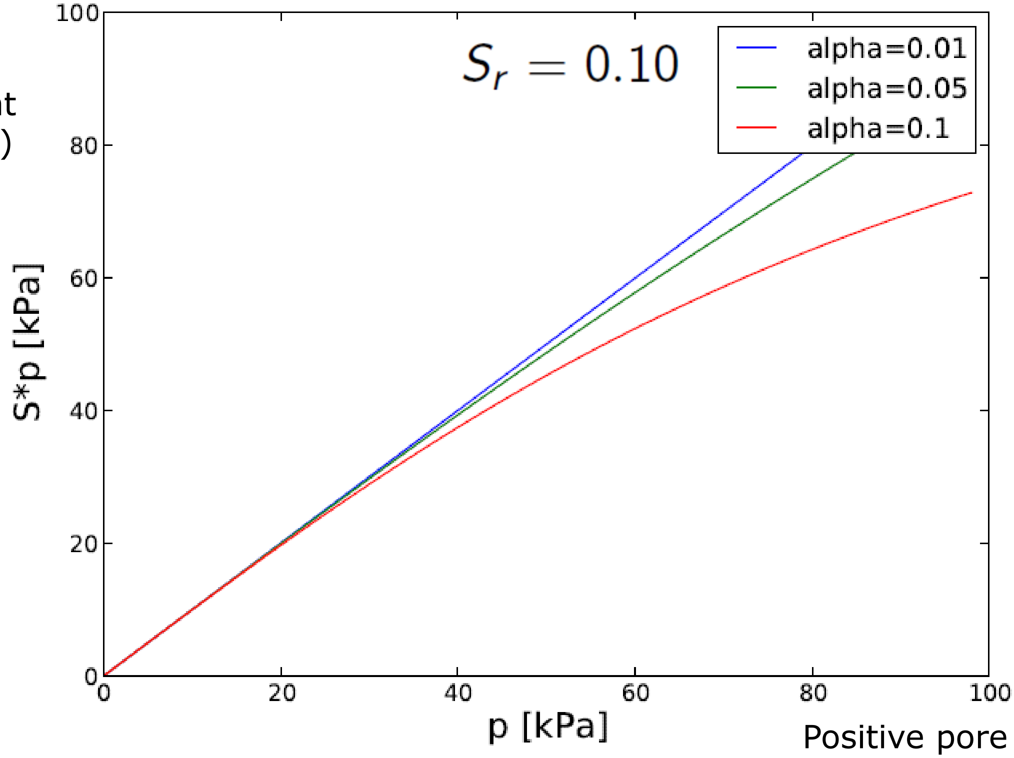
Effect of SWRC constants on soil strength

- Reproducing an apparent cohesion

  - For $S_r > 0.0$ and $p \rightarrow \infty$

$$S \cdot p \ (\text{for } p \rightarrow \infty)$$

Suction (apparent cohesion)

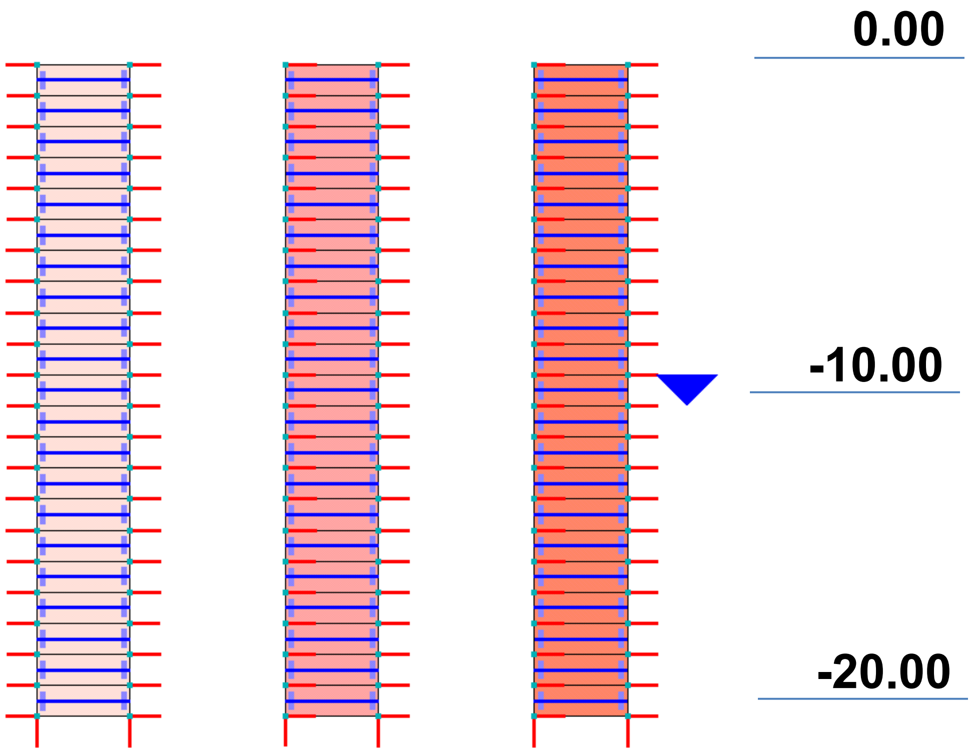


Positive pore pressure above GWT

NB.
To avoid too much apparent cohesion above the GWT set $S_r = 0.0$ and an appropriate α value (e.g. if $\alpha = 1.0$, the apparent cohesion will not exceed 10 kPa)

Exercise 1 – Determining the initial state

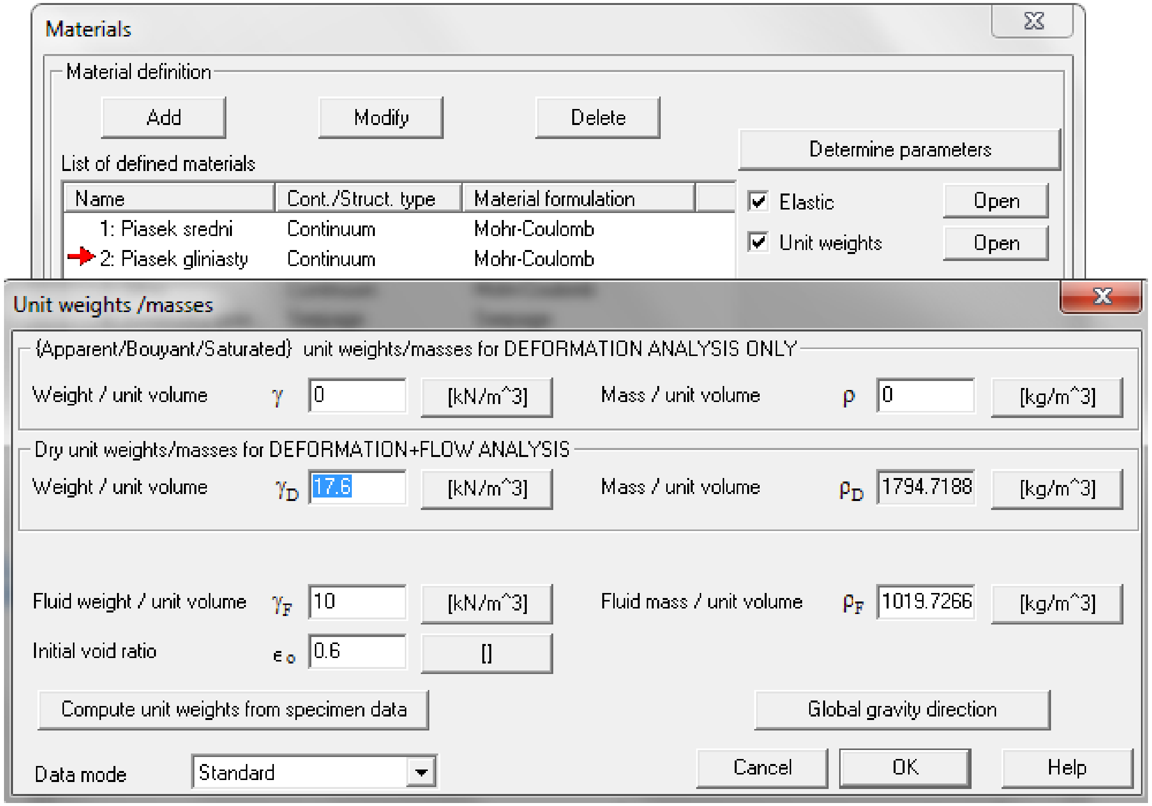
Open file:
Ex_1_InitialState.inp

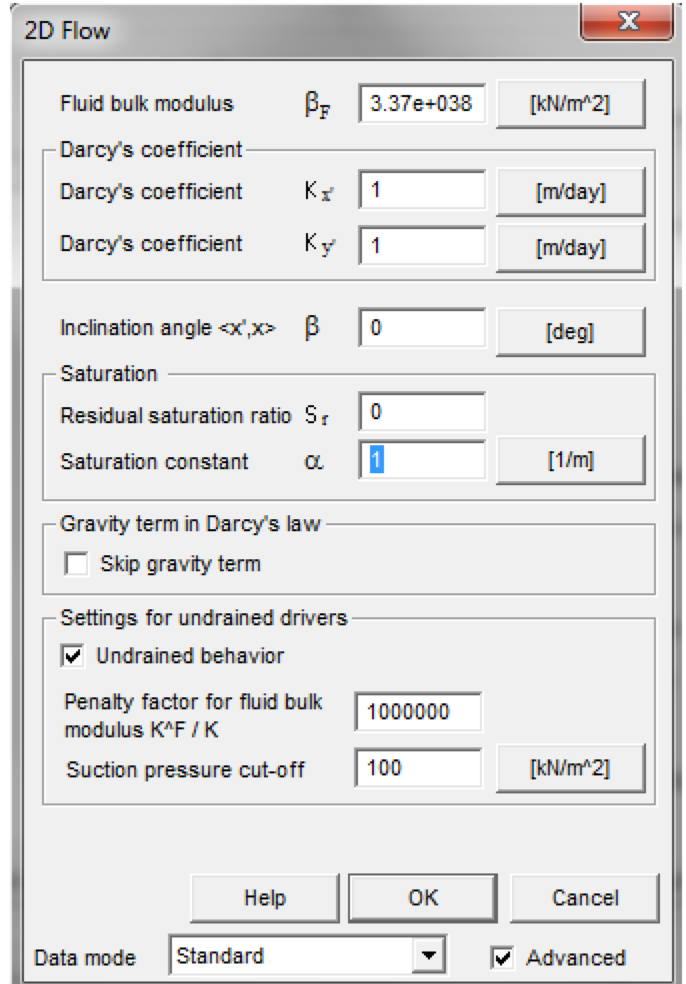


	Medium sand	Clayey sand	Clay
γ_D (γ_{SAT})[kN/m^3]	16.5 (20.3)	17.6 (21.4)	16.5 (20.9)
e_0 [-]	0.6	0.6	0.8
α [1/m]	10	1.0	0.25

Exercise 1 – Determining the initial state

Defining physical properties and hydraulic constants





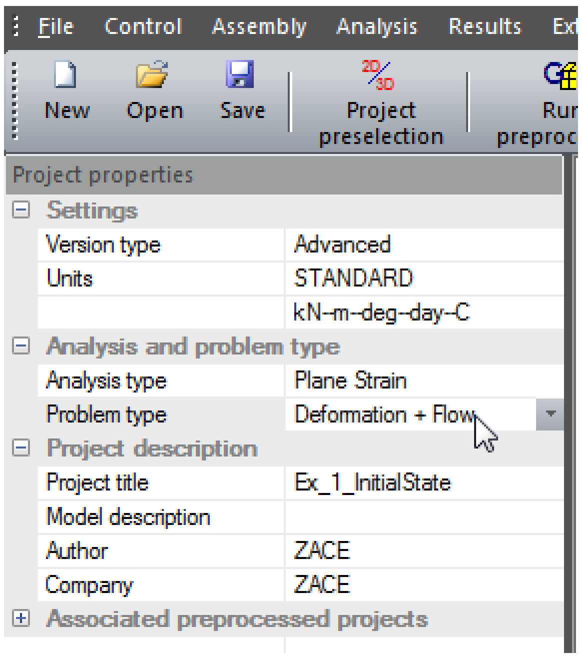
Exercise 1 – Determining the initial state

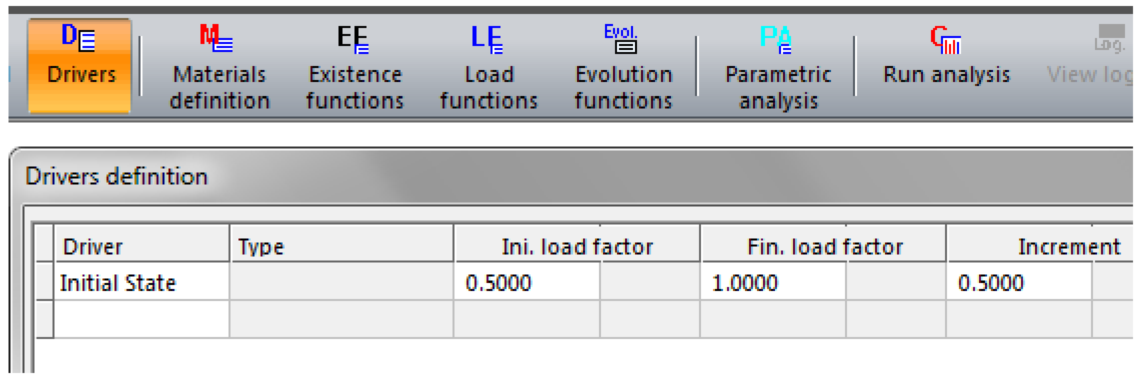
Problem type:

formation+Flow

Analysis type:

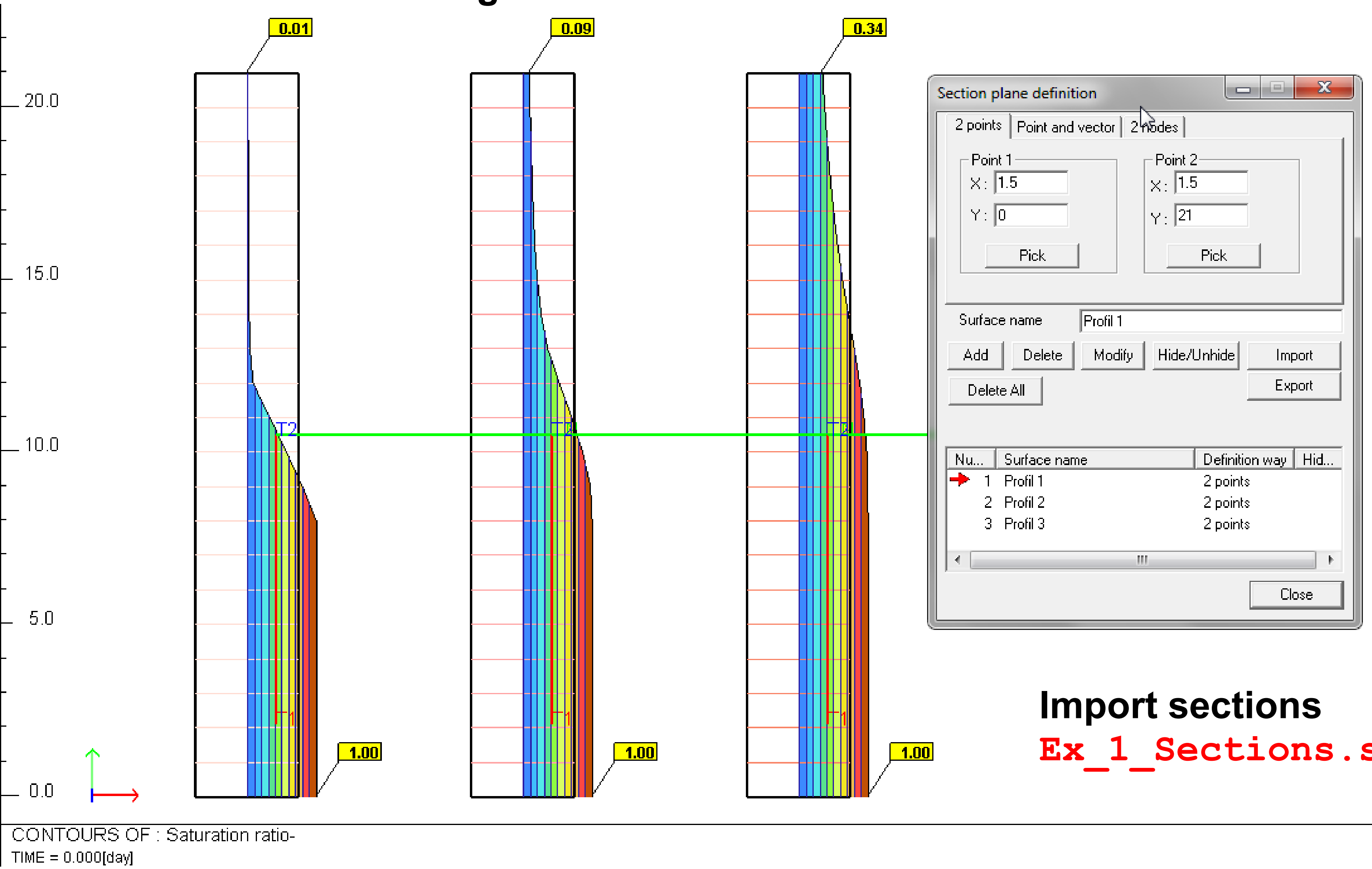
Initial State





Exercise 1 – Determining the initial state

Profiles of saturation degree

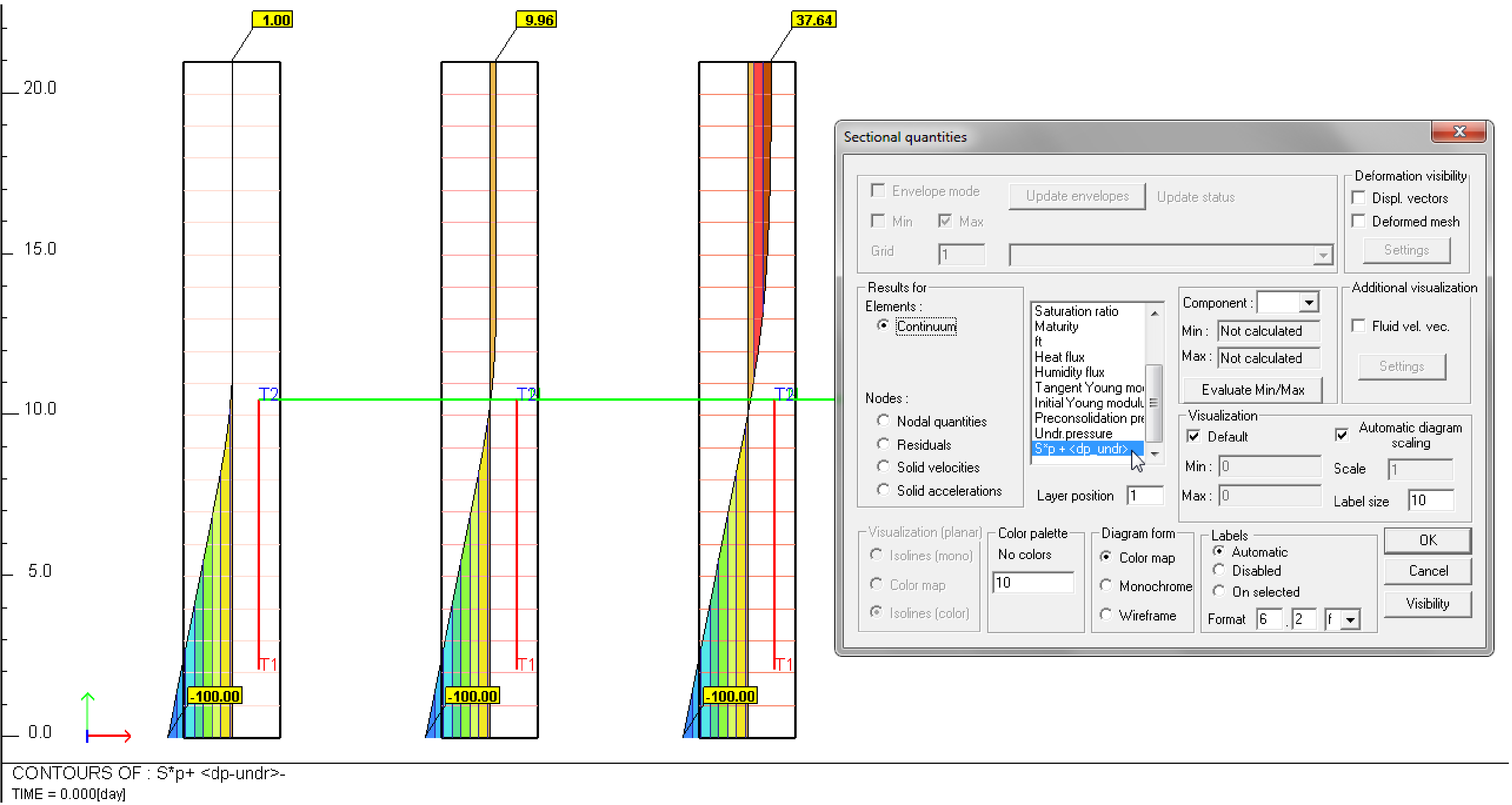


Import sections

Ex_1_Sections.sec

Exercise 1 – Determining the initial state

S*p profiles



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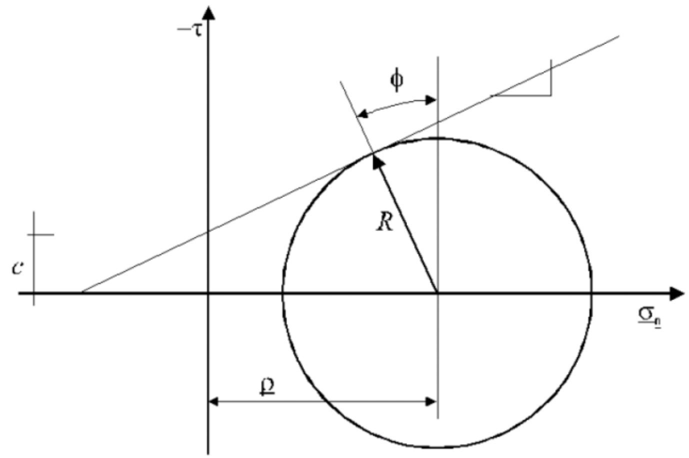
Very short description of selected models

- **Mohr-Coulomb**
- **Drucker-Prager**
- **Cap (+DP)**
- **Modified Cam-Clay**
- **Hardening Soil**



Mohr-Coulomb yield criterion

$$|\ \tau\ | = c + \underline{\sigma}_n \tan\phi$$

Main features

- linear elasticity
- no hardening
- failure described with Mohr-Coulomb criterion
- non-associated flow rule
- "linear" dilatancy

Very short description of selected models

- **Mohr-Coulomb**
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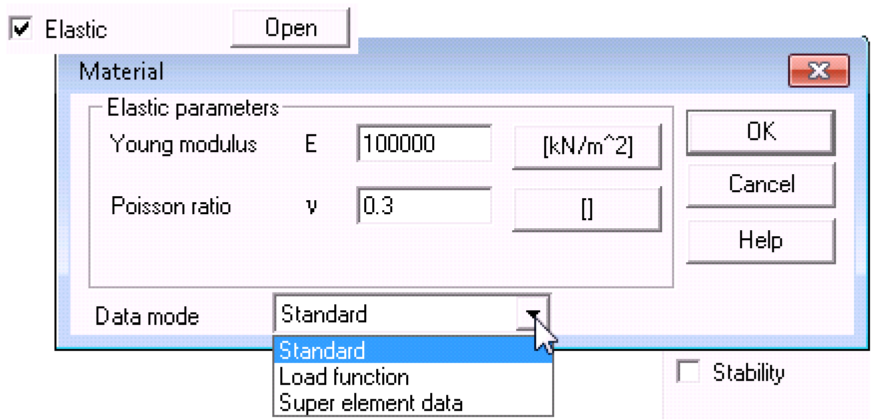

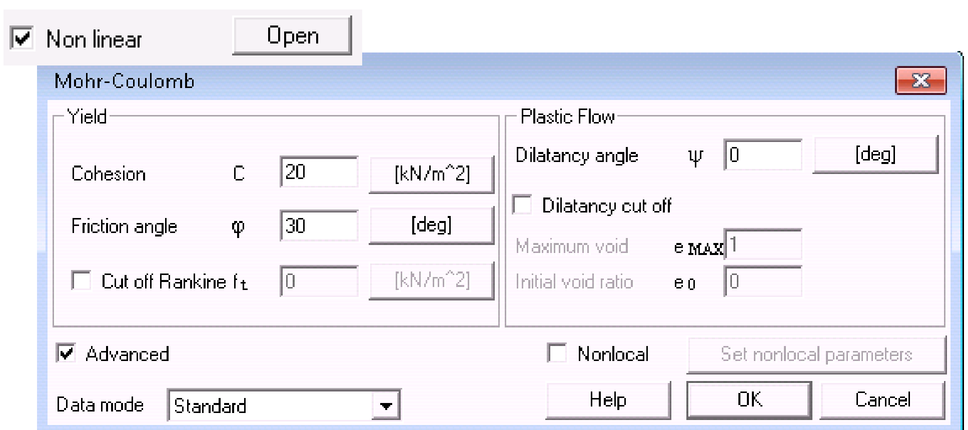

Very short description of selected models

- ➢ **Mohr-Coulomb**
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- ➢ **Modified Cam-Clay**
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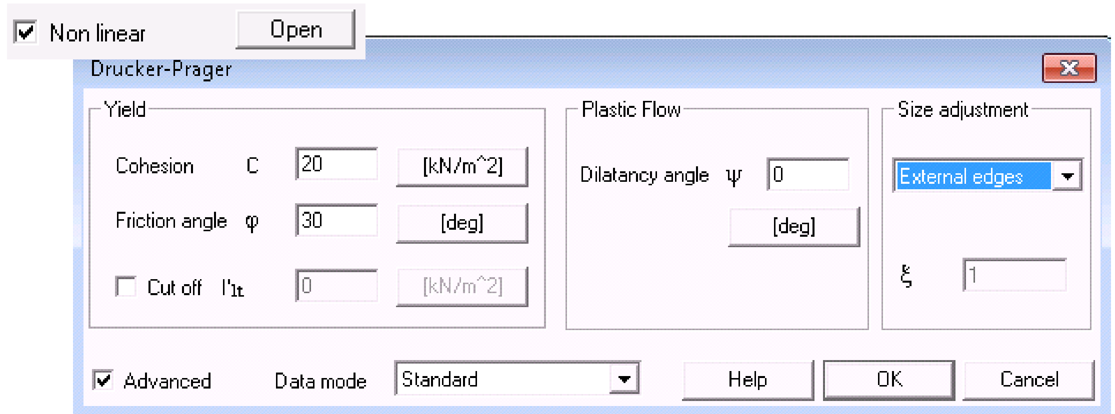
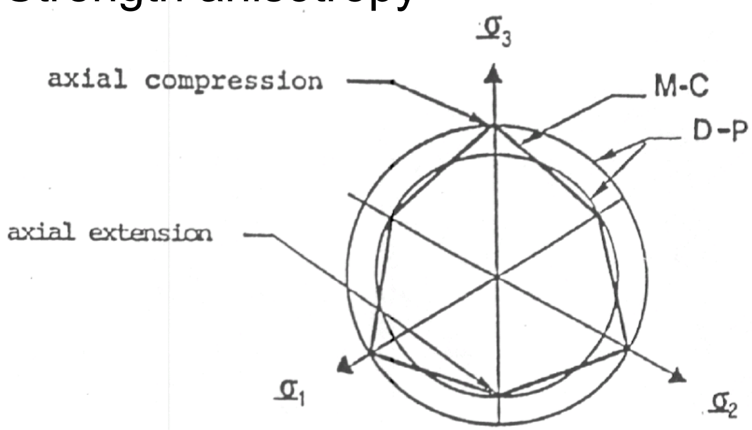


Strength anisotropy



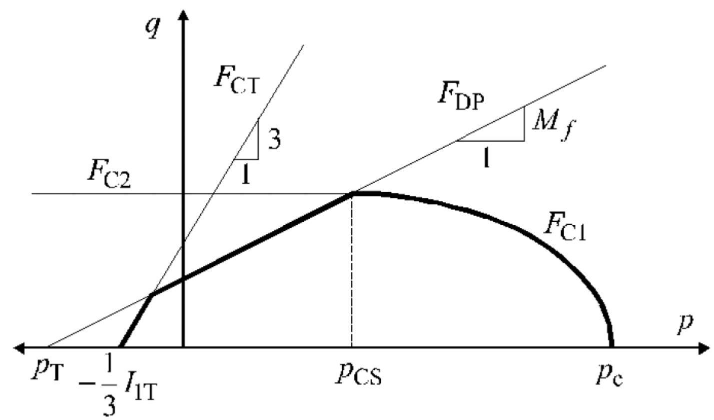
Deviatoric sections of Mohr–Coulomb (M–C) and Drucker–Prager

Main features

- linear elasticity
- no hardening
- no strength anisotropy
- linear dilatancy

Very short description of selected models

- ➢ **Mohr-Coulomb**
- ➢ **Drucker-Prager**
- ➢ **Cap (+DP)**
- ➢ **Modified Cam-Clay**
- ➢ **Hardening Soil**



Yield function criterion

Main features

- linear elasticity
- volumetric hardening (accounting for preconsolidation effects)
- D-P: non-associated flow rule, Cap: associated
- "linear" dilatancy

$$p_{cs} = \frac{p_c + (1 - R)p_T}{R}$$

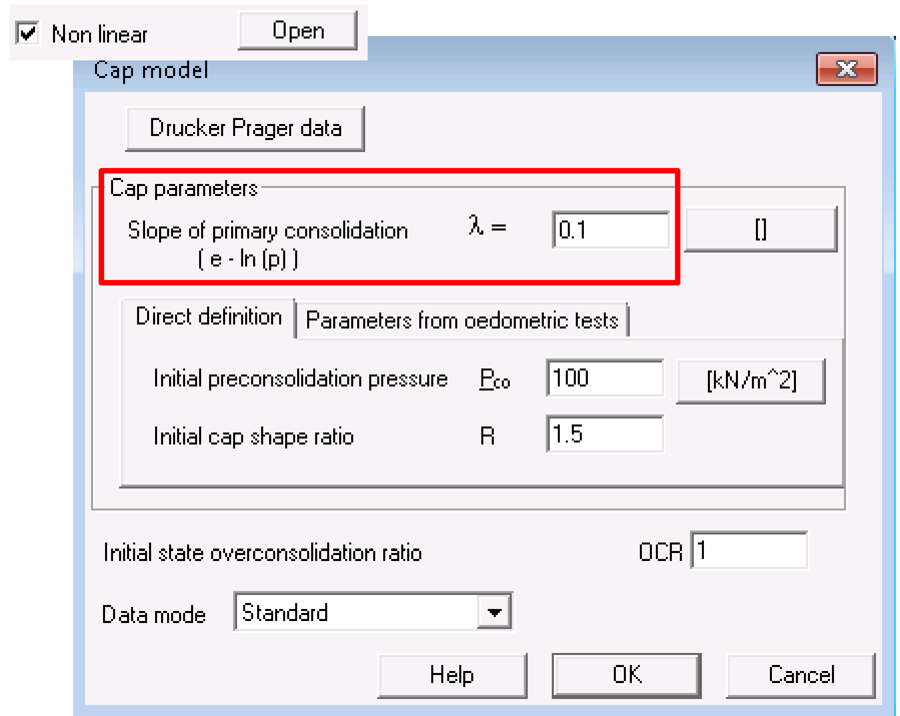
$$p_T = \frac{k}{3a_\phi}$$

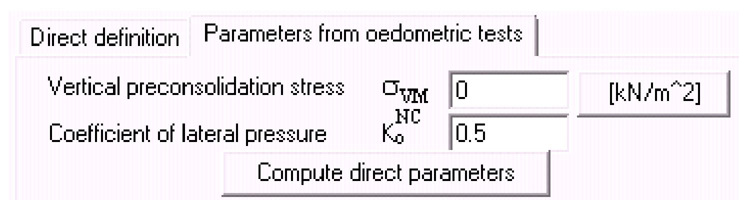
Very short description of selected models

- **Mohr-Coulomb**
- **Drucker-Prager**
- **Cap (+DP)**
- **Modified Cam-Clay**
- **Hardening Soil**

NB. In Cap model, the initial overconsolidation ratio is defined in mean eff. stress

$$OCR^{Cap} = \frac{p'_{c0}}{p'_0}$$





Very short description of selected models

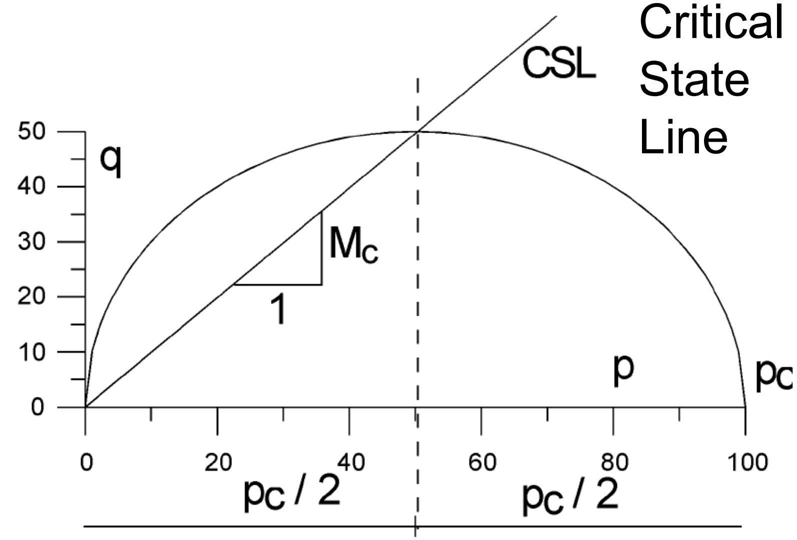
- ➢ **Mohr-Coulomb**
- ➢ **Drucker-Prager**
- ➢ **Cap (+DP)**
- ➢ **Modified Cam-Clay**
- ➢ **Hardening Soil**

Main features

- linear elasticity
- linear stress dependent stiffness
- volumetric hardening (accounting for preconsolidation effects)
- associated flow rule

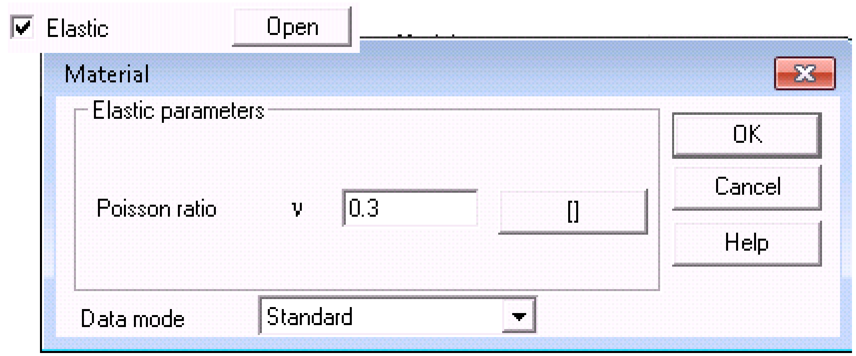
Yield surface

$$F(\sigma, \underline{p}_c) = q^2 + M_c^2\, r^2(\theta)\, p\,(p - \underline{p}_c) = 0$$



Very short description of selected models

- Mohr-Coulomb
- Drucker-Prager
- Cap (+DP)
- **Modified Cam-Clay**
- Hardening Soil



$$K = K(p') = \frac{1+e_o}{\kappa} p'$$

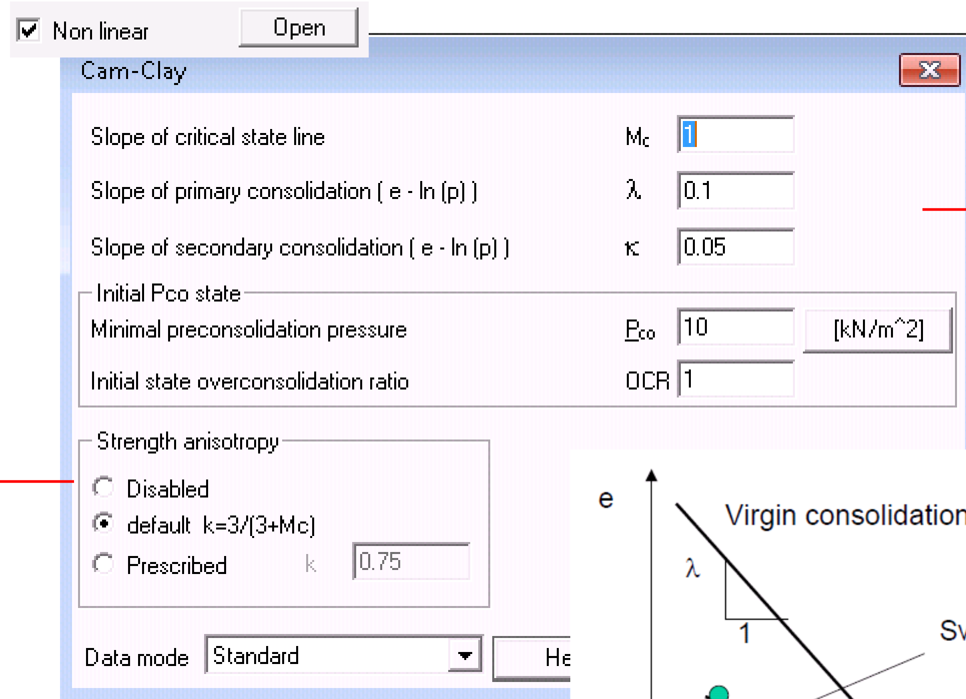
$$\frac{G}{K} = \frac{3}{2}\frac{1-2\nu}{1+\nu} \qquad G/K = const$$

current shear modulus

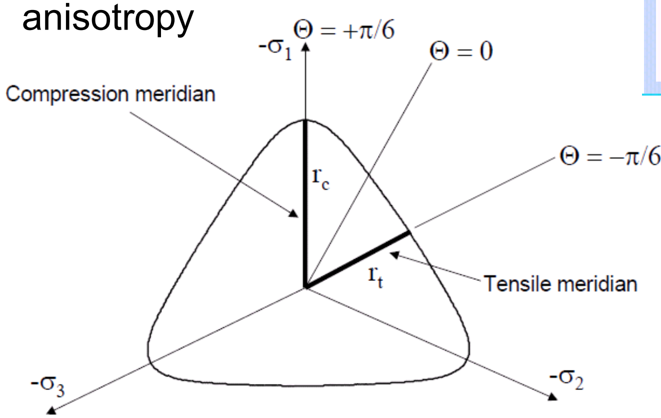
$$G = \frac{3(1-2\nu)(1+e_o)p'}{2(1+\nu)\kappa}$$

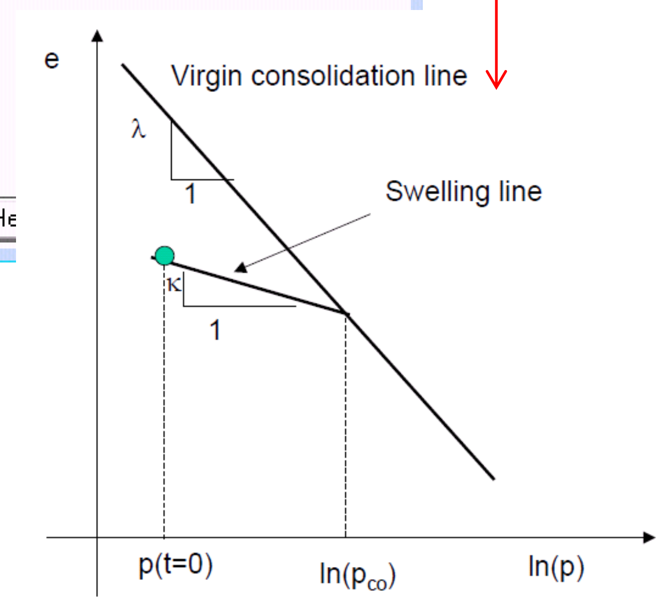
Very short description of selected models

- **Mohr-Coulomb**
- **Drucker-Prager**
- **Cap (+DP)**
- **Modified Cam-Clay**
- **Hardening Soil**



Strength anisotropy





Evolution law

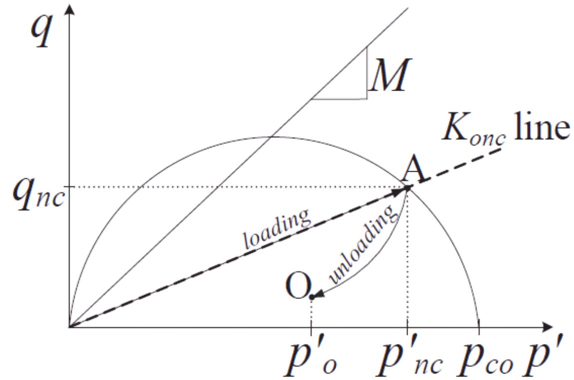
$$\mathrm{d}\underline{p}_c = -\frac{1+e_o}{\lambda-\kappa}\,\underline{p}_c\,\mathrm{d}\varepsilon^p_{kk}$$

Very short description of selected models

- **Mohr-Coulomb**
- **Drucker-Prager**
- **Cap (+DP)**
- **Modified Cam-Clay**
- **Hardening Soil**

Setting overconsolidated state in MCC

- In the MCC, the overconsolidation ratio is defined as $\mathrm{OCR}_{\mathsf{MCC}} = p_{c0}/p'_0$ contrary to the classical definition $\mathrm{OCR} = \sigma_{vc}/\sigma'_{v0}$



- Relationship between $\mathrm{OCR}_{\mathsf{MCC}}$ and OCR can be established using expression for the plastic surface of the MCC:

$$\mathrm{OCR}_{\mathsf{MCC}} = \mathrm{OCR}\frac{9(1-K_0^{\mathsf{NC}})^2 + M^2(1+2K_0^{\mathsf{NC}})^2}{M^2(1+2K_0)(1+2K_0^{\mathsf{NC}})}$$

Very short description of selected models

- ➢ **Mohr-Coulomb**
- ➢ **Drucker-Prager**
- ➢ **Cap (+DP)**
- ➢ **Modified Cam-Clay**
- ➢ **Hardening Soil**

Setting material properties

Restrictions:

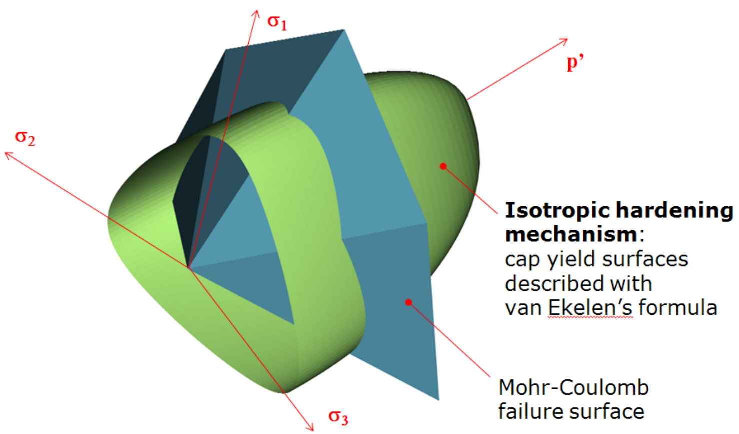
- $0 < \kappa < \lambda \qquad M_c > 0$
- $0.5 < k \leq 1$ and such that $(\alpha = \dfrac{k^{\frac{1}{n}} - 1}{k^{\frac{1}{n}} + 1}) \leq 0.7925$
- minimum value of $\underline{p}_{c0} > 0$ and $\mathrm{OCR} \geq 1.00$
- $e_0 \geq 0$
- $0 \leq \nu \leq 0.49999$

Remarks:

- $\dfrac{\kappa}{\lambda} \to 0$ may yield convergence problem (generally $\dfrac{\kappa}{\lambda} > 0.1$ works well)
- λ parameter can be estimated as $\lambda = 0.36\,(LL - 0.09)$ with liquid limit (Schoefield and Wroth 1968). λ can be also expressed through the oedometric compression index C_c which is the slope of the virgin compression line plotted in $\log \sigma'_v - e$ axes. Since $\log_{10} x = 0.43 \ln x$, one can derive $\lambda = C_c/2.3$
- dito $\kappa = C_s/2.3$ where C_s is the sloped of unloading-reloading line in the oedometric test
- typical ratio between κ and λ can be expressed by the plastic volumetric strain ratio $\Lambda = 1 - \kappa/\lambda$ which for clays is typically between 0.75 and 0.80
- M_C can be calculated from the value of effective residual friction angle ϕ' as $M_C = \dfrac{6 sin(\phi')}{3 - sin(\phi')}$, so $\phi' = 30^o$ gives $M_C = 1.2$
- k=1 gives the original version of the model and it significantly overestimate ultimate limit load or safety factors; hence the use of the default setting for k is recommended)

List of soil models and description of the most versatile one

- ➢ **Mohr-Coulomb**
- ➢ **Drucker-Prager**
- ➢ **Cap (+DP)**
- ➢ **Modified Cam-Clay**
- ➢ **Hardening Soil SmallStrain**



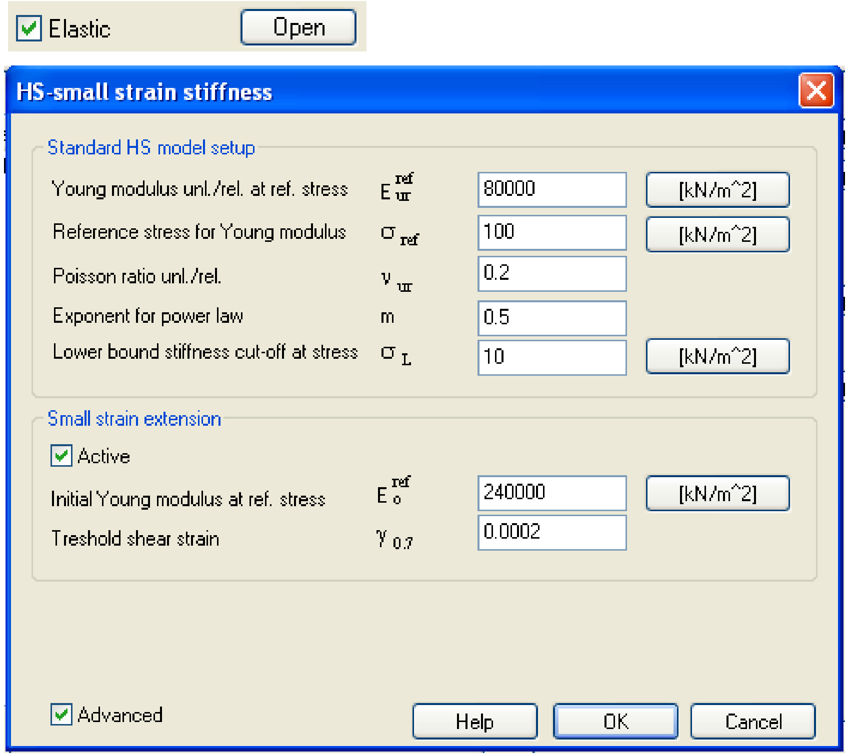
Main features

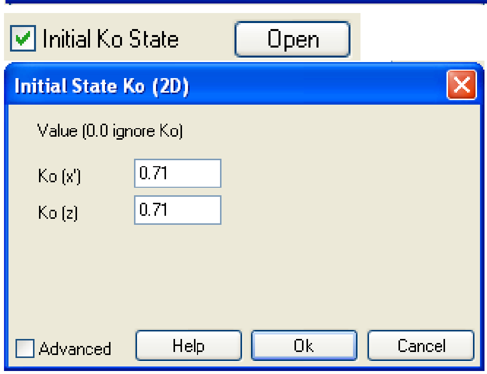
- non-linear elasticity including hysteretic behavior
- strong stiffness variation for very small strains
- stress dependent stiffness (power law)
- volumetric hardening (accounting for preconsolidation effects)
- deviatoric hardening (prefailure nonlinearities before reaching ultimate state)
- failure: Mohr-Coulomb
- Rowe's dilatancy
- flow rule

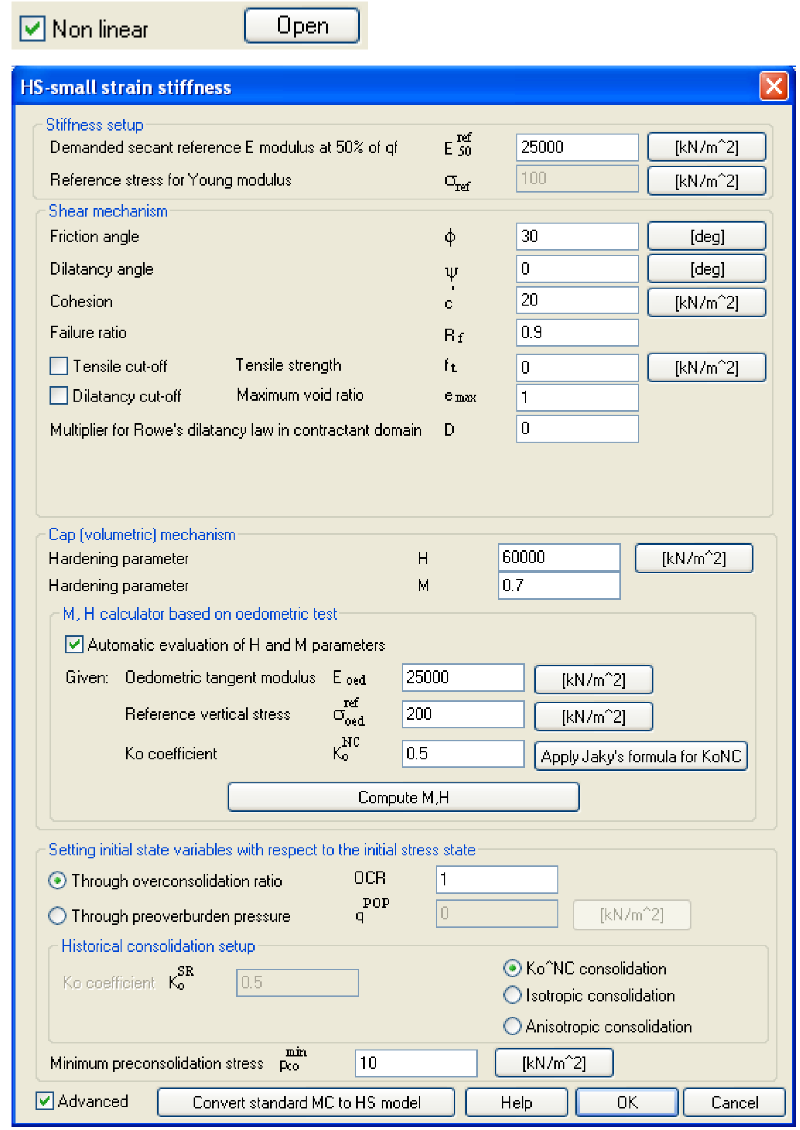
  non-associated for deviatoric mechanism

  associated for isotropic mechanism

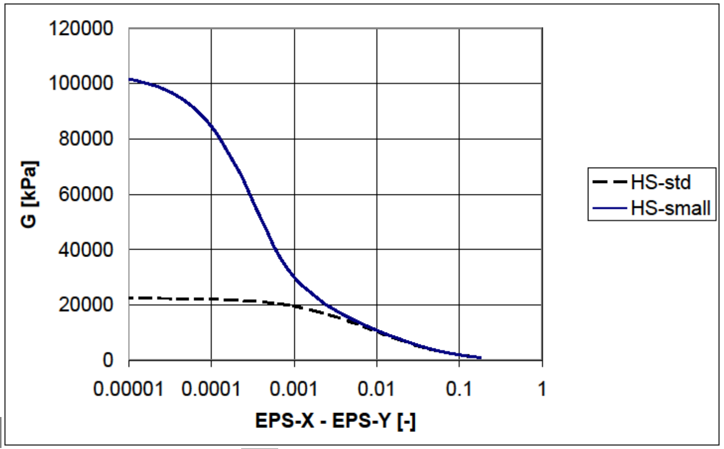
HSM – Dialog windows







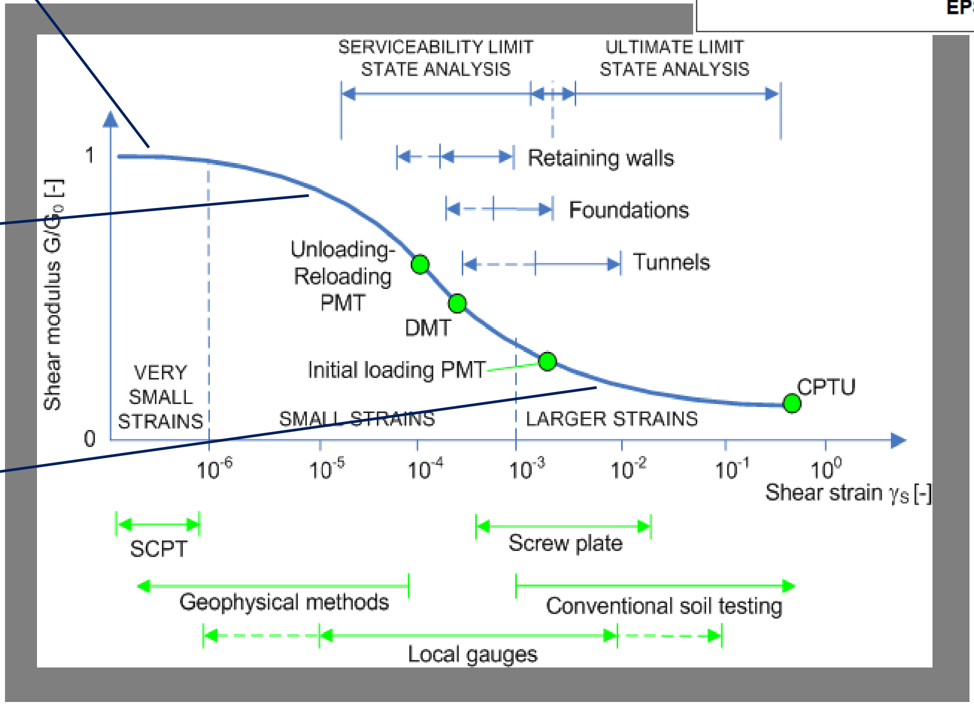
Small strain stiffness in geotechnical practice



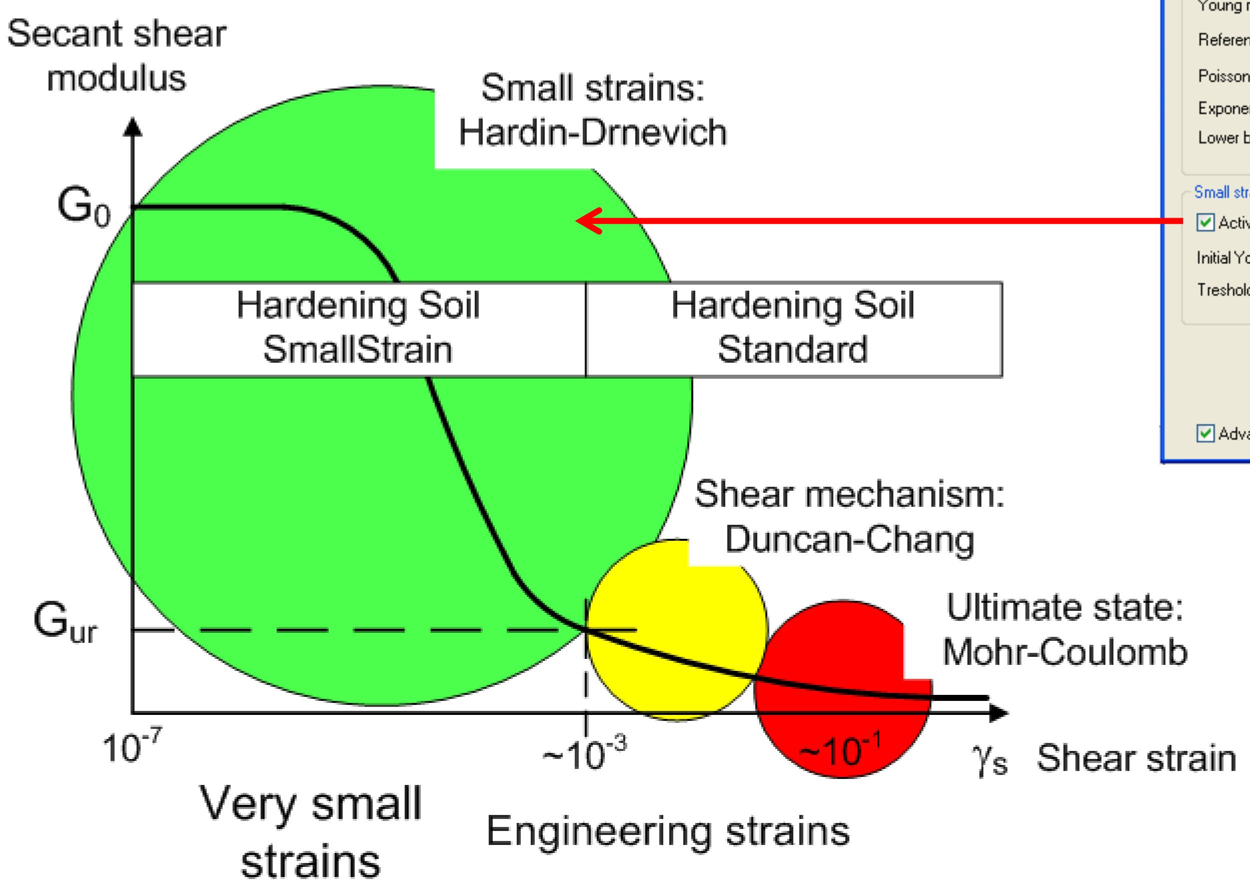
Strain range in which soils can be considered truly elastic is very small

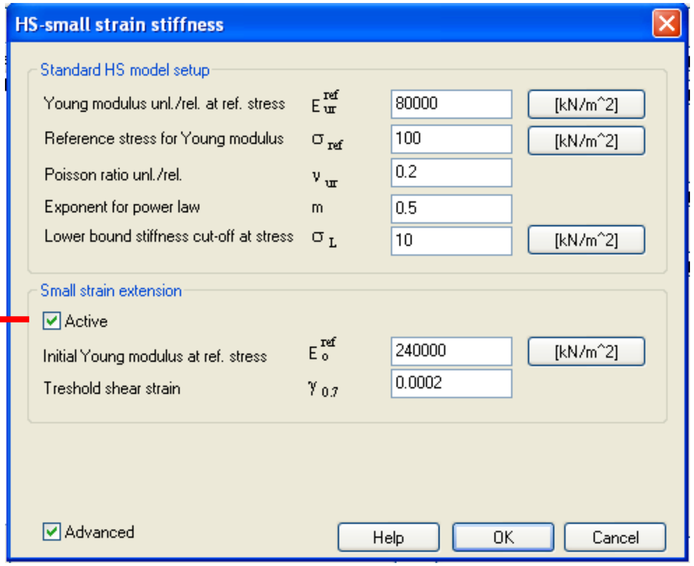
Once a certain shear strain threshold is reached a strong stiffness degradation is observed

On exceeding the domain of non-linear elasticity, plastic (irreversible) strains develop

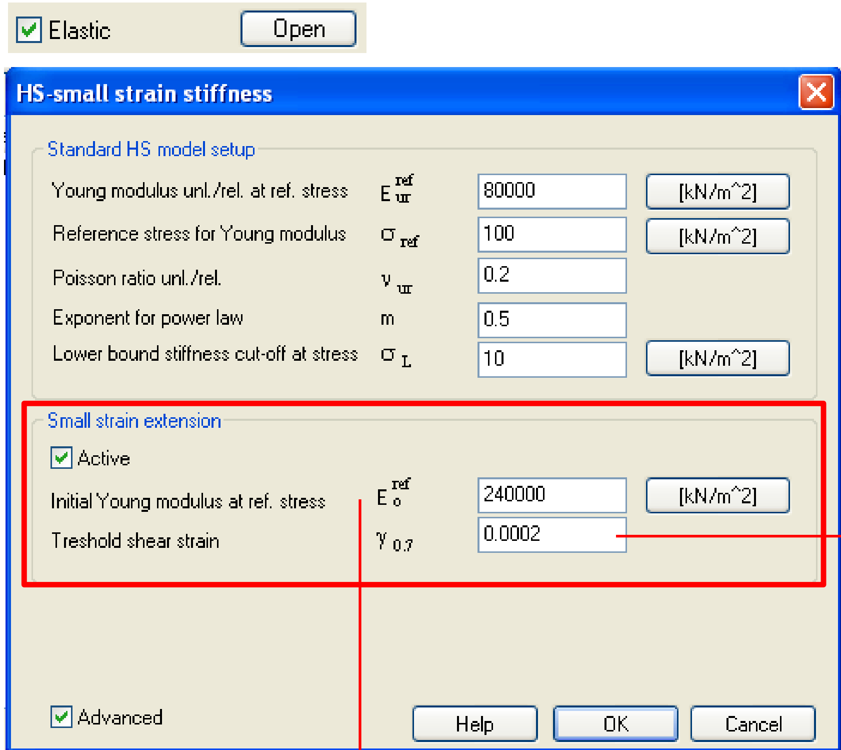


Hardening Soil model framework

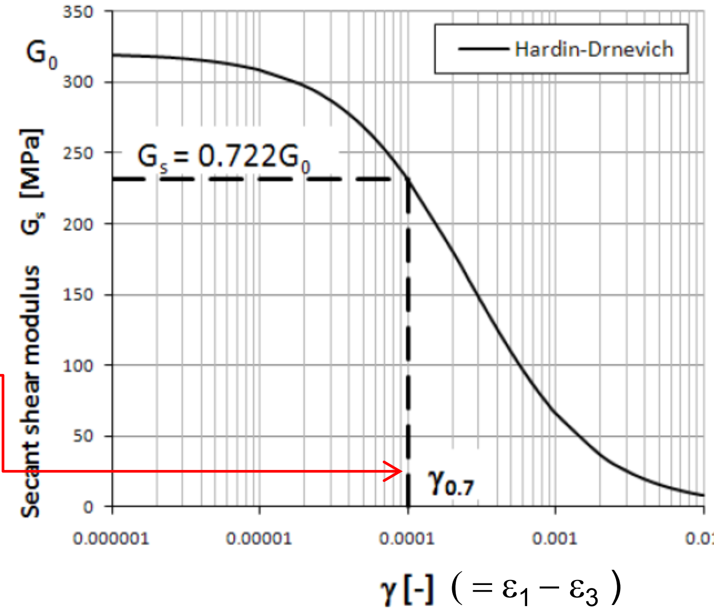




HS-SmallStrain: Non-linear elasticity for very small strains



Hyperbolic Hardin-Drnevich relation to describe S-shaped stiffness



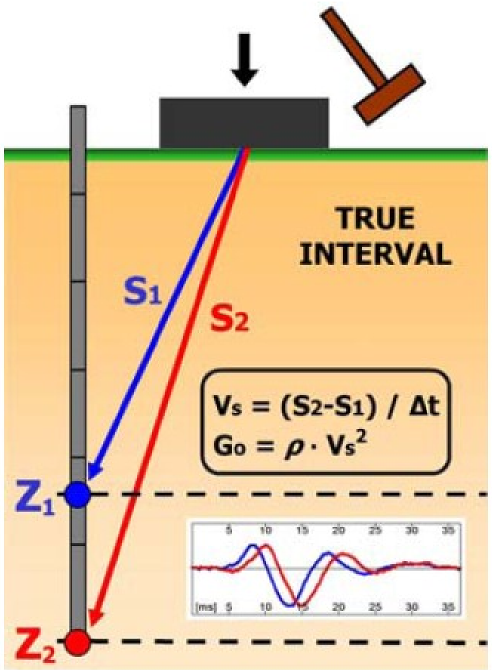
$$E_0 = 2(1 + \nu_{ur})G_0$$

HS-SmallStrain: Non-linear elasticity– estimation of E_0

Determination of G_0 from geophysical tests, SCPT, SDMT and others

$$G_0 = \rho V_s^2 \qquad\qquad E_0 = 2(1+\nu_{ur})G_0$$

with ρ denoting density of soil and V_s shear wave velocity. (ν=0.15..0.25 for small strains)



Seismic dilatometer (Monaco and Marchetti, 2007)

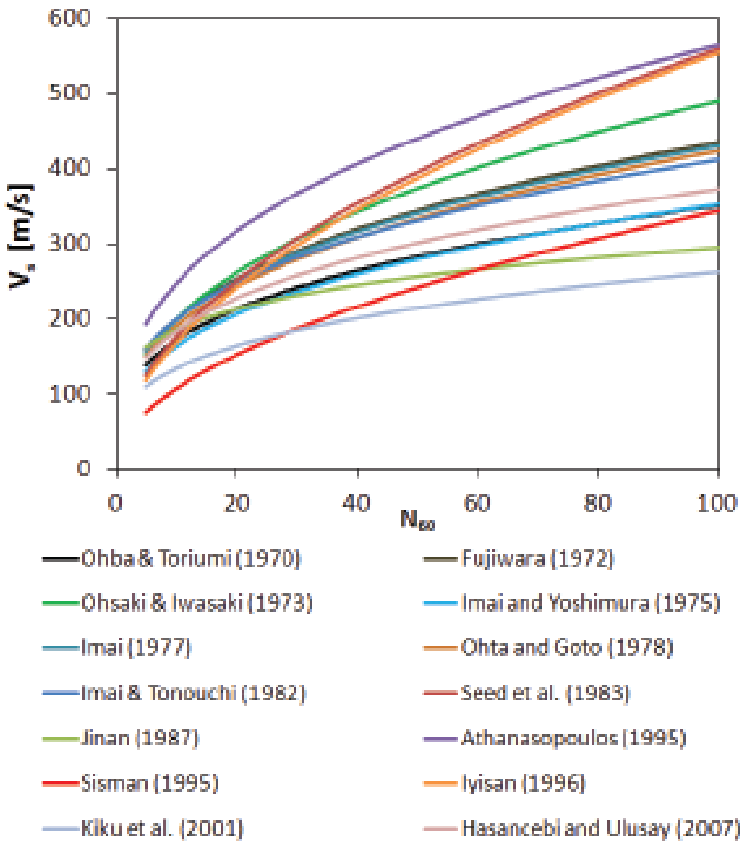
In situ tests with seismic sensors:

- **seismic piezocone testing (SCPTU)** (Campanella et al. 1986)
- **seismic flat dilatometer test (SDMT)** (Mlynarek et al. 2006, Marchetti et al. 2008)
- **cross hole, down hole seismic tests**

Geophysical tests: (see review by Long 2008)

- **continuous surface waves (CSW)**
- **spectral analysis of surface waves (SASW)**
- **multi channel analysis of surface waves (MASW)**
- **frequency wave number (f-k) spectrum method**

HS-SmallStrain: Non-linear elasticity– estimation of V_s from SPT



(correlations for all types of soil)

$$G_0 = \rho \cdot V_s^2$$

V_s measured at given depth at σ'_{v0}

$$G_0 = G_0(\sigma'_3)$$

where:

$$\sigma'_3 = min\,(\sigma'_{v0} \cdot K_0,\ \sigma'_{v0})$$

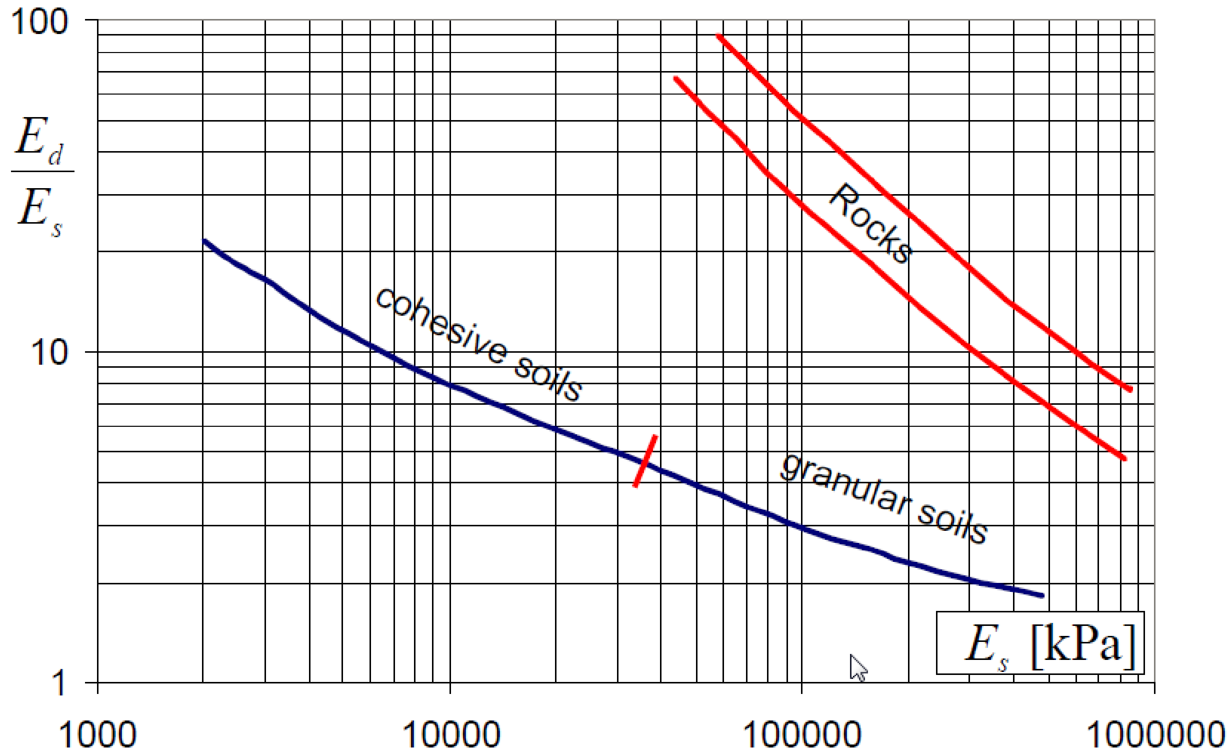
then G_0 can be scaled to σ_{ref} :

$$G_0^{ref}(\sigma_{ref}) = \frac{G_0(\sigma'_3)}{\left(\frac{\sigma'_3 + a}{\sigma_{ref} + a}\right)^m}$$

with : $a = c' \cdot \cot \phi'$

For correlations see **report on HS model in Zsoil Help**

HS-SmallStrain: Non-linear elasticity– estimation of E_0



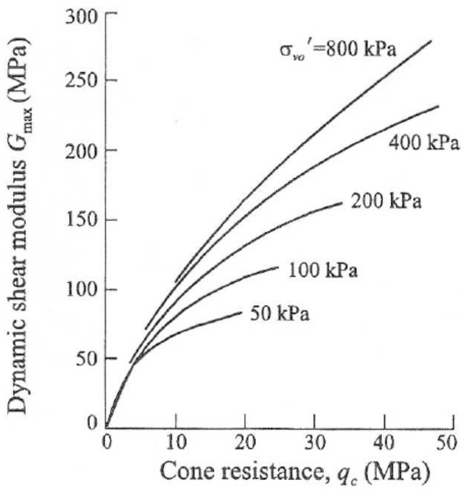
Approximative relation between "static" E_s and "dynamic“ modulus E_d corresponding to E_0 proposed by Alpan (1970)

HS-SmallStrain: Non-linear elasticity– estimation of E_0

Determination of G_0 for sands from CPT

NB. $E_0 = 2(1+\nu_{ur})G_0$

after Robertson and Campanella (1983)



Rix & Stokoe (1992)

$$\left(\frac{G_0}{q_c}\right)_{\text{avg}} = 1634\left(\frac{q_c}{\sqrt{\sigma'_{v0}}}\right)^{-0.75}$$

$$\text{Range} = \text{Average} \pm \frac{\text{Average}}{2}$$

with G_0, q_c and σ'_{v0} in kPa

Typically for soils, $G_0/G_{ur} = 2 \div 10$

Average ratios:

granular soils - $(G_0/G_{ur})_{avg} = 3 \div 4$

clays - $(G_0/G_{ur})_{avg} = 6$

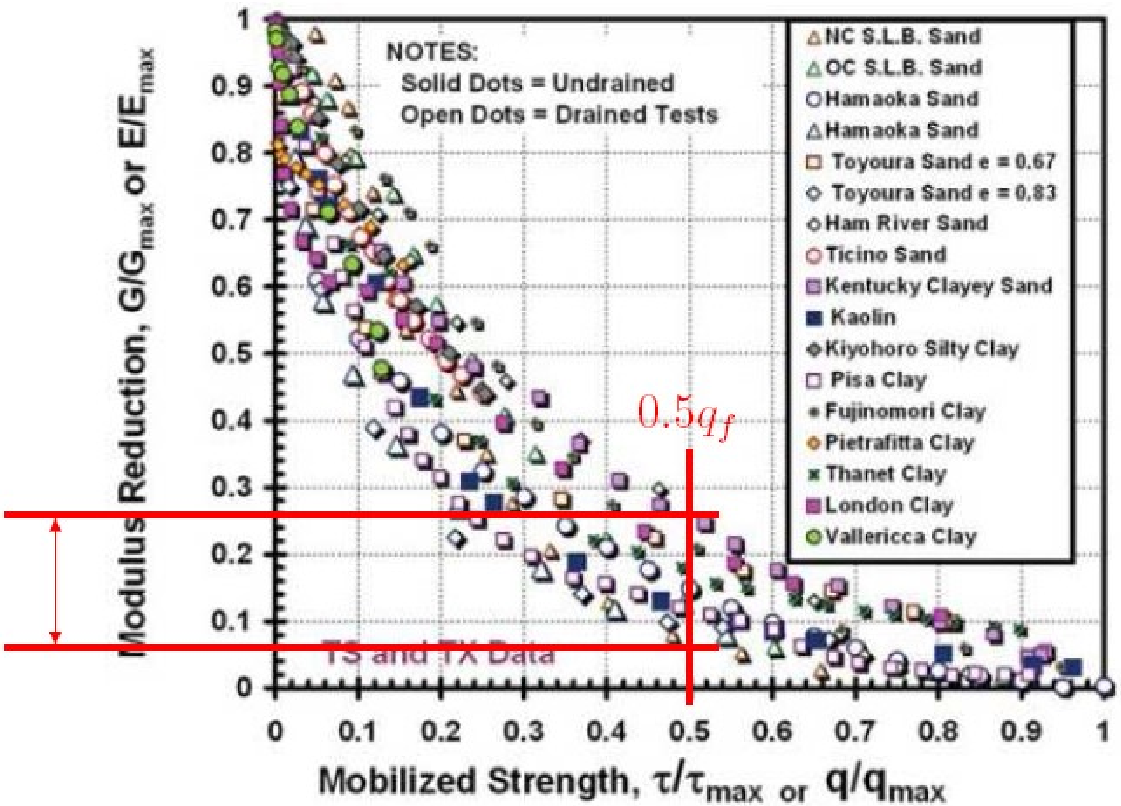
HS-SmallStrain: Non-linear elasticity– estimation of E_0

Natural clays

$E_0 / E_{50} = 5 \div 30$

higher values for the ratio are suggested for aged, cemented and structured clays, whereas the lower ones for insensitive, unstructured and remoulded clays

Relation between $\mathbf{E_0}$ and $\mathbf{E_{50}}$



Observed secant stiffness modulus reduction curves from static torsional and triaxial shear data on clays and sands (from Mayne, 2007).

Granular soils

$E_0 / E_{50} = 4 \div 18$

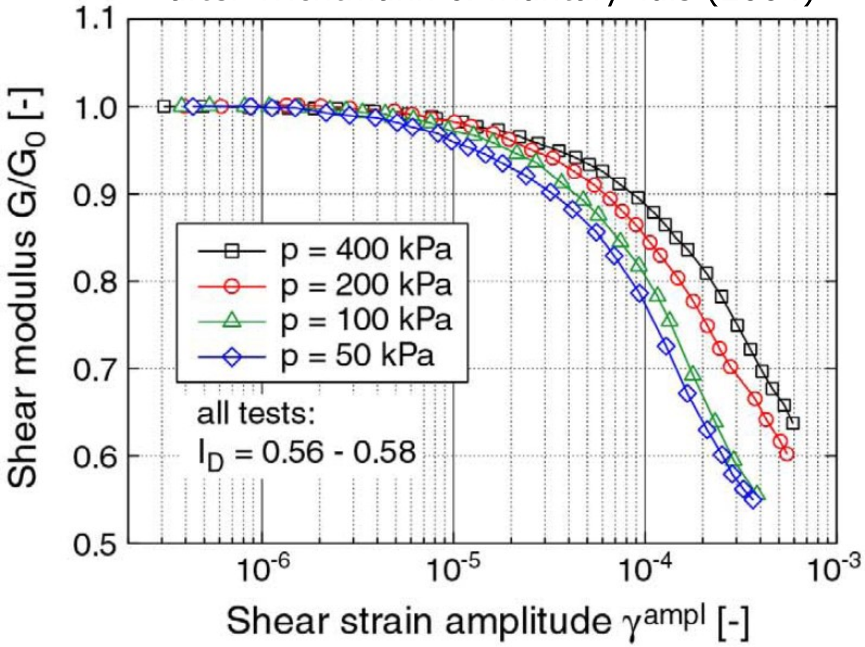
higher values are suggested for normally-consolidated soils

See HS Report

HS-SmallStrain: Non-linear elasticity at small strains

Granular soils $\gamma_{0.7}$ mainly depends on magnitude of mean eff. stress p'

after Wichtmann & Triantafyllidis (2004)



$$\gamma_{0.7} = 8.75 \cdot 10^{-5} \frac{p'}{p^{\mathrm{ref}}} + \gamma_{0.7}^{\mathrm{ref}}$$

$$\gamma_{0.7}^{\mathrm{ref}}\left(p^{\mathrm{ref}}\right) = 1.0 \cdot 10^{-4}$$

$$p^{\mathrm{ref}} = 100\mathrm{kPa}$$

Typically for clean sands and p^{ref} = 100kPa: $8{\cdot}10^{-5} < \gamma_{0.7} < 2 \cdot 10^{-4}$

For more correlations see **report on HS model in Zsoil Help**

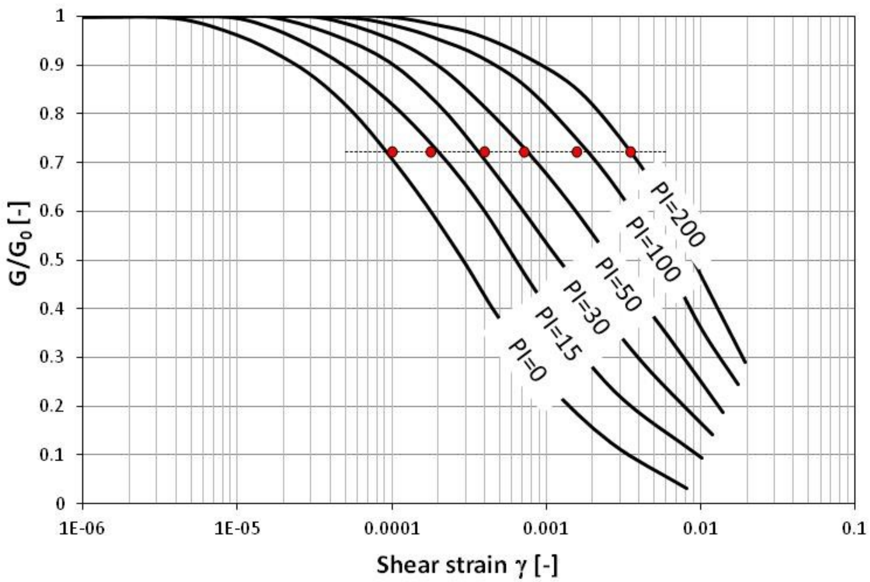
HS-SmallStrain: Non-linear elasticity for small strains

Cohesive soils

$\gamma_{0.7}$ depends on soil plasticity PI, w_{L}, w_P

Influence of soil plasticity

after Vucetic&Dobry (1991)



$$\gamma_{0.7} = \gamma_{0.7}^{\mathrm{ret}} + 5 \cdot 10^{-6} I_P \quad \text{for } I_P < 15$$
$$\gamma_{0.7} = 10^{1.15 \log(I_P) - 5.1} \quad \text{for } I_P \geq 15$$

Stokoe *et al.* 2004

$$\gamma_{0.7} = \gamma_{0.7}^{\mathrm{ref}} + 5 \cdot 10^{-6} I_P \mathrm{OCR}^{0.3}$$

$$\gamma_{0.7}^{\mathrm{ref}}(I_P = 0, \mathrm{OCR} \models 1) = 1 \cdot 10^{-4}$$

For more correlations see **report on HS model in Zsoil Help**

Hardening Soil: Double hardening model

Why do we need two hardening mechanisms?

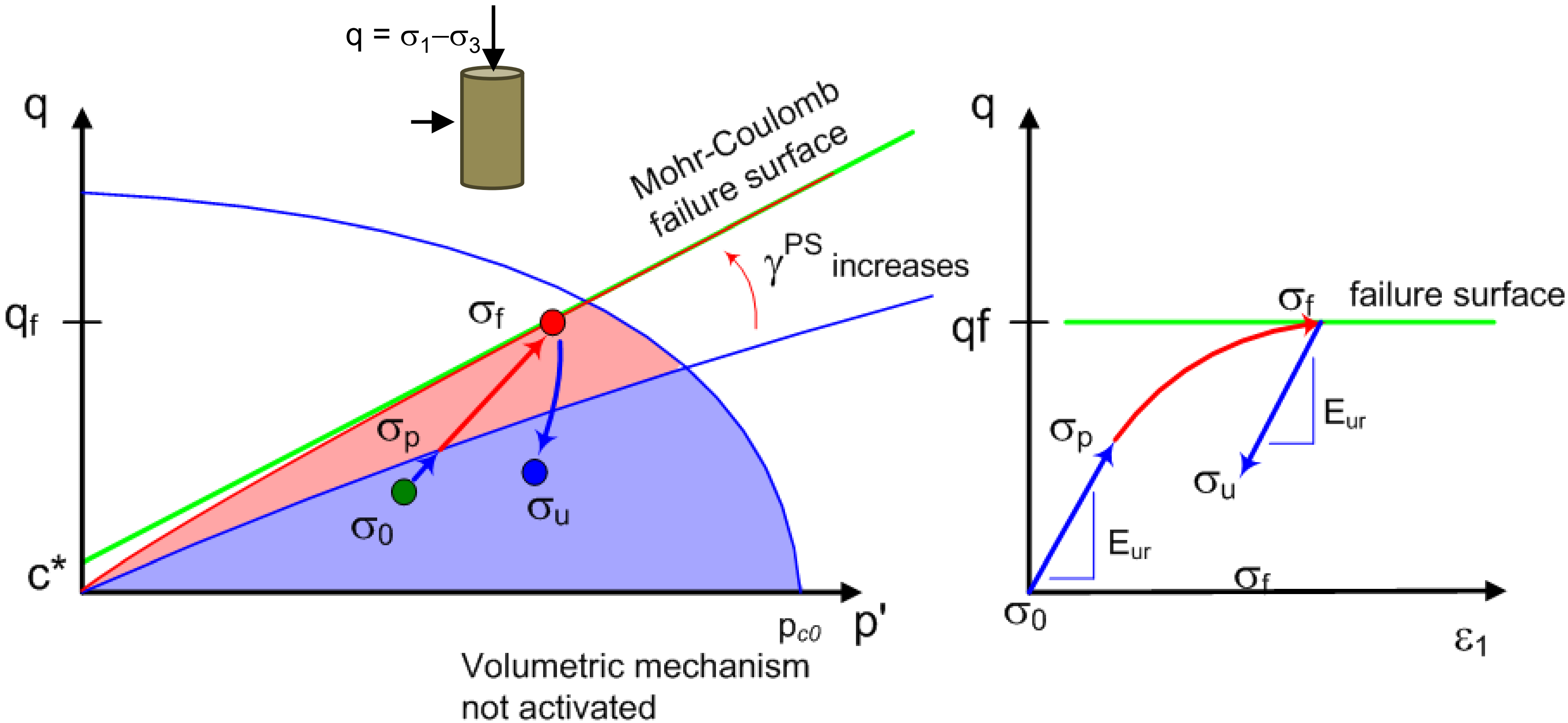
1. **Volumetric plastic strains** are often dominant in normally consolidated clays and loose sands

2. **Deviatoric (shear) plastic strains** are dominant in overconsolidated and dense sands

3. In practice, all depends on stress paths …

Hardening Soil: Double hardening – shear hardening

Pure deviatoric shear in overconsolidated material with **HS Standard model**



σ_0 Initial stress state

$\sigma_0 - \sigma_p$ Linear elastic

$\sigma_p - \sigma_f$ Elasto-plastic

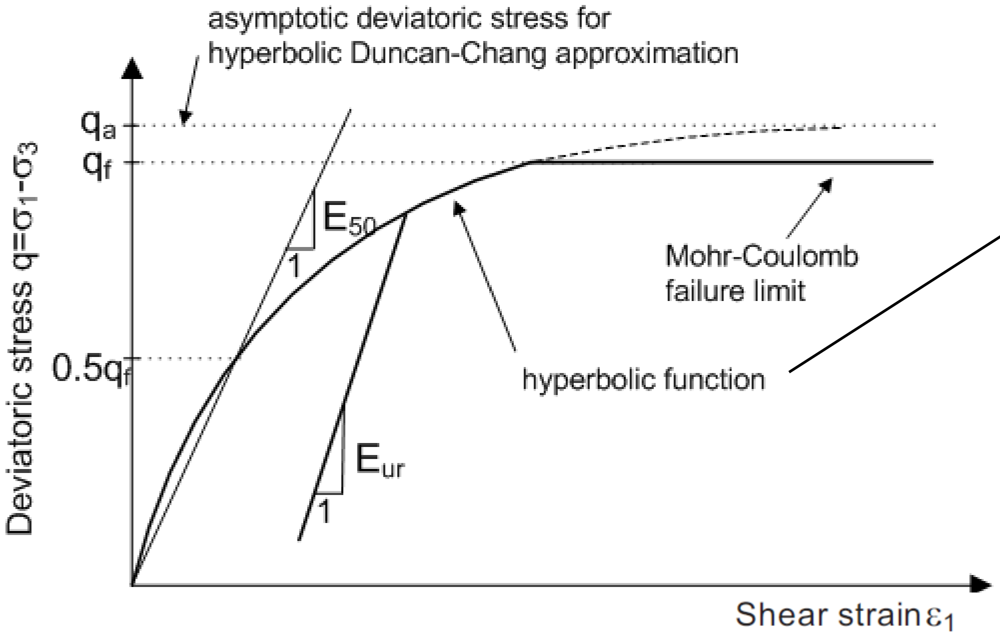
σ_f On failure surface

$\sigma_f - \sigma_u$ Linear elastic

Hardening Soil: Hyperbolic approximation of the stress-strain

E_{50} – secant stiffness modulus corresponding to 50% of the ultimate deviatoric stress q_f described by Mohr-Coulomb criterion

E_{ur} – unloading/reloading modulus



hardening parameter which tracks the evolution of the deviatoric mechanism that evolves with the deviatoric plastic strains

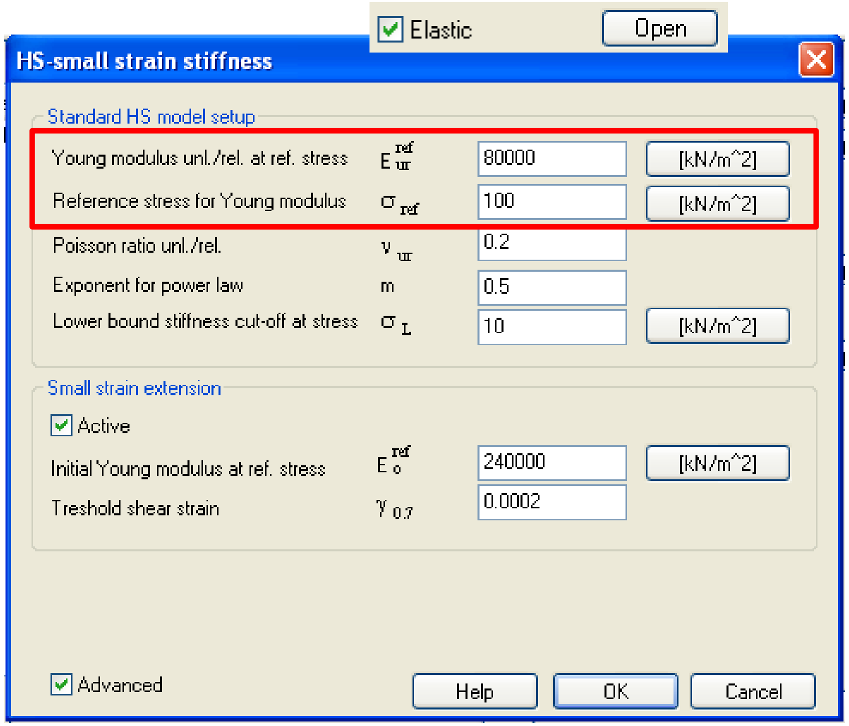
$$f_1 = \frac{q_a}{E_{50}} \frac{q}{q_a - q} - 2\frac{q}{E_{ur}} - \boxed{\gamma^{PS}} \quad \text{for } q < q_f$$

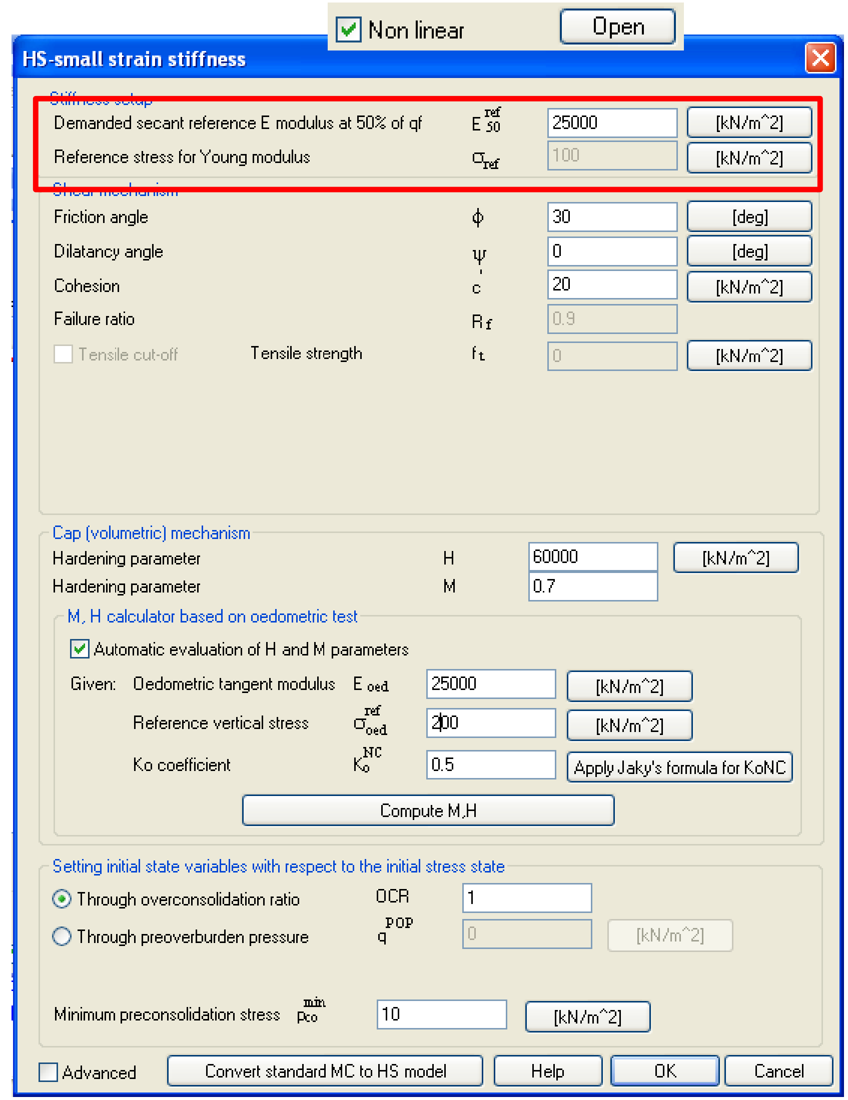
$$q_a = \frac{q_f}{R_f}$$

by default $R_f = 0.9$

For most soils R_f falls between 0.75 and 1

Hardening Soil: Stiffness moduli





Typically for most soils:

$$E_{ur} / E_{50} = 2 \div 6$$

Must be satisfied: $E_{ur} / E_{50} > 2$

Hardening Soil: Stiffness moduli

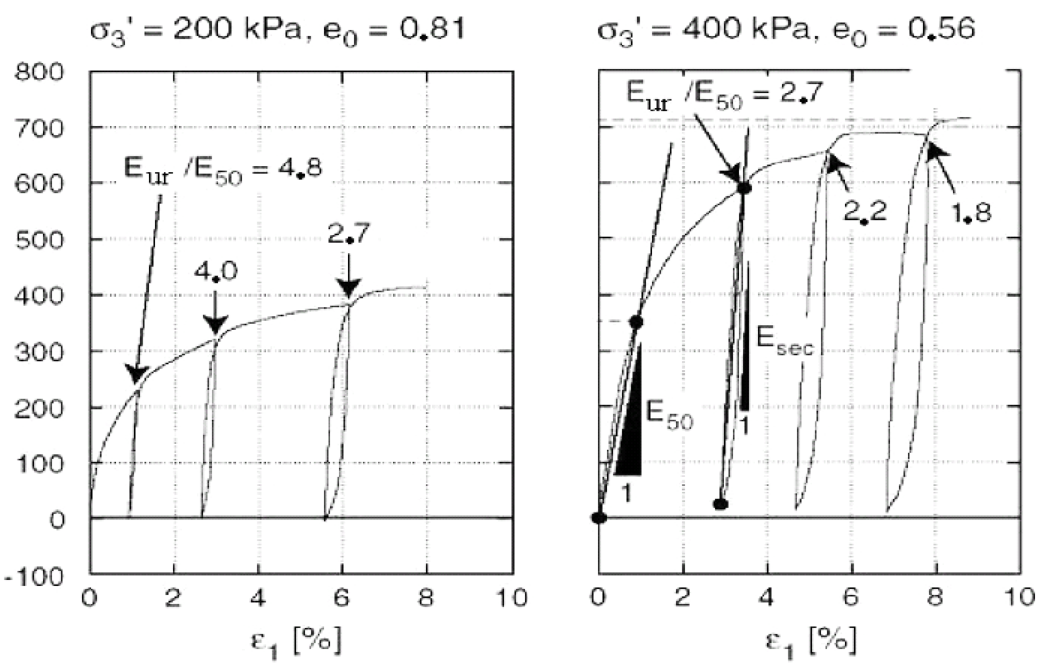
Secant vs unloading-reloading modulus in drained test on **sand**

In the case of **cohesive** soils the analogy to C_s/C_c can be considered

loose sands:
$E_{ur} / E_{50} = 3 \div 5$

dense sands:
$E_{ur} / E_{50} = 2 \div 3$

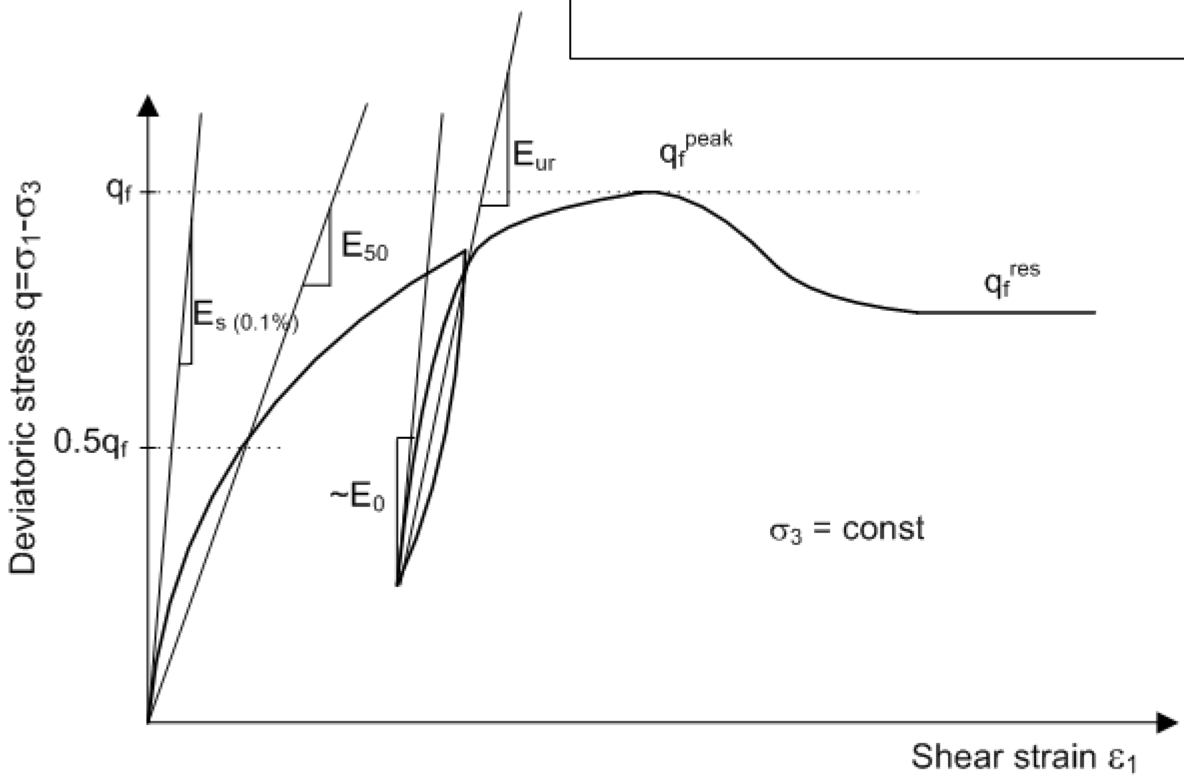
typical C_s/C_c ratios are $3 \div 6$



HSM: Parameter identification based on drained triaxial test

Stiffness characteristics

$$E_{50} < E_s(\varepsilon_1 = 0.1\%) < E_{ur}$$

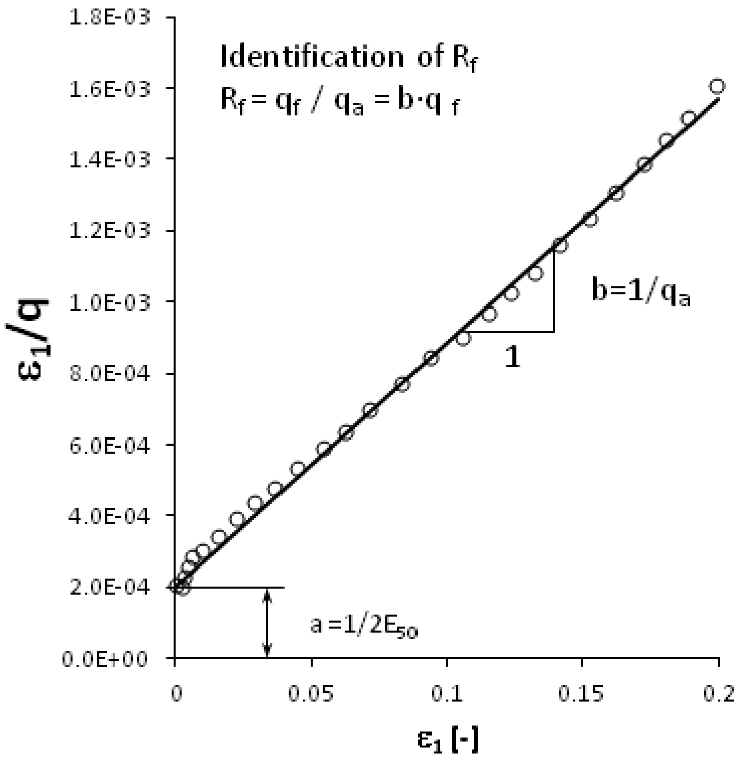


0.1% being resolution of a standard triaxial apparatus

HSM: Identifying E_{50} from drained triaxial test

Identification of E_{50} and R_f

$$q = \frac{\varepsilon_1}{\dfrac{1}{2E_{50}} + \dfrac{\varepsilon_1 R_f}{q_f}}$$



HSM: Identifying E_{50} based on known E_s

Classical, geotechnical modulus
so-called « static » modulus

E_s corresponds to $\varepsilon_1 = 0.1\%$

$E_{50} < E_s < E_{ur}$

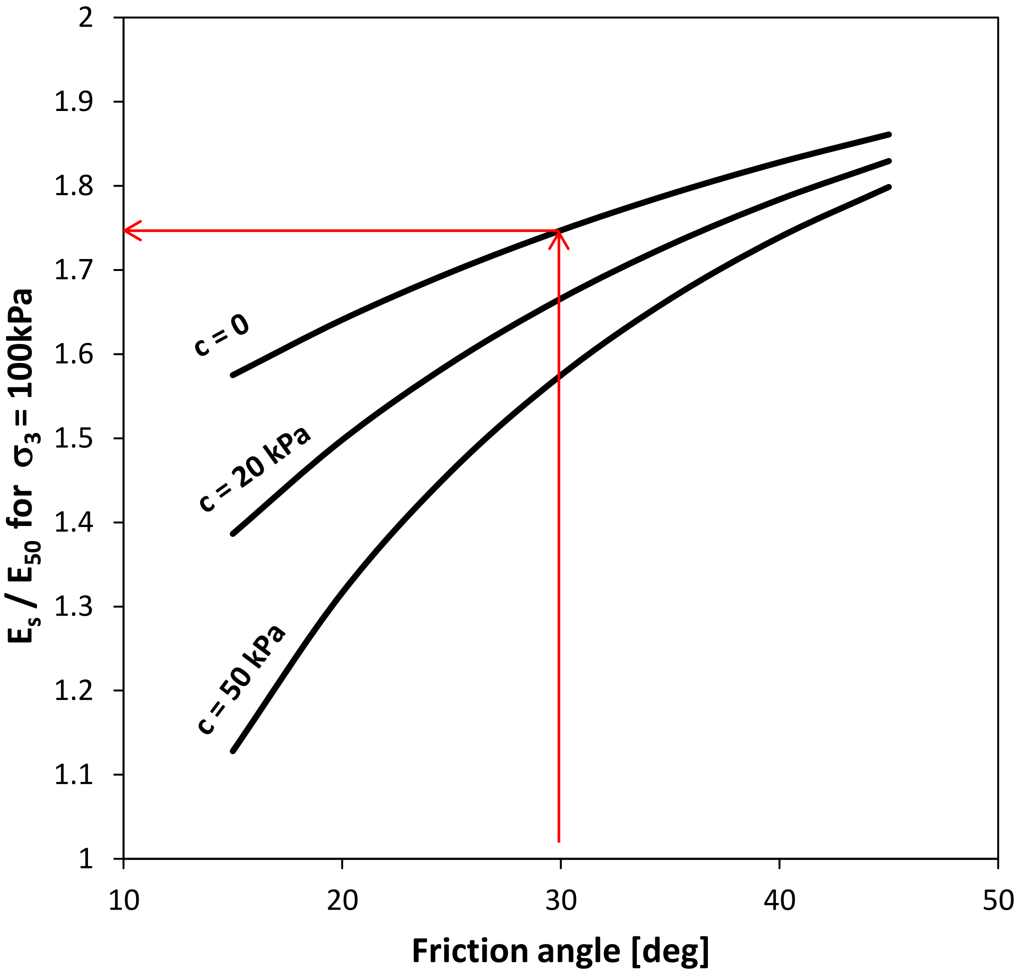
Shear strain-shear stress
hyperbolic relation in HSM

$$q = \frac{\varepsilon_1}{\dfrac{1}{2E_{50}} + \dfrac{\varepsilon_1 R_f}{q_f}}$$

Assumption $\varepsilon_1 = 0.1\%$

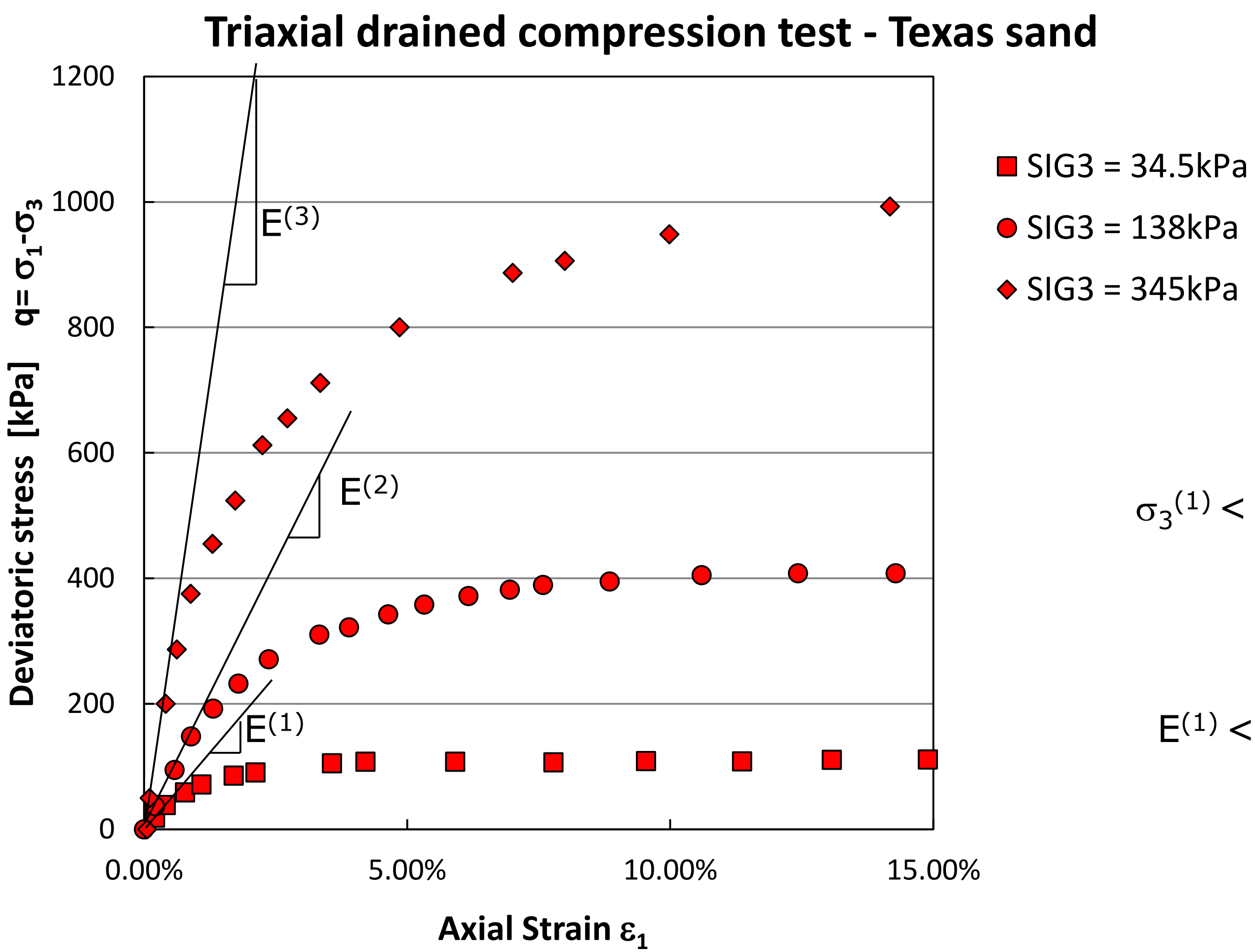
$$E_s = \boxed{\frac{q}{0.001}} = \frac{1}{\dfrac{1}{2E_{50}} + \dfrac{0.001 \cdot R_f}{q_f}}$$

HSM: Identifying E_{50} based on known E_s



$$E_s = \frac{1}{\dfrac{1}{2E_{50}} + \dfrac{0.001 \cdot R_f}{q_f(\phi, c)}}$$

Hardening Soil: Stress dependent stiffness



$\sigma_3^{(1)} < \sigma_3^{(2)} < \sigma_3^{(3)}$

$E^{(1)} < E^{(2)} < E^{(3)}$

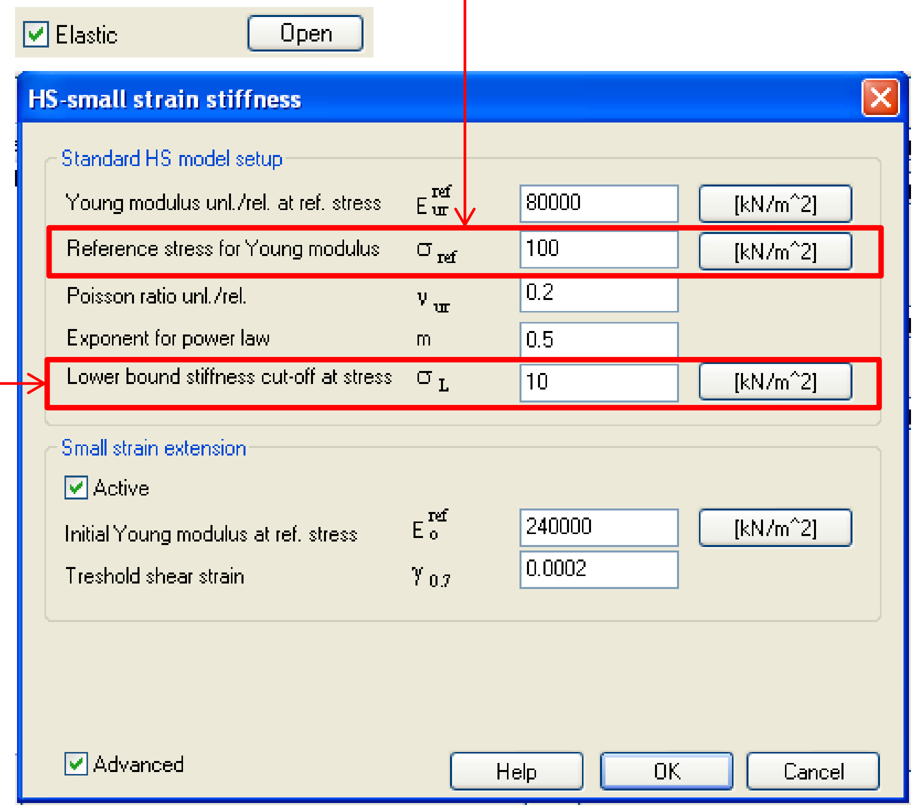
Hardening Soil: Stress dependent stiffness

E_0^{ref} , E_{50}^{ref}, , E_{ur}^{ref} correspond to reference minor stress σ^{ref}

$$E_{50} = E_{50}^{\mathrm{ref}} \left(\frac{\sigma_3^* + c\cot\phi}{\sigma^{\mathrm{ref}} + c\cot\phi} \right)^m$$

where $\sigma_3^* = \max\left(\sigma_3, \sigma_L\right)$

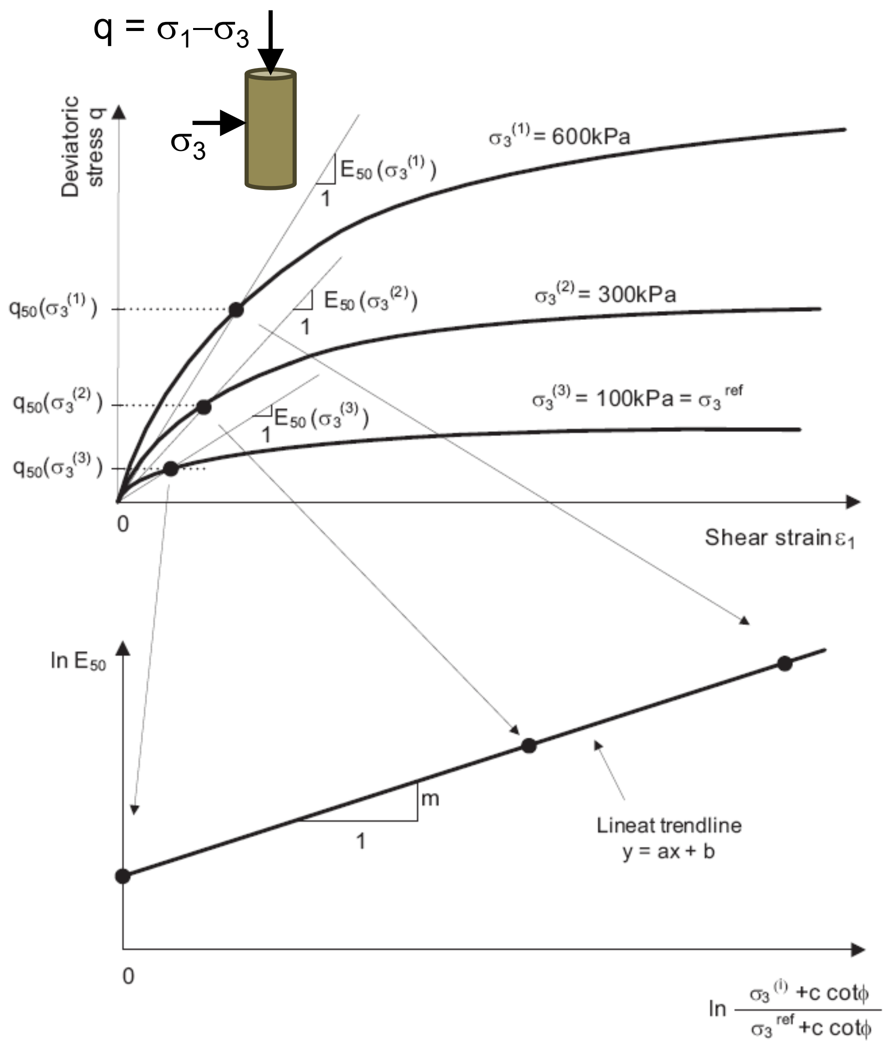
i.e. stiffness degrades with decreasing σ_3 up to the limit minor stress σ_L

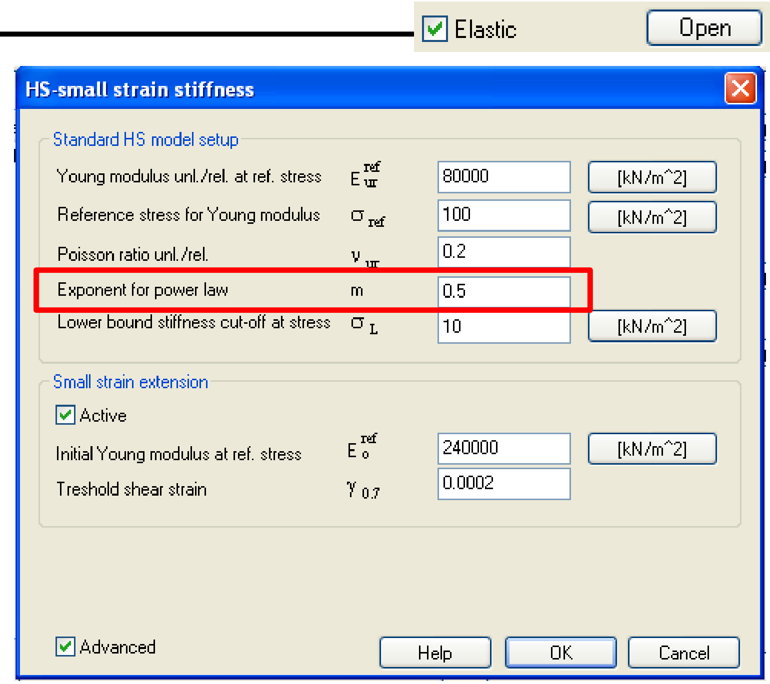


NB.Setting m=0 -> constant stiffness like in the standard M-C model

HSM: Identyfikacja parametru m

II identification method for E_{50}





1. **Find three values of $E_{50}^{(i)}$ corresponding $\sigma_3^{(i)}$ to respectively**

2. **Find a trend line $y = ax + b$ by assigning variables**

$$y = \ln E_{50}^{(i)}$$

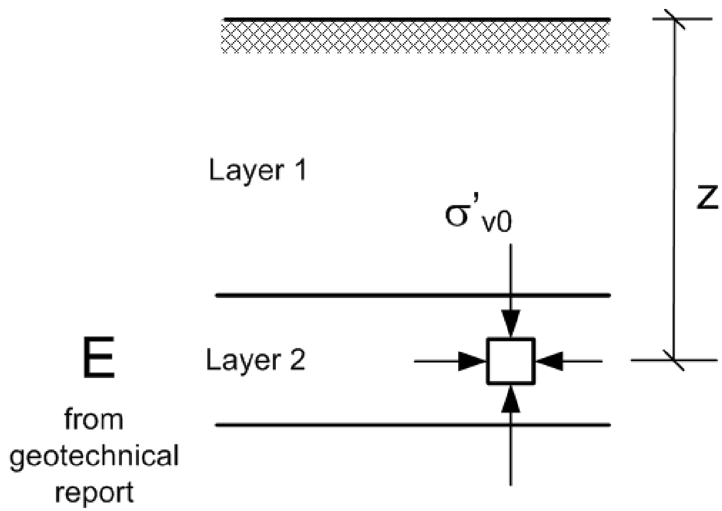
$$x = \ln\left(\frac{\sigma^{(i)} + c\cot\phi}{\sigma^{\mathrm{ref}} + c\cot\phi}\right)$$

3. **Then the determined slope of the trendline a is the parameter m**

Hardening Soil: Stress dependent stiffness – Setting σ_{ref}

Let's assume that the geotechnical report suggests assuming the stiffness modulus E for a given layer which is located at given depth …

1. Define what is given E with respect to E_{50} and E_{ur}
2. Evaluate reference stress σ_{ref}



$$\sigma_{ref} = \min\left(\sigma'_{v0},\ K_0 \cdot \sigma'_{v0}\right)$$

reference level

$$E_{ref}(\sigma_{ref}) = E$$

σ^{ref} -reference minor stress so if $K_0 < 1$ then $\sigma'_3 = \sigma'_h$

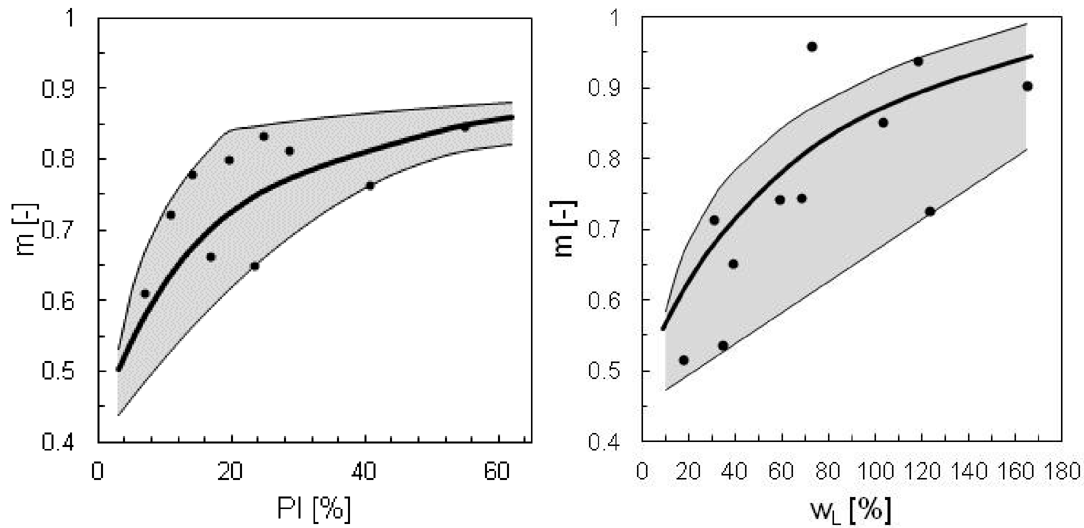
Hardening Soil: Stiffness exponent *m*

Coarse-grained soils

Soil tested	m [-]
Kenya carbonate sand	0.45-0.52
Quiou carbonate sand	0.62
Ottawa sand No.20-30	0.50
SLB sand (subround)	0.44-0.53
Toyoura sand (subangular)	0.41-0.51
Toyoura sand (subangular)	0.50-0.57
Toyoura sand (subangular)	0.45
Ticino sand (subangular)	0.44-0.53
Ticino sand (subangular)	0.43
Ticino sand (subangular)	0.43-0.48
H.River sand (subangular)	0.5-0.52
Silica sand (subangular)	0.5
Hostun sand (angular)	0.47
Silica sand (angular)	0.50
Silica sand	0.42
Hime gravel (subround)	0.45-0.51
Chiba gravel (subround)	0.50

Typically m = 0.4 to 0.6

Cohesive soils

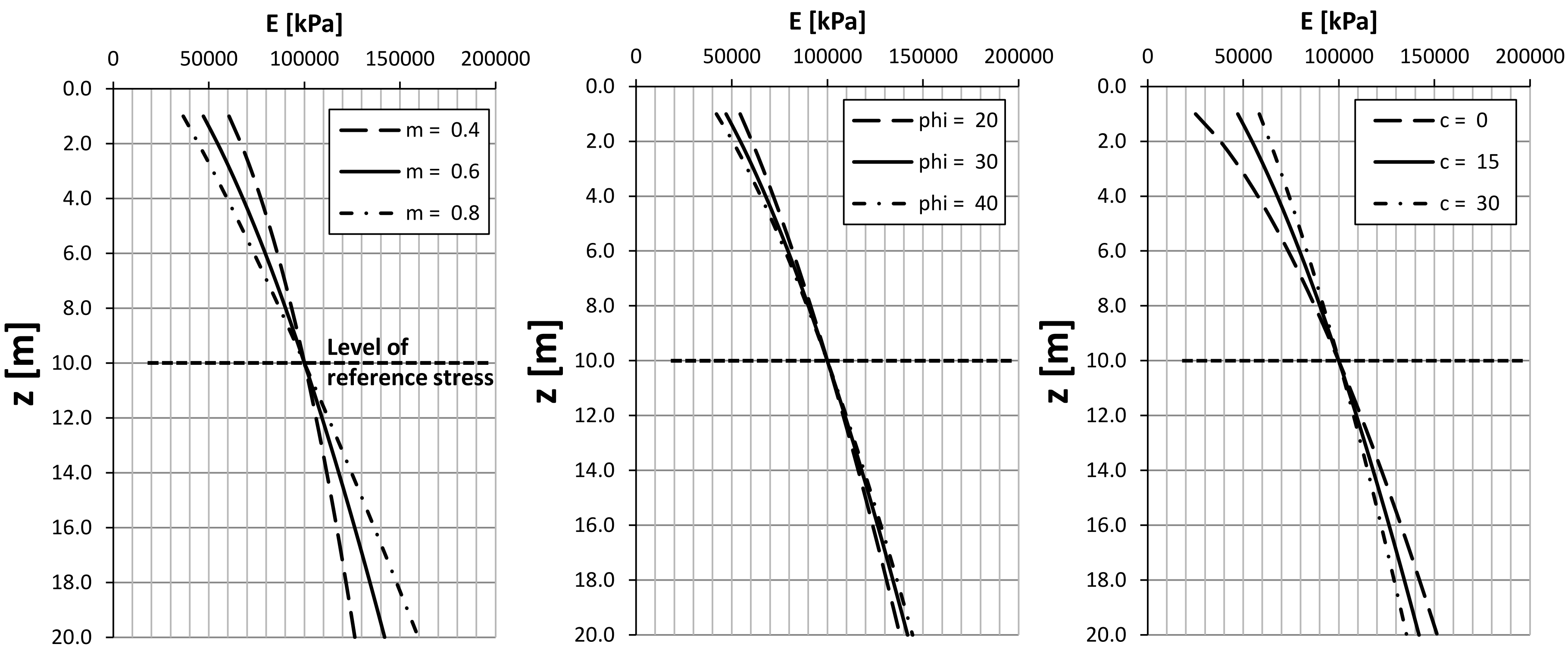


after Viggiani&Atkinson (1995) , and Hicher (1996)

Typically m > 0.5

Hardening Soil: Stress dependent stiffness at initial state

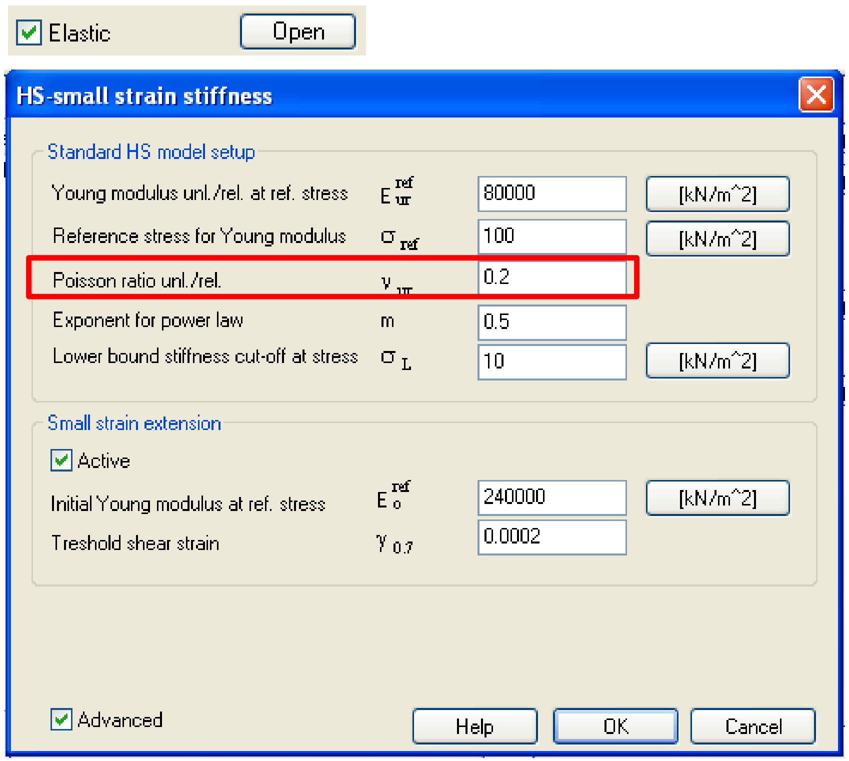
$$E_{50} = E_{50}^{\text{ref}} \left(\frac{\sigma_3^* + c \cot\phi}{\sigma^{\text{ref}} + c \cot\phi} \right)^m$$



Stiffness moduli will evolve with the stress levels

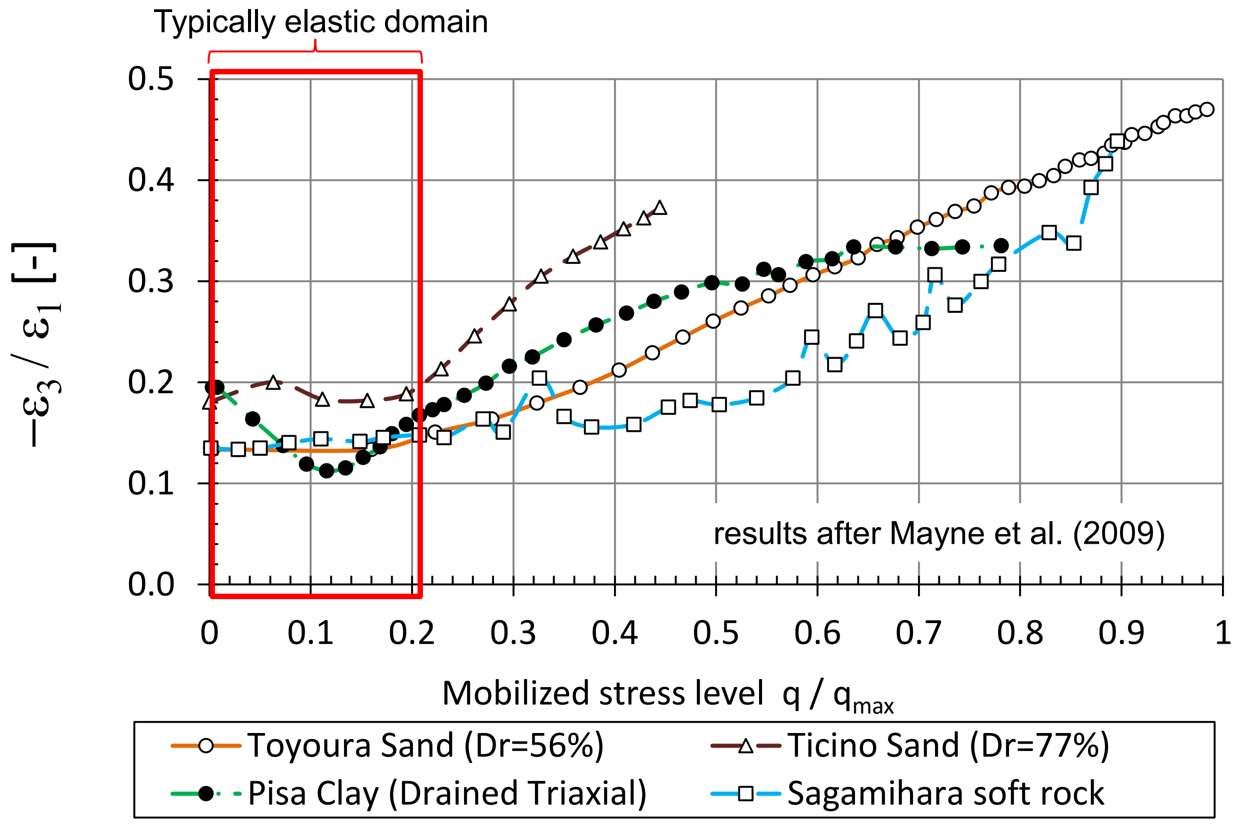
Hardening Soil: Unloading/reloading Poisson's ratio ν_{ur}

A typical value for the elastic unloading/reloading Poisson's ratio of ν_{ur} = 0.2 can be adopted for most soils.

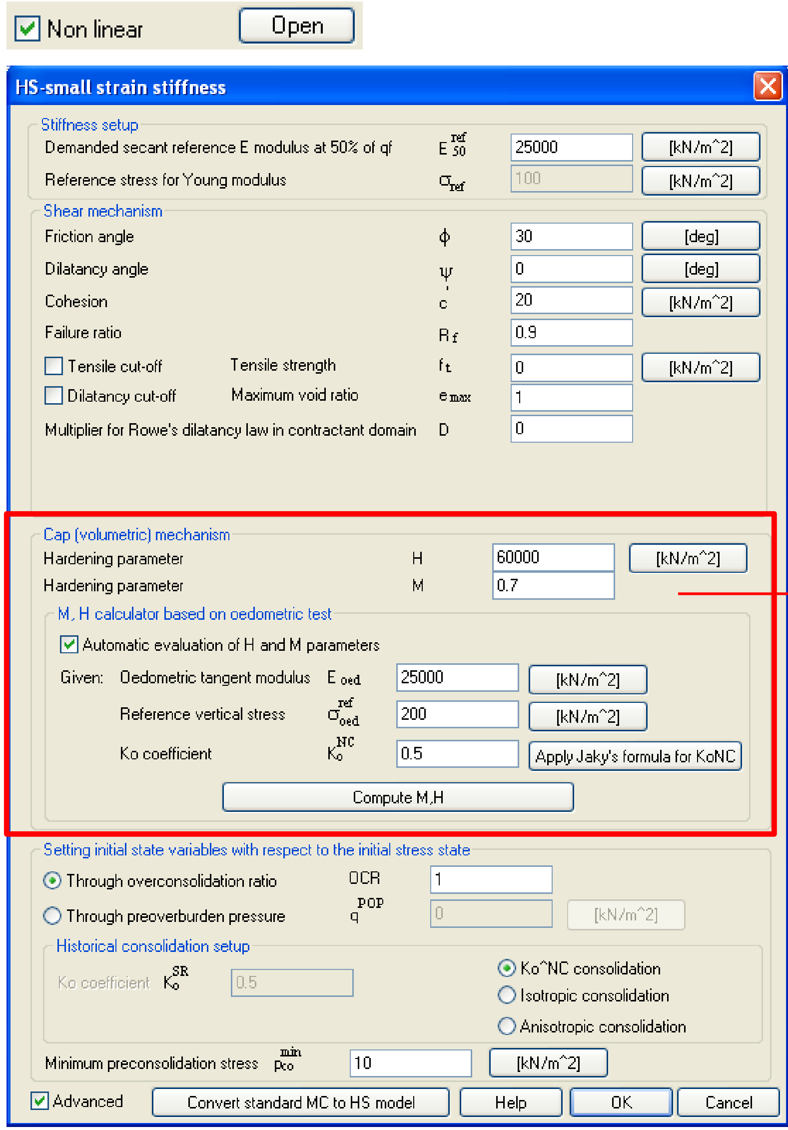


Hardening Soil: Unloading/reloading Poisson's ratio ν_{ur}

Experimental measurements from local strain gauges show that the initial values of Poisson's ratio in terms of small mobilized stress levels q/q_{max} varies between 0.1 and 0.2 for clays, sands.



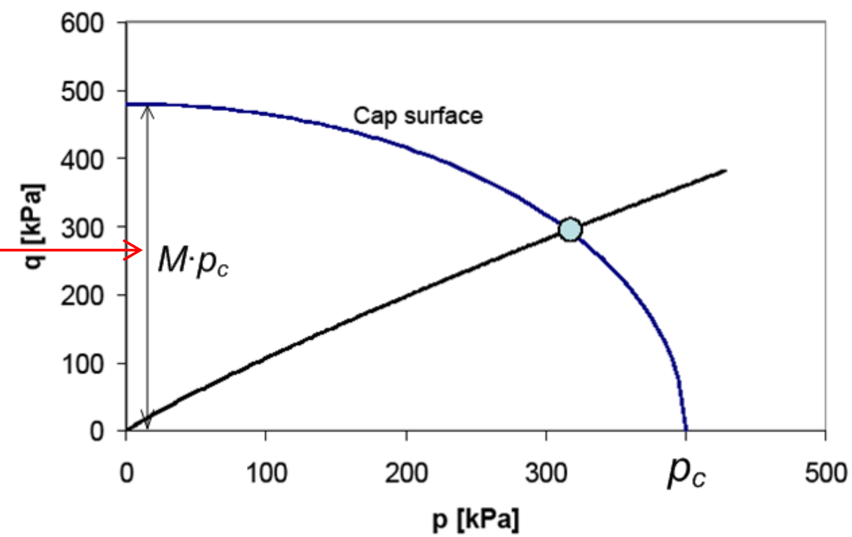
Hardening Soil: Parameters *M* and *H*



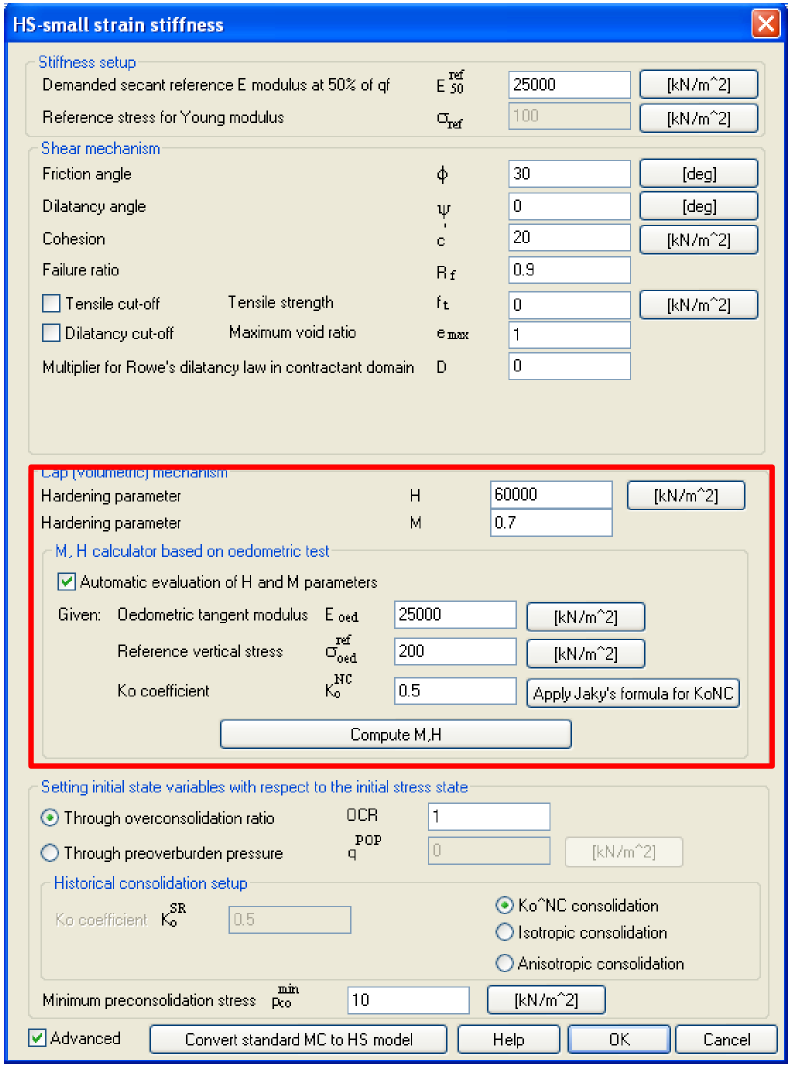
Evolution of the hardening parameter p_c :

$$dp_c = -H\left(\frac{p_c + c\cot\phi}{\sigma_{\mathrm{ref}} + c\cot\phi}\right)^m d\varepsilon_v^p$$

H controls the rate of volumetric plastic strains



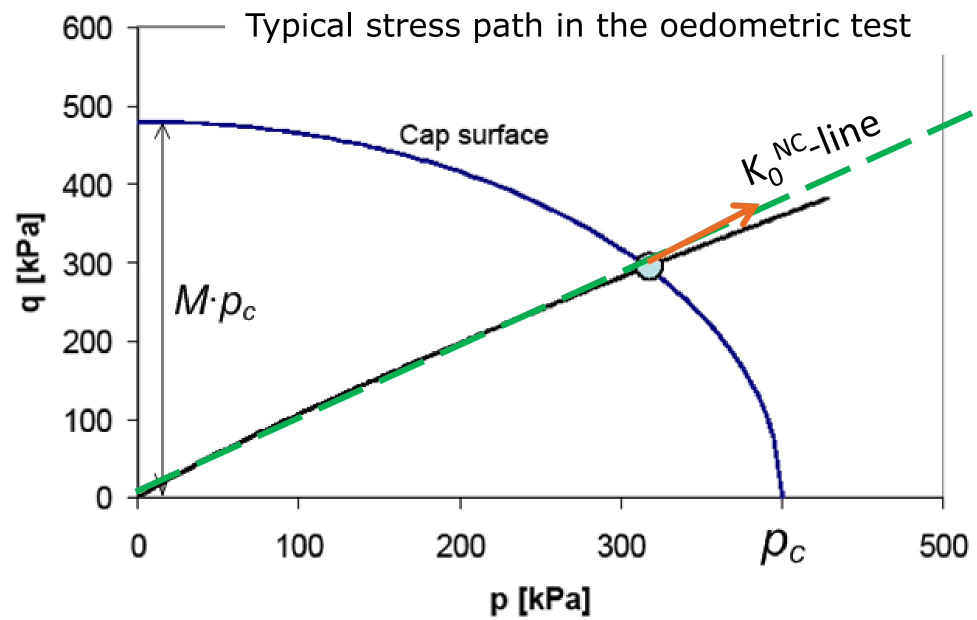
Hardening Soil: Parameters *M* and *H*



M and H must fulfil two conditions:

1. K_0^{NC} produced by the model in oedometric conditions is the same as K_0^{NC} specified by the user

2. E_{oed} generated by the model is the same E_{oed}^{ref} specified by the user



Hardening Soil: selection of oedometric modulus E_{oed}

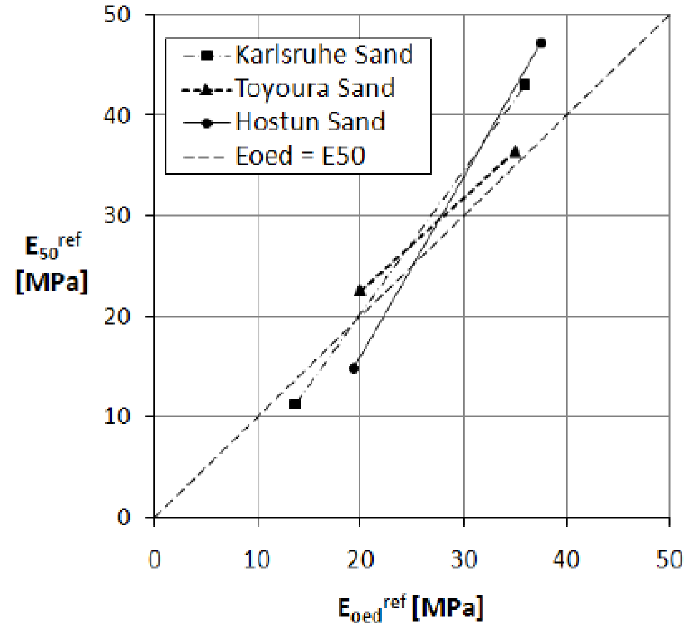
In case of lack of oedometric test data for granular material the oedometric modulus can approximately be taken as:

$$E_{oed}^{ref} \approx E_{50}^{ref}$$

if so σ_{oed}^{ref} should be matched to the reference minor stress σ^{ref} since the latter typically corresponds to the confining (horizontal) pressure

$$E_{oed}^{ref} \qquad\qquad E_{50}^{ref}$$

$$\sigma_{oed}^{ref} = \sigma^{ref} / K_0^{NC} \qquad\qquad \sigma^{ref}$$



On the other hand, when defining $\sigma_{oed}^{ref} = \sigma^{ref}$ in the model, the following relationship should be taken:

$$E_{oed}^{ref} \approx E_{50}^{ref} (K_0^{NC})^m$$

Table 3.7: Relationship between triaxial stiffness moduli and oedometric moduli for three lacustrine clays in Germany, from Kempfert (2006).

	E_i/E_{oed}	E_{50}/E_{oed}	$E_{ur}/E_{oed,ur}$	$E_{oed,ur}/E_{oed}$
Soil 1	2.08	1.03	2.33-2.52	2.60
Soil 2	1.63	0.77	1.29-2.09	3.63
Soil 3	2.82	1.45	1.32-2.51	6.65
Average	2.17	1.08		4.29

E_i was derived from the initial slope of the triaxial curve $\varepsilon_1 - q$
$E_{oed,ur}$ denotes unloading/reloading oedometer modulus

Hardening Soil: Tangent oedometric modulus E_{oed}

$$E_{\text{oed}} = \left(\frac{1+e^{\text{ref}}}{C_c}\right)\sigma^*$$

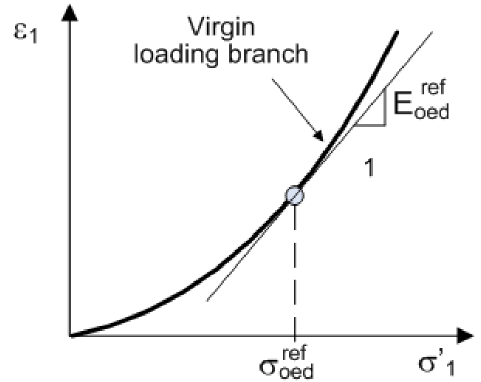
where C_c is the compression index

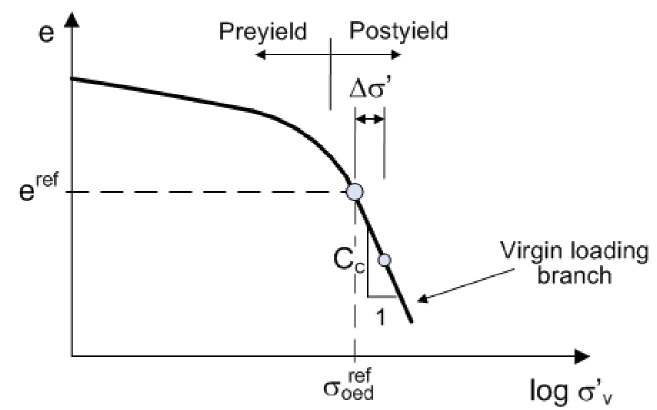
$$\sigma^* = \frac{\Delta\sigma'}{\log_{10}\left(\frac{\sigma_{\text{oed}}^{\text{ref}}+\Delta\sigma'}{\sigma_{\text{oed}}^{\text{ref}}}\right)}$$

Since we look for tangent E_{oed}, $\Delta\sigma' \rightarrow 0$, and

$\sigma^* = 2.303\sigma_{oed}^{ref}$

$$\boxed{E_{\text{oed}} = \frac{2.3(1+e^{\text{ref}})}{C_c}\sigma_{\text{oed}}^{\text{ref}}}$$





Initial state variables - *OCR*

Notion of overconsolidation ratio:

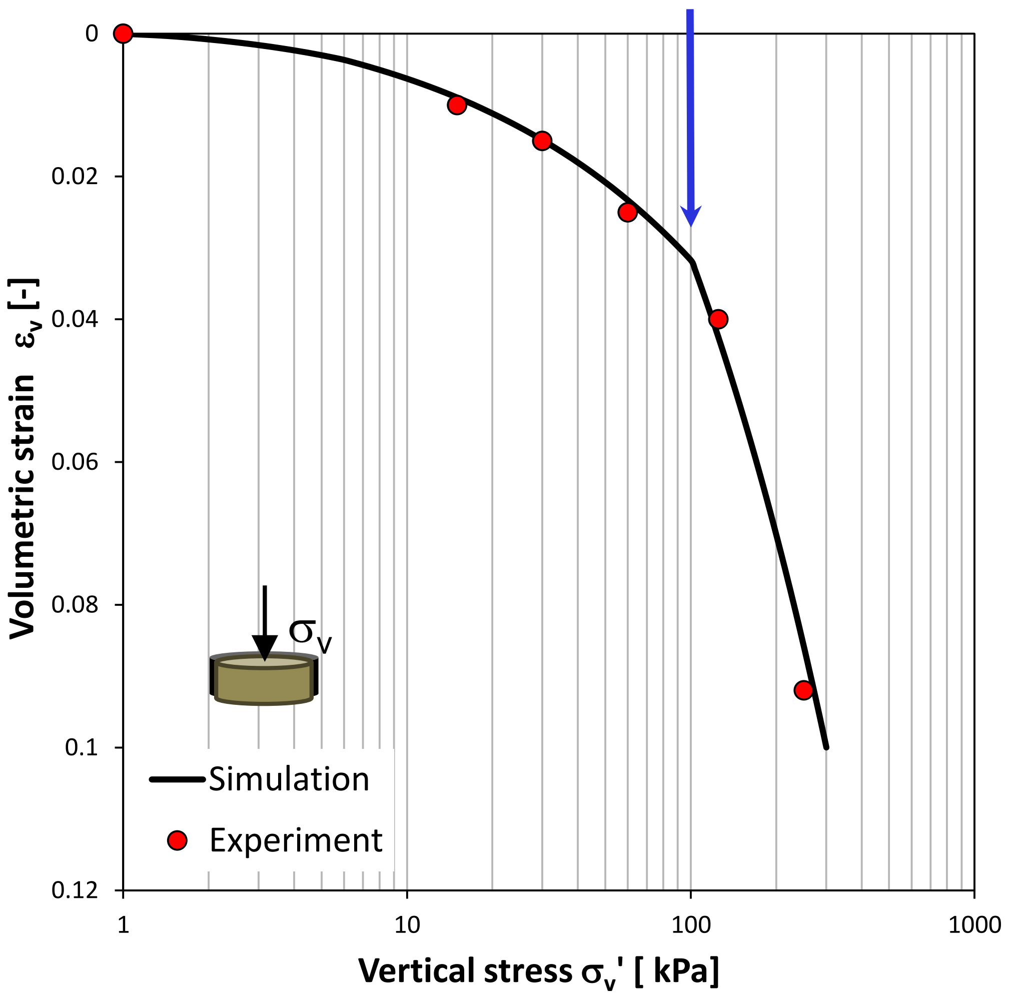
$$\mathbf{OCR} = \frac{\sigma'_{vc}}{\sigma'_{v0}}$$

σ'_{v0} – current in situ stress

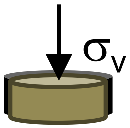
σ'_{vc} – past vertical preconsolidation pressure

NB. In natural soils, overconsolidation may stem from mechanical unloading such as erosion, excavation, **changes in ground water level, or due to other phenomena such as desiccation, melting of ice cover,** compression and cementation.

Typical oedometer test



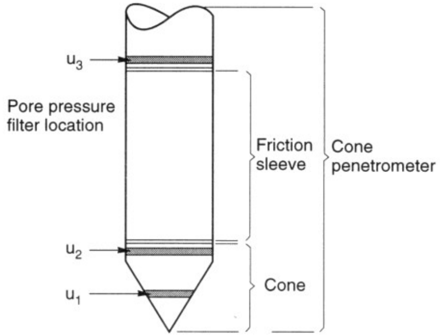
Estimation of preconsolidation pressure and OCR



Laboratory:

- **Oedometer test** (Casagrande's method, Pacheco Silva method (1970), cf. **report on HS model**)

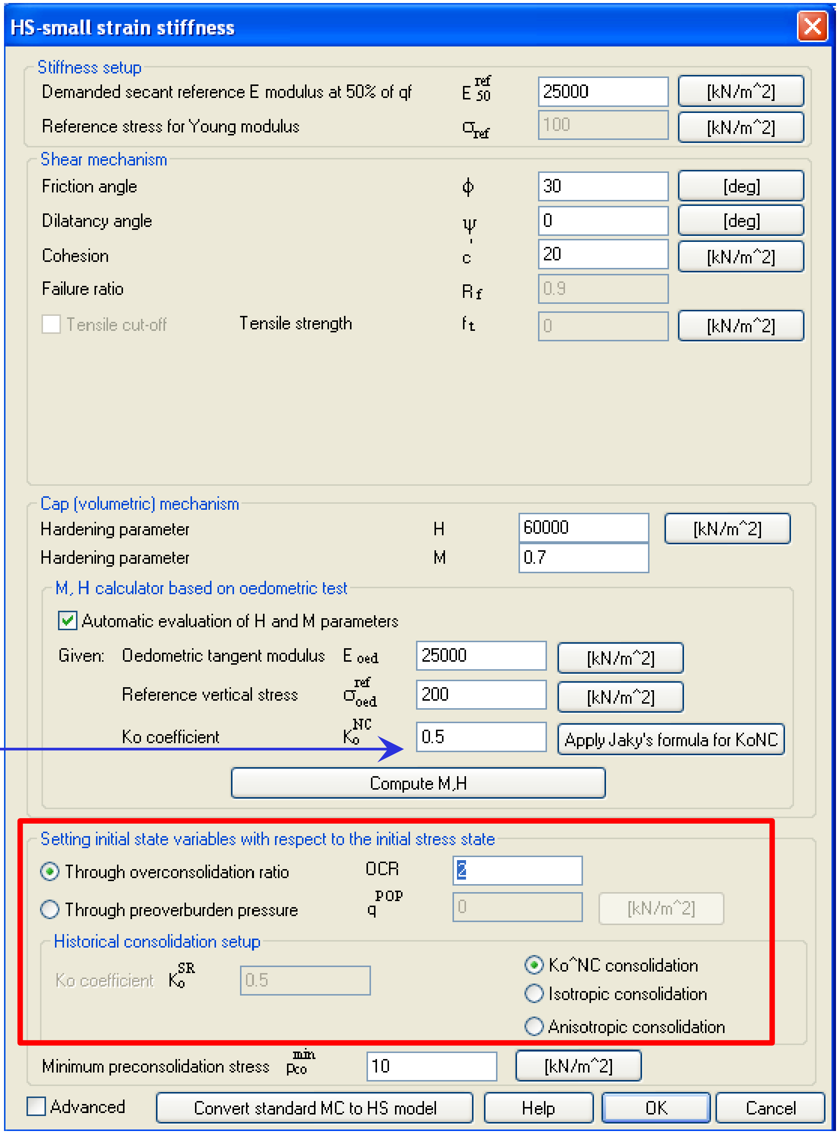
Field tests:



- **Static piezocone penetration (CPTU)** (for correlations see **report on HS model in Zsoil Help**)

- **Marchetti flat dilatometer (DMT)** (correlations by Marchetti (1980), Lacasse and Lunne, 1988), see **report on HS model in Zsoil Help**)

Initial state variables – *OCR* and K_0

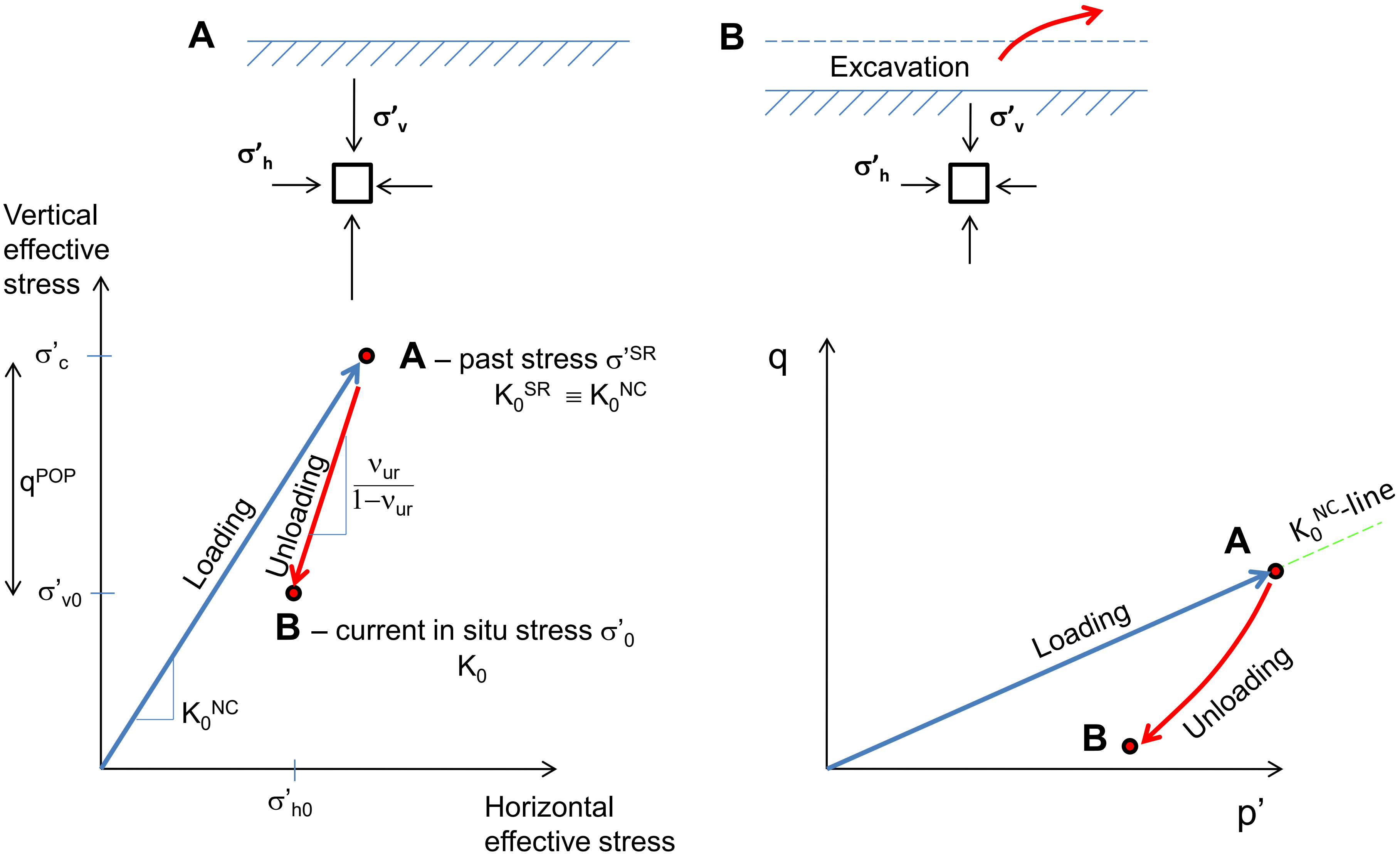


K_0^{NC} consolidation – most cases of natural soils; the value is automatically copied from

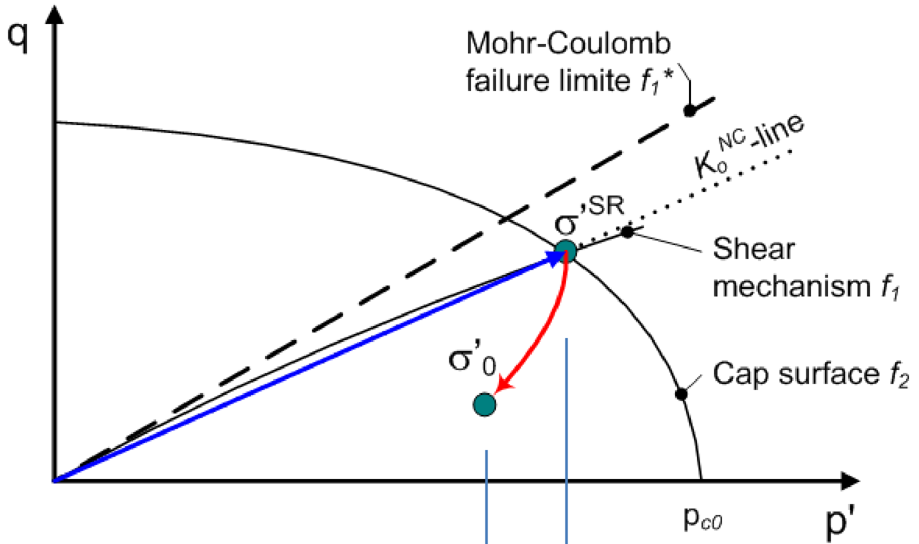
Isotropic consolidation – for running triaxial compression test after isotropic consolidation

Anisotropic consolidation – for running triaxial compression test after anisotropic consolidation

Initial state variables – K_0 and K_0^{NC}

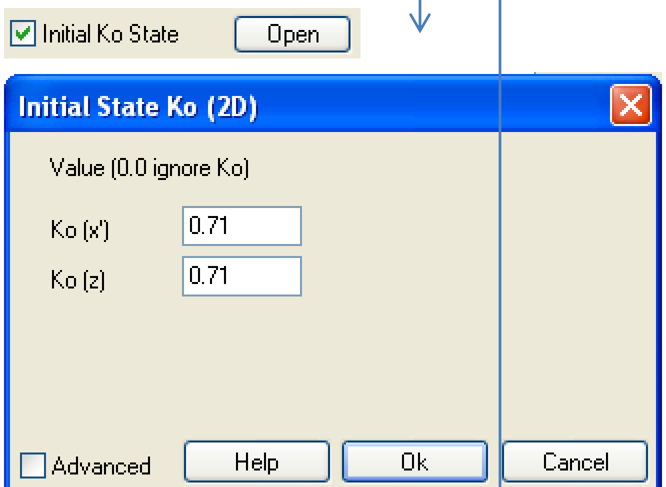


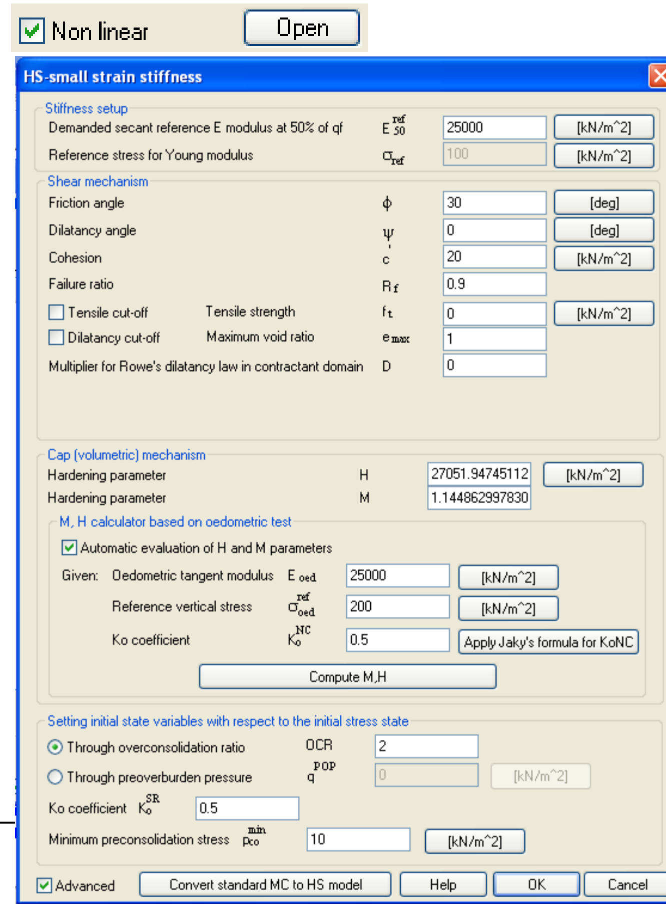
Initial state variables – K_0 and K_0^{NC}



$\sigma'^{\,SR}$ Preconsolidation stress configuration corresponding to K_0^{NC}

σ'_0 Current *in situ* stress configuration corresponding to K_0 (at the beginning of numerical simulation)

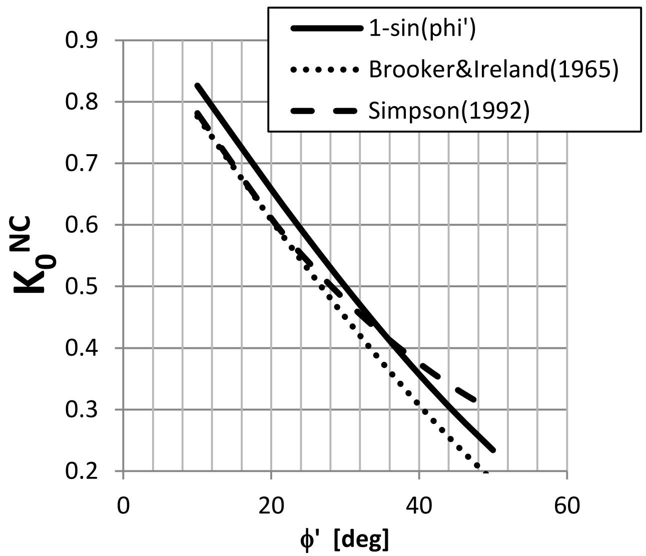




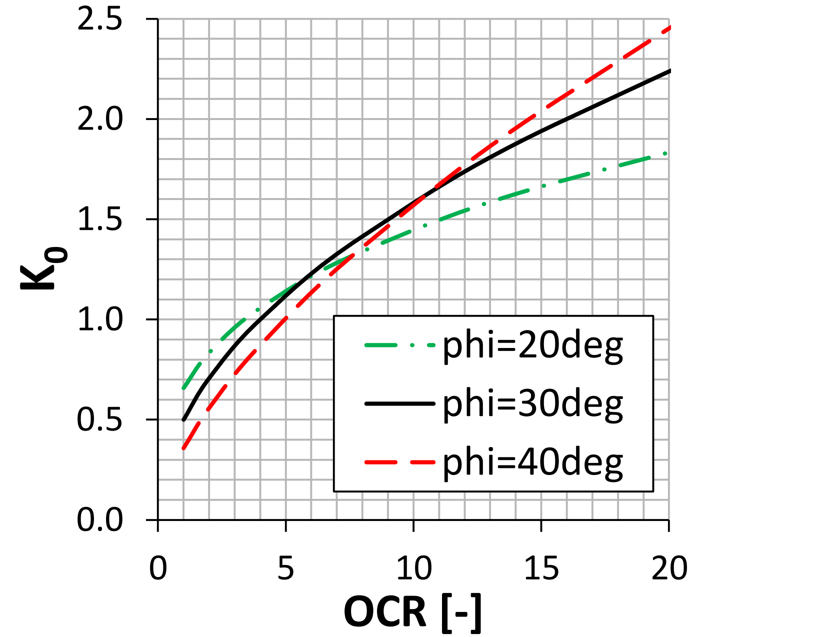
Initial state variables – K_0 vs. OCR

Estimation of earth pressure at rest

Normally-consolidated soils



Overconsolidated soils



$K_0^{\mathrm{NC}} = 1 - \sin\phi'$ so-called (Jaky's formula)

$K_0^{\mathrm{NC}} = (\sqrt{2} - \sin\phi')/(\sqrt{2} + \sin\phi')$ (Simpson, 1992)

$K_0^{\mathrm{NC}} = 0.95 - \sin\phi'$ Brooker & Ireland (1965)

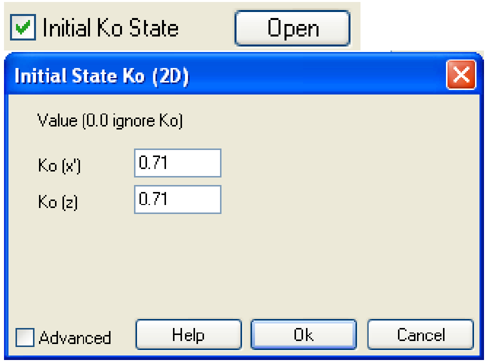
$$K_0 = K_0^{\mathrm{NC}}\mathrm{OCR}^m$$

$m = 0.5$ suggested by Meyerhof (1976)

$m = \sin\phi'$ suggested in Kulhawy & Mayne (1982)

Setting initial effective stresses

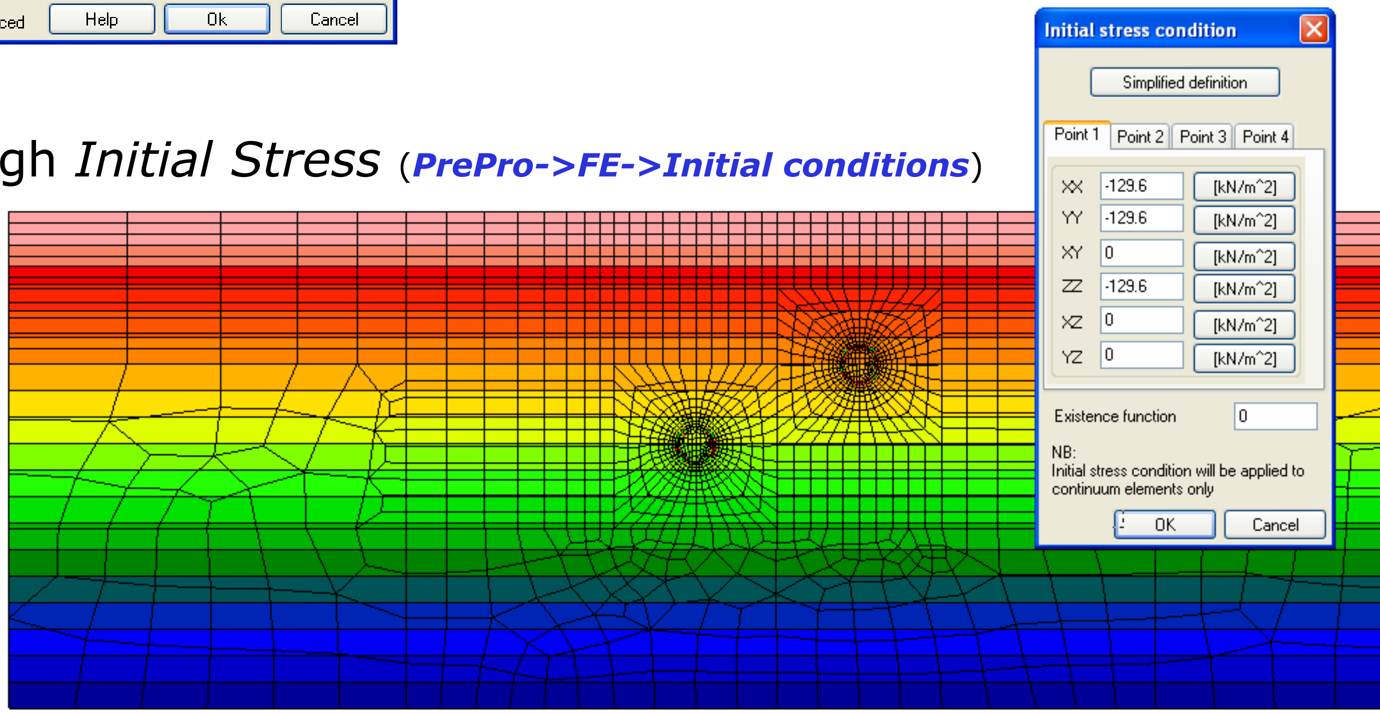
1. Through K_0 (***Materials***)



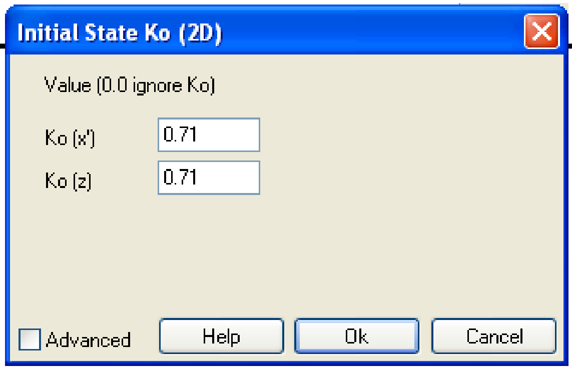
$$\sigma_y = \gamma \cdot \text{“depth”}$$

$$\sigma_x = \sigma_x \cdot K_0(x)$$

2. Through *Initial Stress* (***PrePro->FE->Initial conditions***)

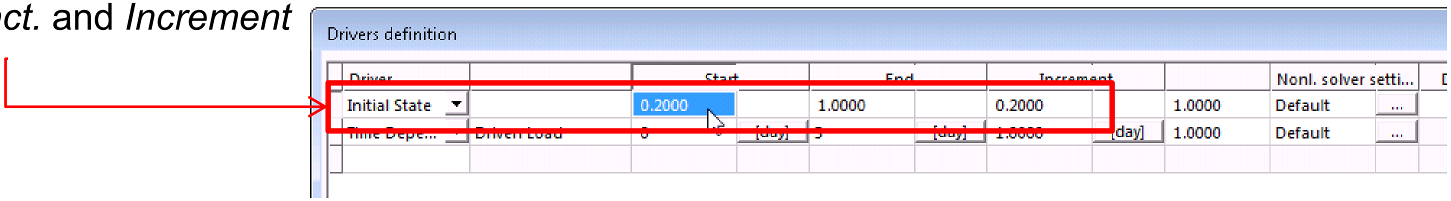


Setting initial state variables - troubleshooting

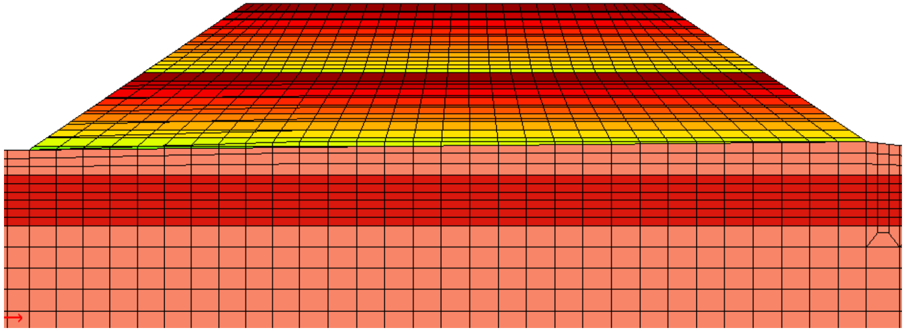


What to do if a model does not converge at *Initial State* even though for a specified K_0

1. try to start *Initial State* analysis from a small *Initial loading Fact.* and *Increment*



2. otherwise, define initial conditions through *Initial Stresses* option

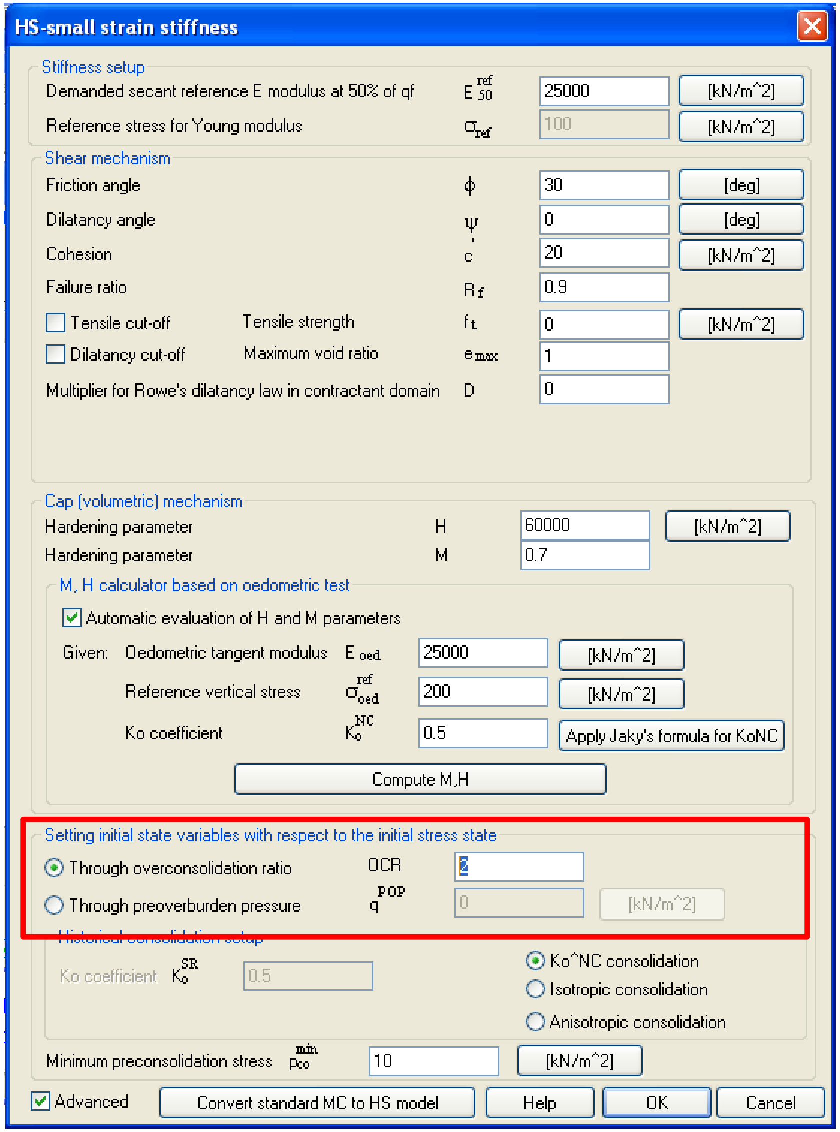


embanking – troubleshooting by means of *Initial stresses*

Numerics, like engineers, they like regularity and elegancy.

Privilege regular meshes to avoid spourious oscillations and lack of convergence.

Initial state variables – preconsolidation effect



1. through *OCR* (gives constant OCR profile)

At the beginning of FE analysis, Zsoil sets the stress reversal point (SR) with:

$$\sigma_y^{\text{'SR}} = \sigma_y \cdot \text{OCR} \quad \text{or} \quad \sigma_y^{\text{'SR}} = \sigma'_{y0} + q^{\text{POP}}$$

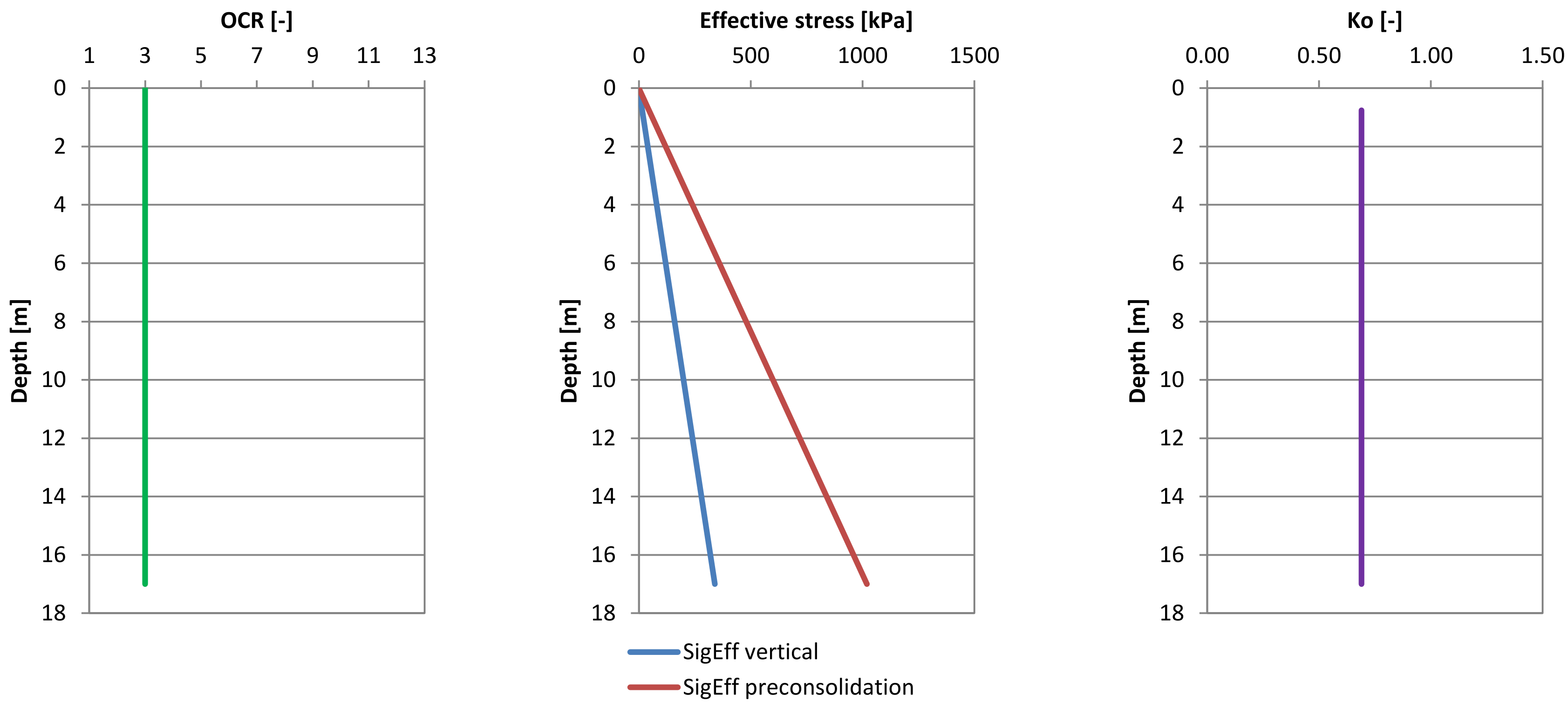
$$\text{and}$$

$$\sigma_x^{\text{'SR}} = \sigma_y^{\text{'SR}} K_0^{\text{SR}} \quad \text{and} \quad \sigma_z^{\text{'SR}} = \sigma_y^{\text{'SR}} K_0^{\text{SR}}$$

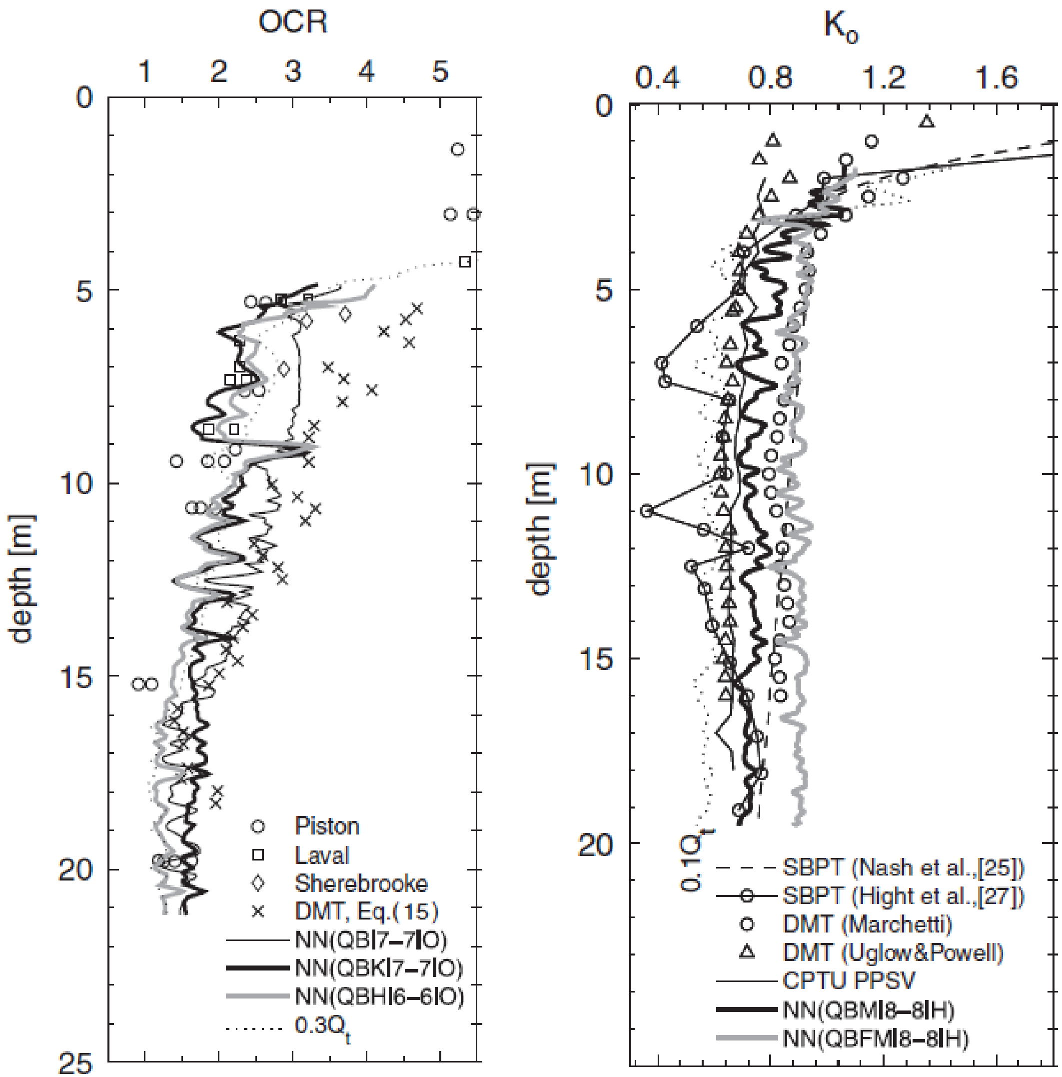
2. through q^{POP} (gives variable OCR profile)

Stress history through OCR

Deposits with constant OCR over the depth (typically deeply bedded soil layers)

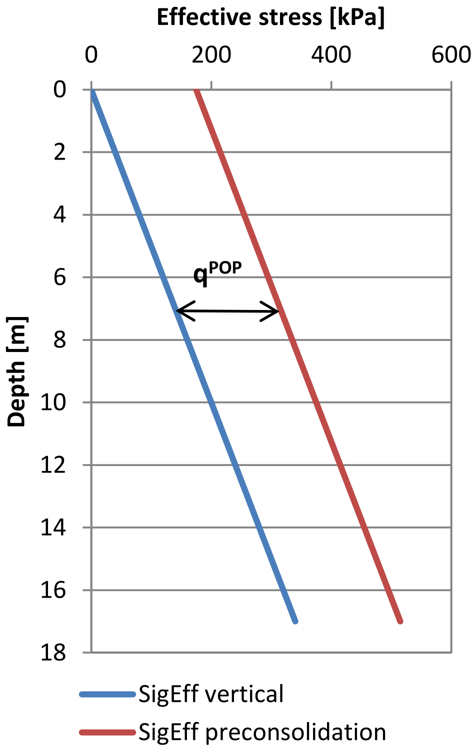


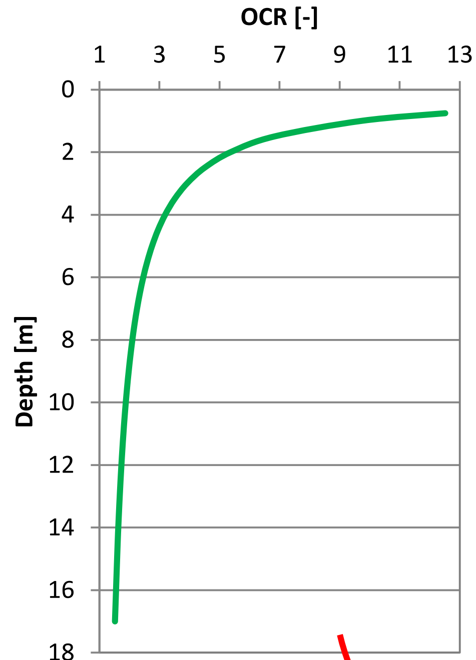
Deposits with varying OCR over the depth (typically superficial soil layers)

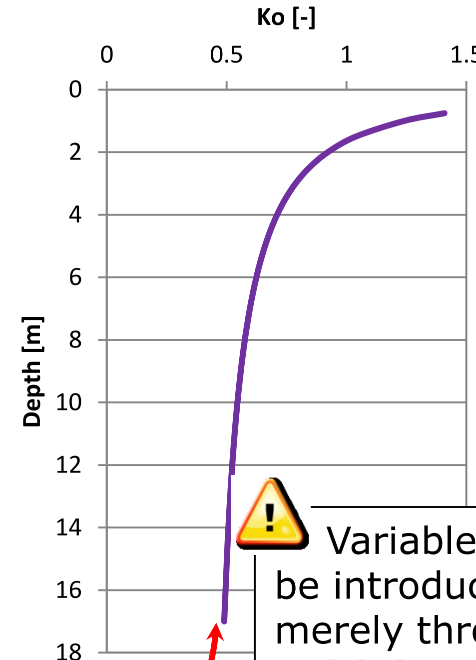


Profiles for Bothkennar clay

Deposits with varying OCR over the depth (typically superficial soil layers)







Variable K_0 can be introduced merely through ***Initial Stress*** option

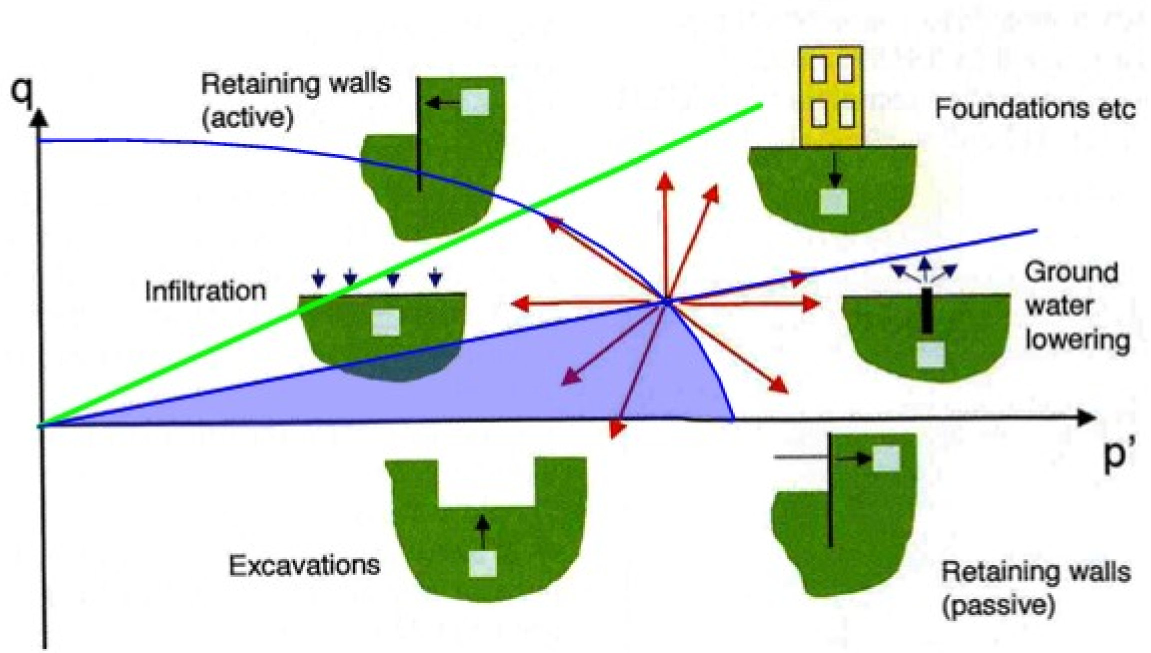
$$K_0^{OC} = K_0^{NC} \sqrt{OCR}$$

$$K_0^{OC} = K_0^{NC} \, OCR^{\sin(\phi)}$$

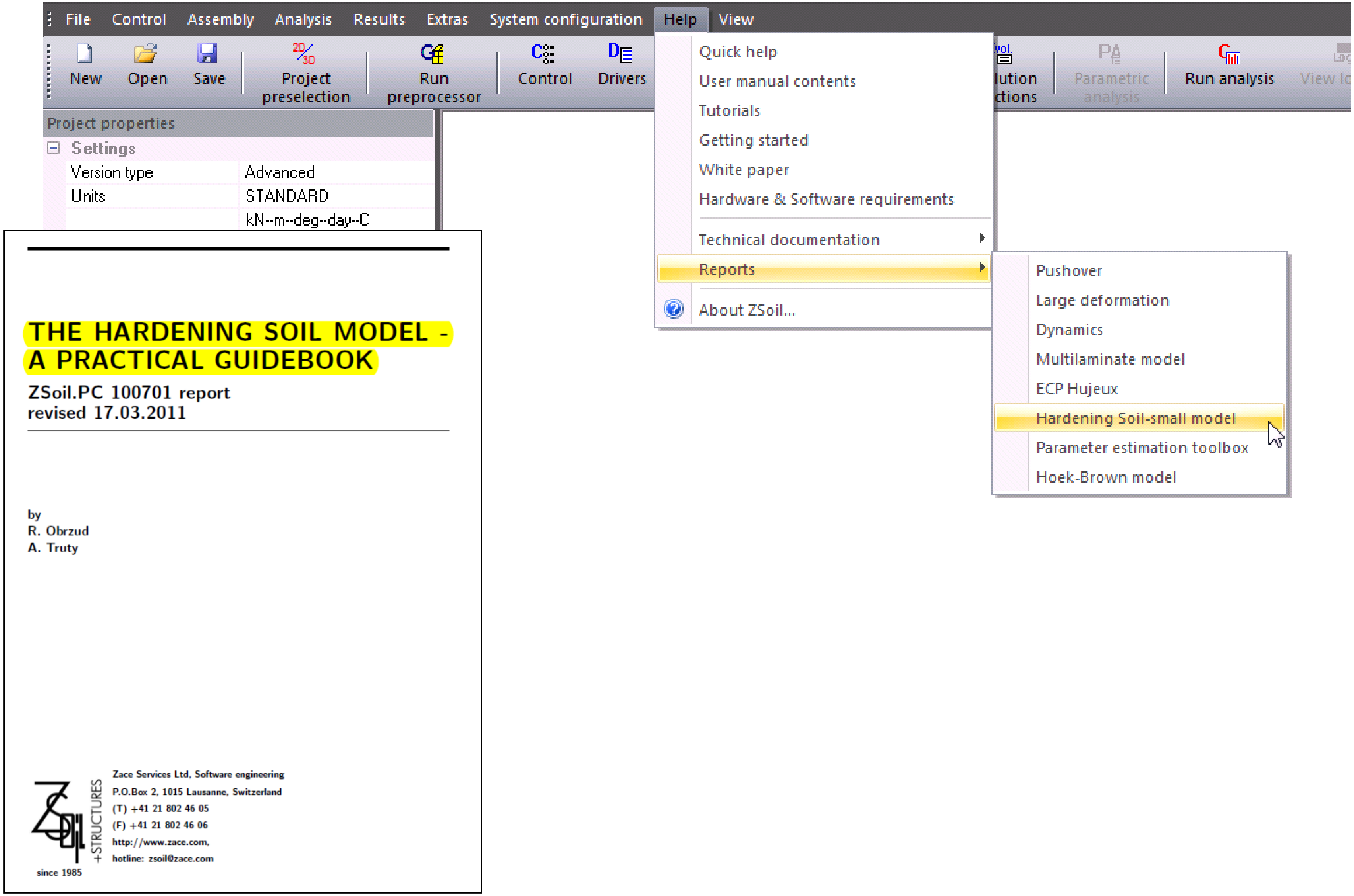
(Mayne&Kulhawy 1982)

Versatility of Hardening Soil model

Good approximation of stress-strain relation for different and complex stress paths that can be encountered in geotechnical engineering



Hardening Soil Model - Report



Report contents

- **Short introduction to the HS models** (theory)
- **Parameter determination**
  - ✓ Experimental testing requirements for direct parameter identification
  - ✓ Alternative parameter estimation for **granular materials**
  - ✓ Alternative parameter estimation for **cohesive materials**
- **Benchmarks**
- **Case studies** including parameter determination
  - ✓ retaining wall excavation
  - ✓ tunnel excavation
  - ✓ shallow footing

Contents

Undrained behavior analysis (1st approach)

Input parameters

- Hardening Soil model is formulated in **effective stresses** (σ'_1, σ'_2, σ'_3 and p') and therefore it requires:
  - **Effective stiffness** parameters E'_0, E'_{ur}, E'_{50}, ν'_{ur}
  - **Effective strength** parameters ϕ', c'
- **Undrained** or **Partially drained** conditions can be obtained by in **`Deformation+Flow, Consolidation`** type analysis depending on action time and adequate permeability coefficients

Drivers definition

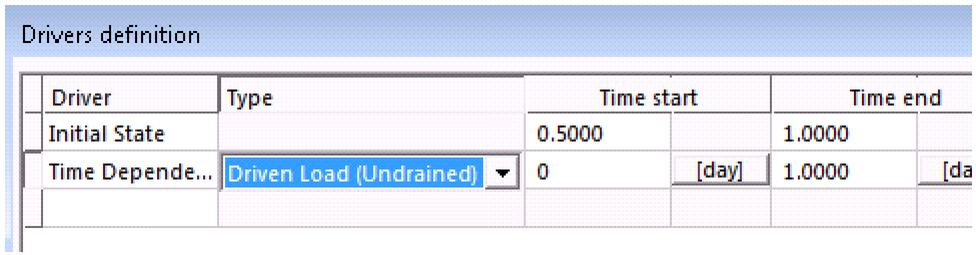
Driver	Type	Time start		Time end		Increment		M
Initial State		0.5000		1.0000		0.1000		
Time Depende...	Consolidation	0	[day]	1.0000	[day]	1.0000	[day]	1

Advantages:

1. Partial saturation effects included
2. Possibility of running any analysis type after consolidation analysis

Undrained behavior analysis (2nd approach)

- **Undrained behavior** can be simulated in **effective stress analysis**
  `Deformation+Flow, Driven Load (Undrained)`



It follows any other like `Driven load+Steady state`, `Driven load+Transient` or `Consolidation` then the condition for the suction pressure is verified for pressure

$$p = S(p_0)\,p_0 + \Delta p$$

(Δp is produced exclusively by the undrained driver.). In order to trace the evolution of the pore pressure values stored at the element integration point must be used so:

Disadvantages

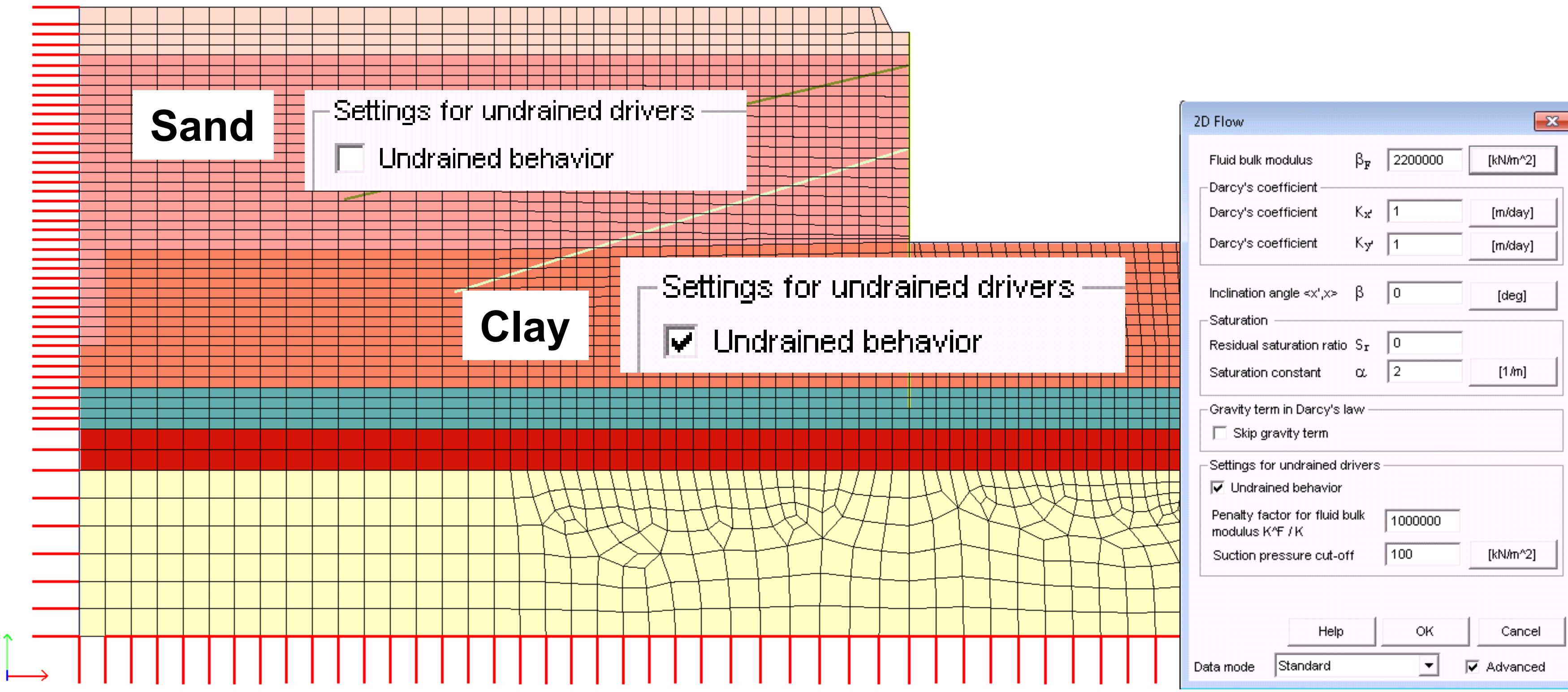
1. **Undrained driver cannot be followed by any other driver**

Advantages:

1. Fully undrained behavior (no volume change) with effective stress parameters
2. Useful for dynamics

Setting undrained behaviour for `Driven Load(Undrained)`

Mixed drainage conditions in terms of sublayers setup



Undrained behavior analysis (3rd approach - simplified)

- **Undrained behavior** in **total stress** analysis can also be performed for the HS model using total stress strength parameters, i.e.
  - $\phi = 0°$, $c = S_u$, $(\psi = 0°)$
  - E'_0, E'_{ur}, E_{50}^{ud}, ν=0.499 **(undrained conditions)**
  - **Set high OCR, e.g. 1000 to disable cap mechanism (no plastic volumetric deformations)**

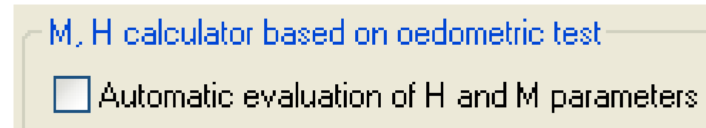
  however

- **sequence of parameter setup** should be followed (given below)
- **appropriate analysis type** should be selected (**Deformation**)
- Limitations …

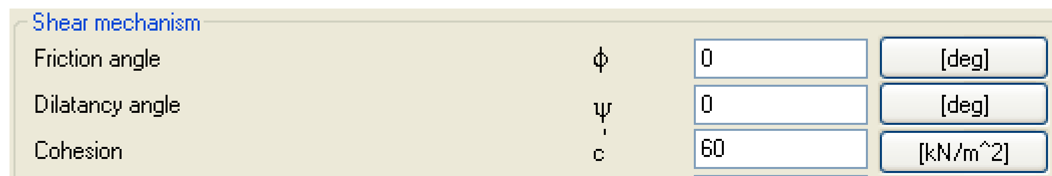
Undrained behavior analysis – single phase analysis using HS

Parameter setup for undrained simulation with single phase analysis:

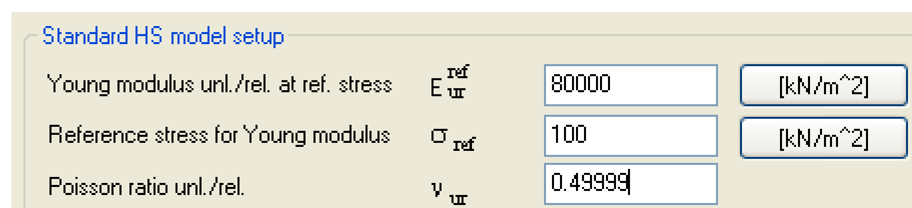
1. Insert effective parameters E'_0, E'_{ur} in `Elastic` menu.
2. Disable Automatic evaluation of H and M parameters (to avoid autoeval. while closing the dialog and errors due to null friction angle)



3. Set high OCR, e.g.1000, to disable cap mechanism (no plastic volumetric deformations should be produced due to assumed undrained conditions)
4. Change ϕ' and c' into "undrained" parameters ϕ = 0° and $c = S_u$. In order to ensure stability of numerical computing, specify ψ = 0°



5. Considering that the undrained conditions imply $\sigma_1 = \sigma_3$, change ν'_{ur} into ν_{ur} = 0.4999 (the "undrained" stiffness behavior will be obtained in the analysis by recomputing the stiffness tensor with ν_{ur} = 0.4999



Parameter setup for undrained simulation with single phase analysis:

6. Set input E_{50} as undrained E_{50}^{ud} (but not E_{ur} and E_0 which should remain as effective ones)

shear modulus is not affected by the drainage condition so one can write:

$$\frac{E_u}{2(1+\nu_u)} = G_u = G = \frac{E}{2(1+\nu)}$$

where ν_u is the Poisson's coffecient in undrained conditions equalk to 0.499.

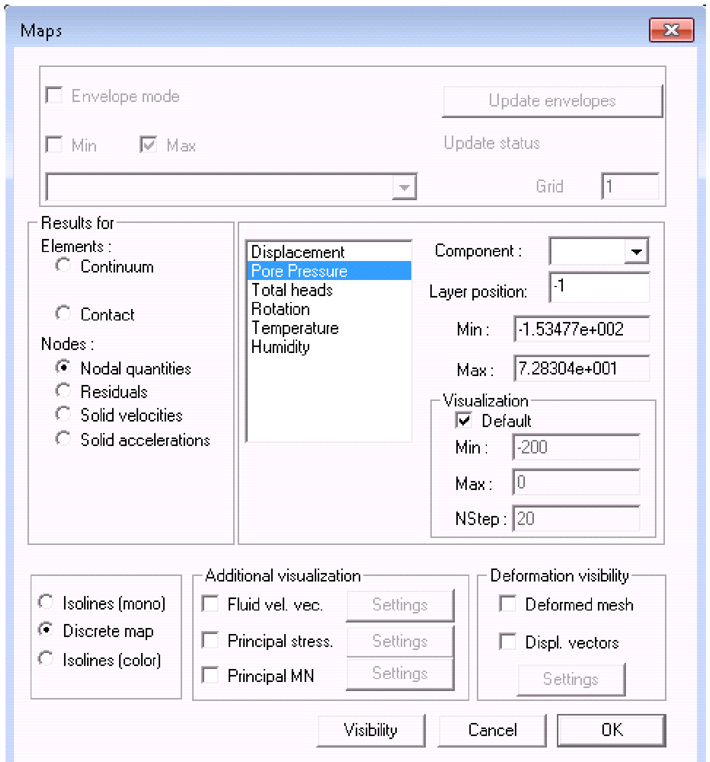
So the above equation can be rewritten for E_{50} as:

$$E_{50}^{u} = \frac{3E'_{50}}{2(1+\upsilon)}$$

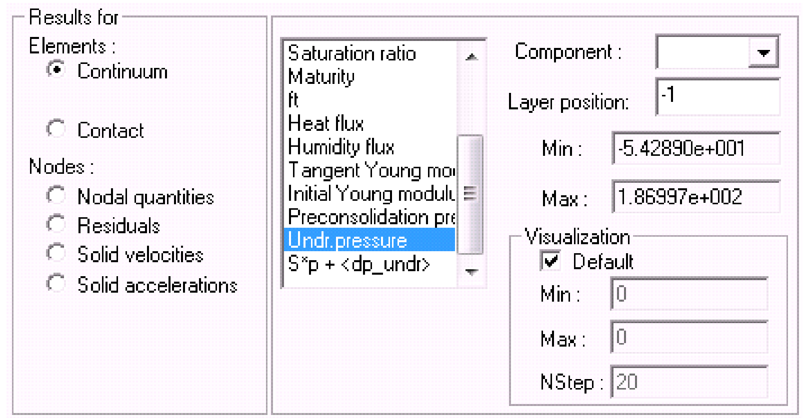
where ν should be considered as that corresponding to E'_{50} , i.e. $\nu \approx 0.3$ (plastic deformations)

Post-processing

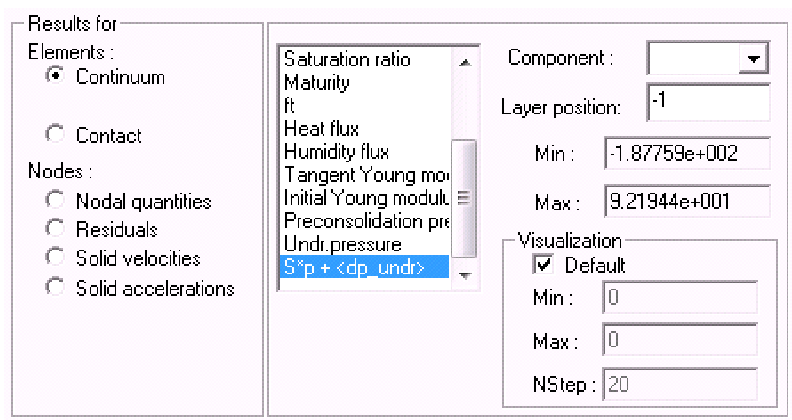
Pore pressure (p) only for
Deformation+Flow



"Undrained" pressure only for
Driven Load (Undrained)



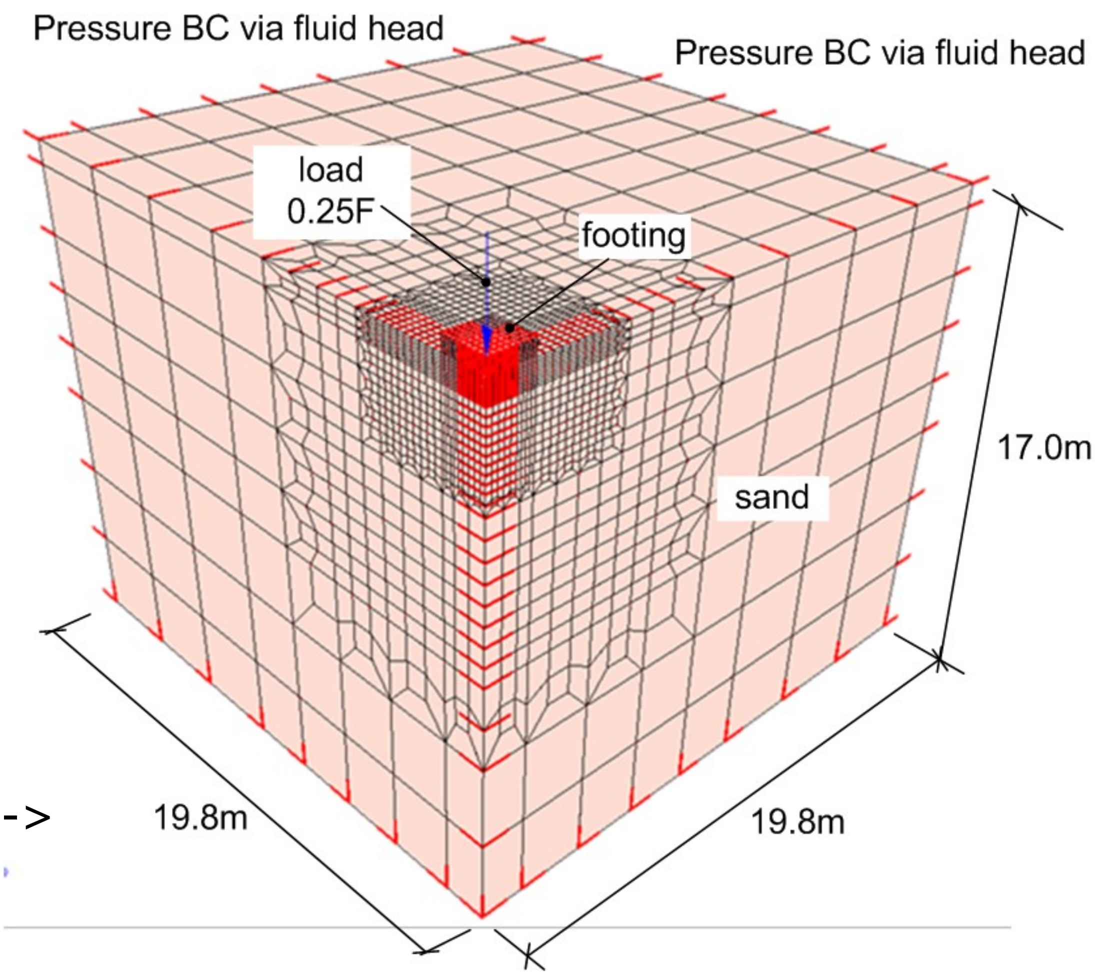
Suction pressure (S*p)



Contents

Practical applications – Shallow footing on an overconsolidated sand

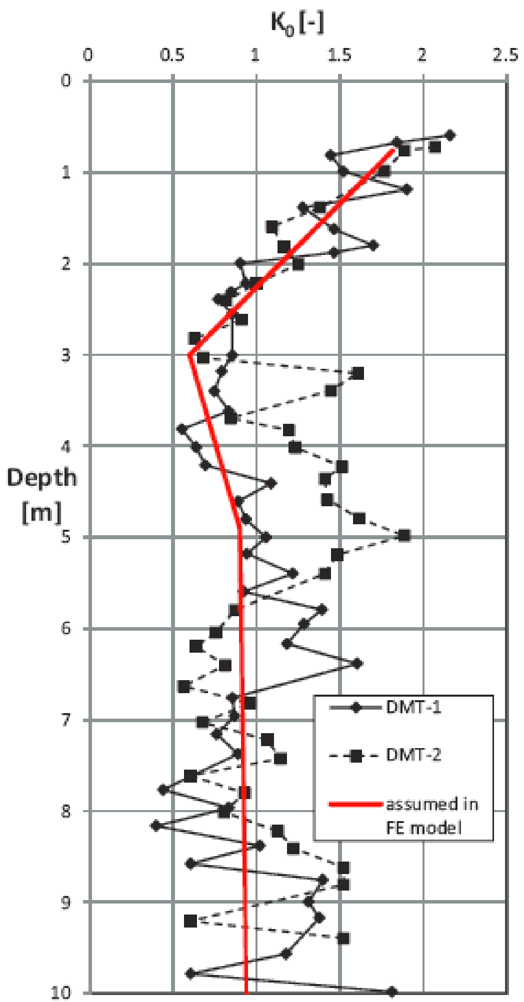
Input files:
`HS-small-Footing-Texas-Sand-2phase.inp`

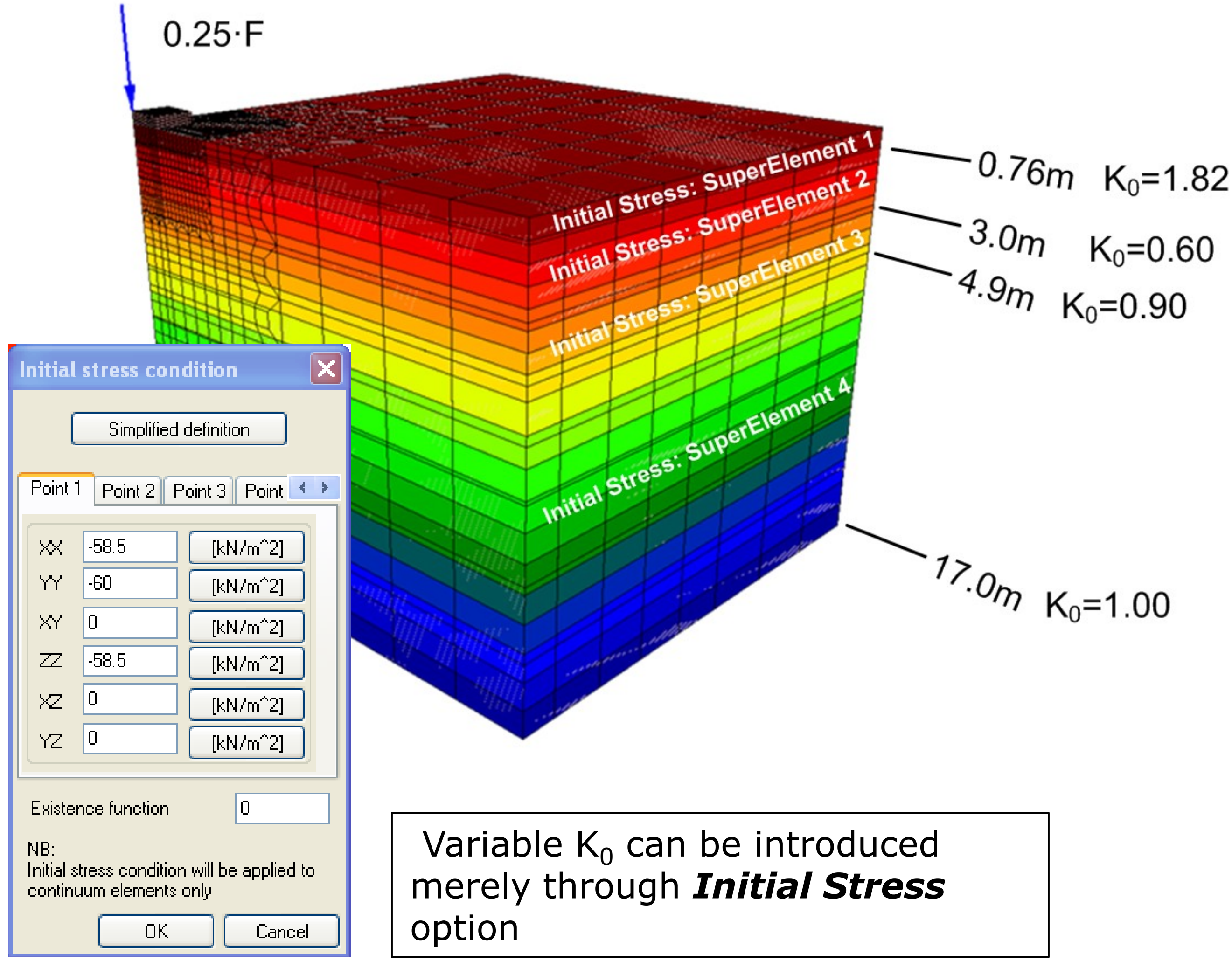


(Help/Reports/Hardening Soil small -> Benchmarks/Spread footing on overconsolidated sand)

Practical applications – Shallow footing on an overconsolidated sand

5. Interpreting and Setting K_0

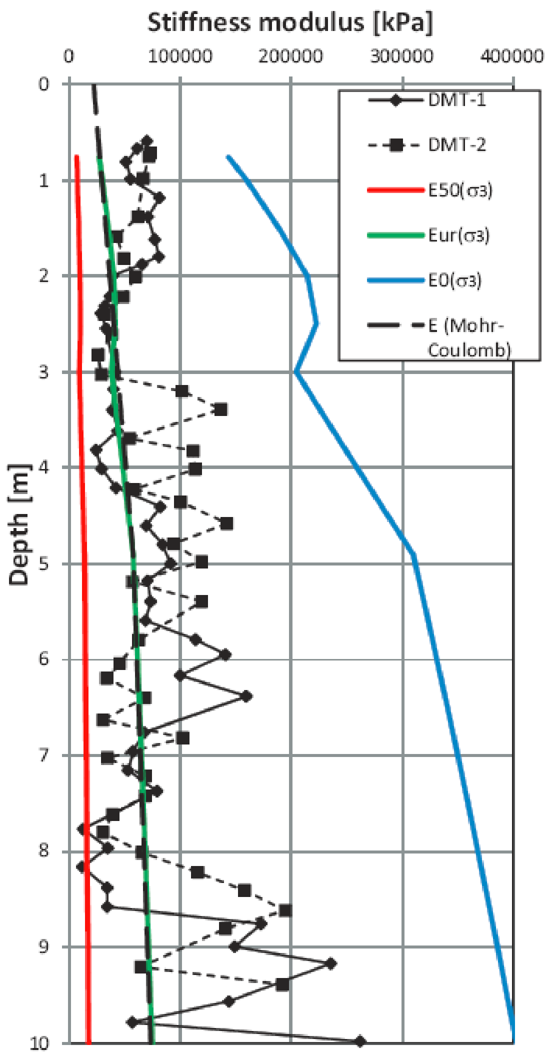




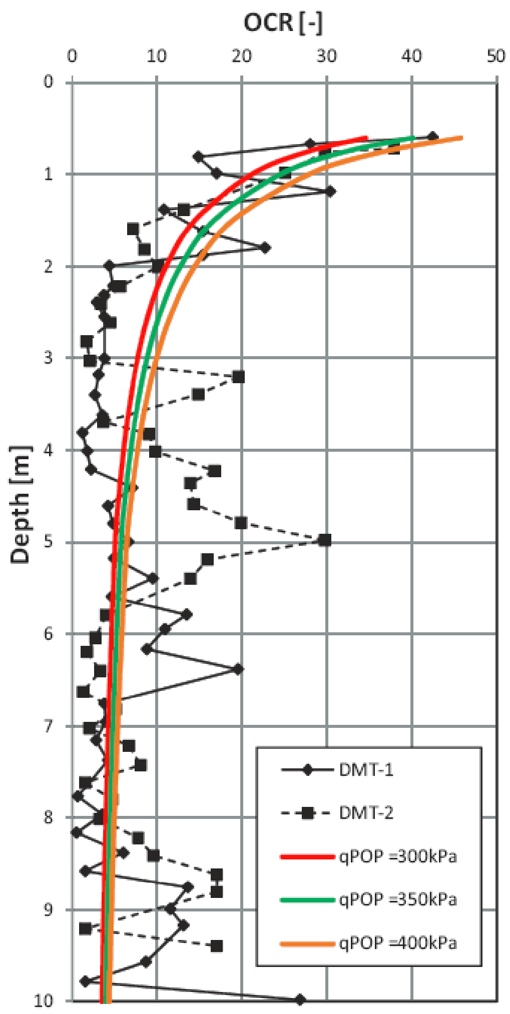
Variable K_0 can be introduced merely through ***Initial Stress*** option

Practical applications – Shallow footing on an overconsolidated sand

4. Selecting q^{POP}



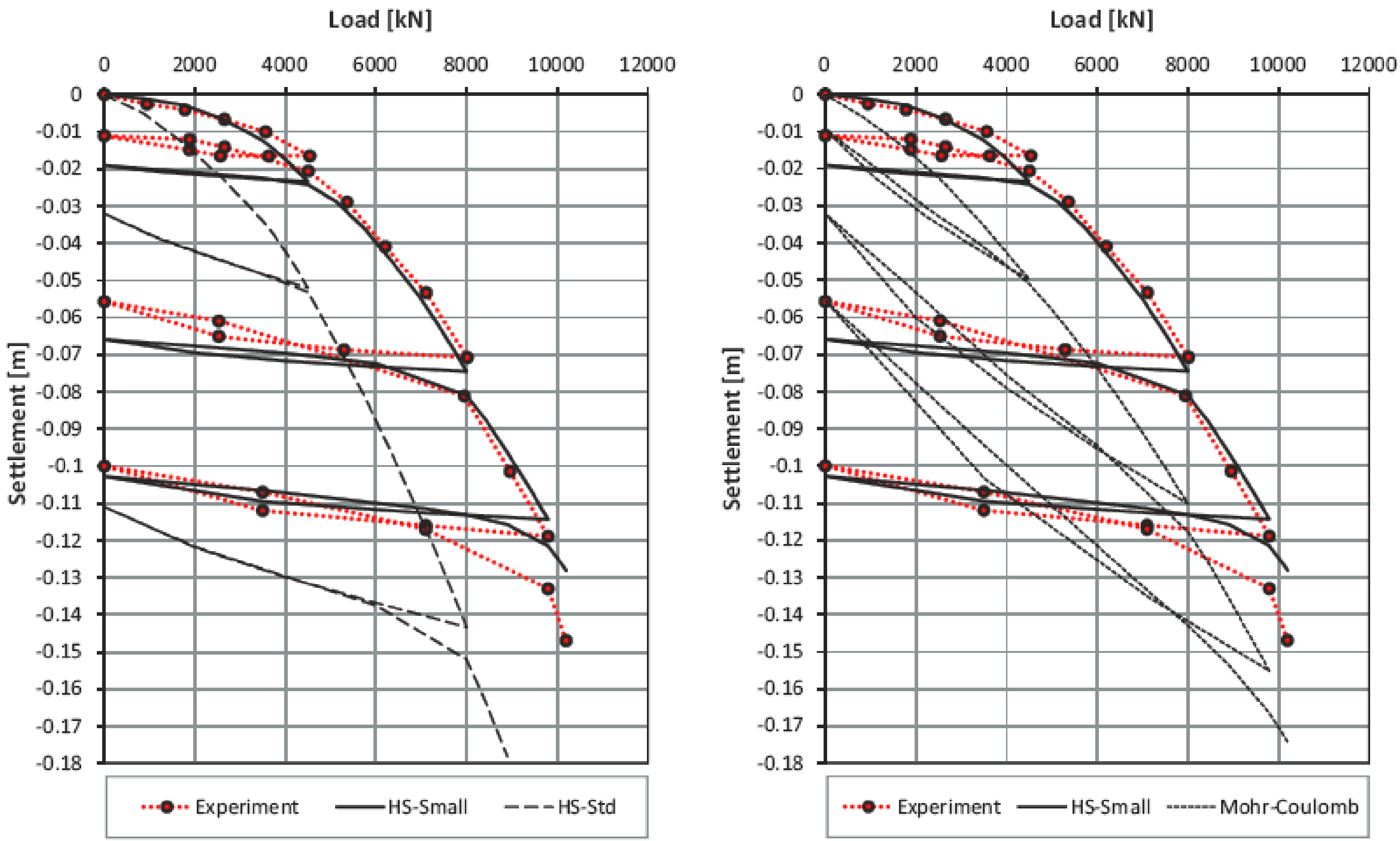
5. OCR based on q^{POP}



Variable OCR profile

Practical applications – Shallow footing on an overconsolidated sand

Comparison of models: HS-SmallStrain, HS-Std vs Mohr-Coulomb

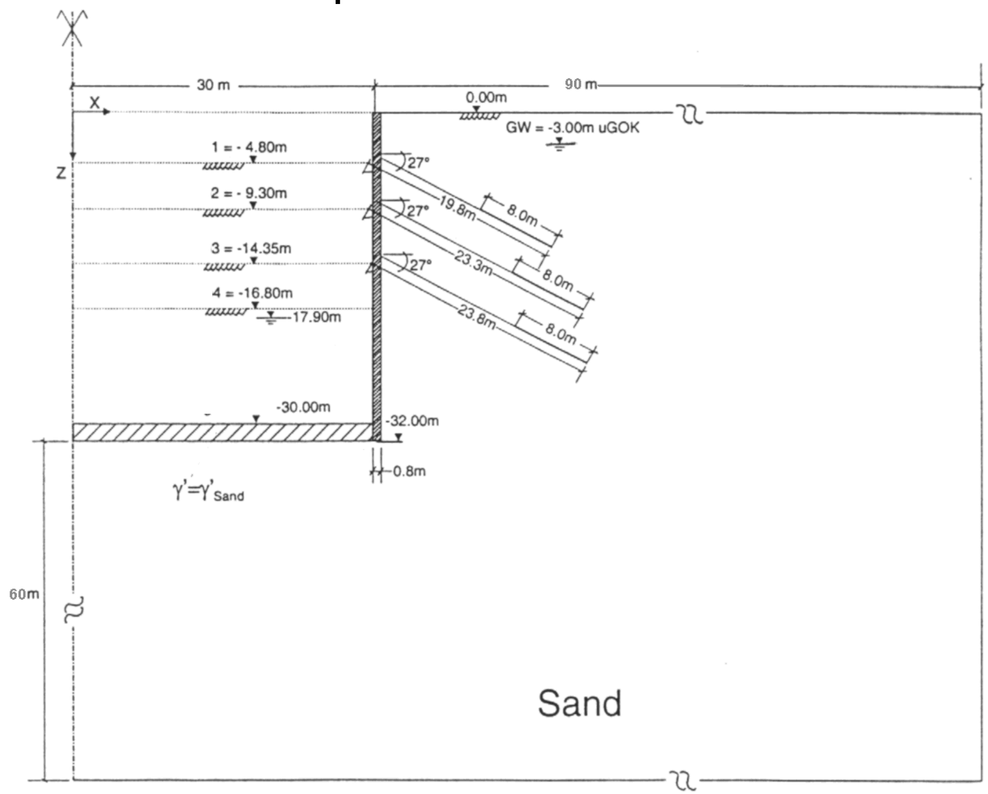


(a) HS-SmallStrain vs HS-Std

(b) HS-SmallStrain vs Mohr-Coulomb

Practical applications – Excavation in Berlin sand

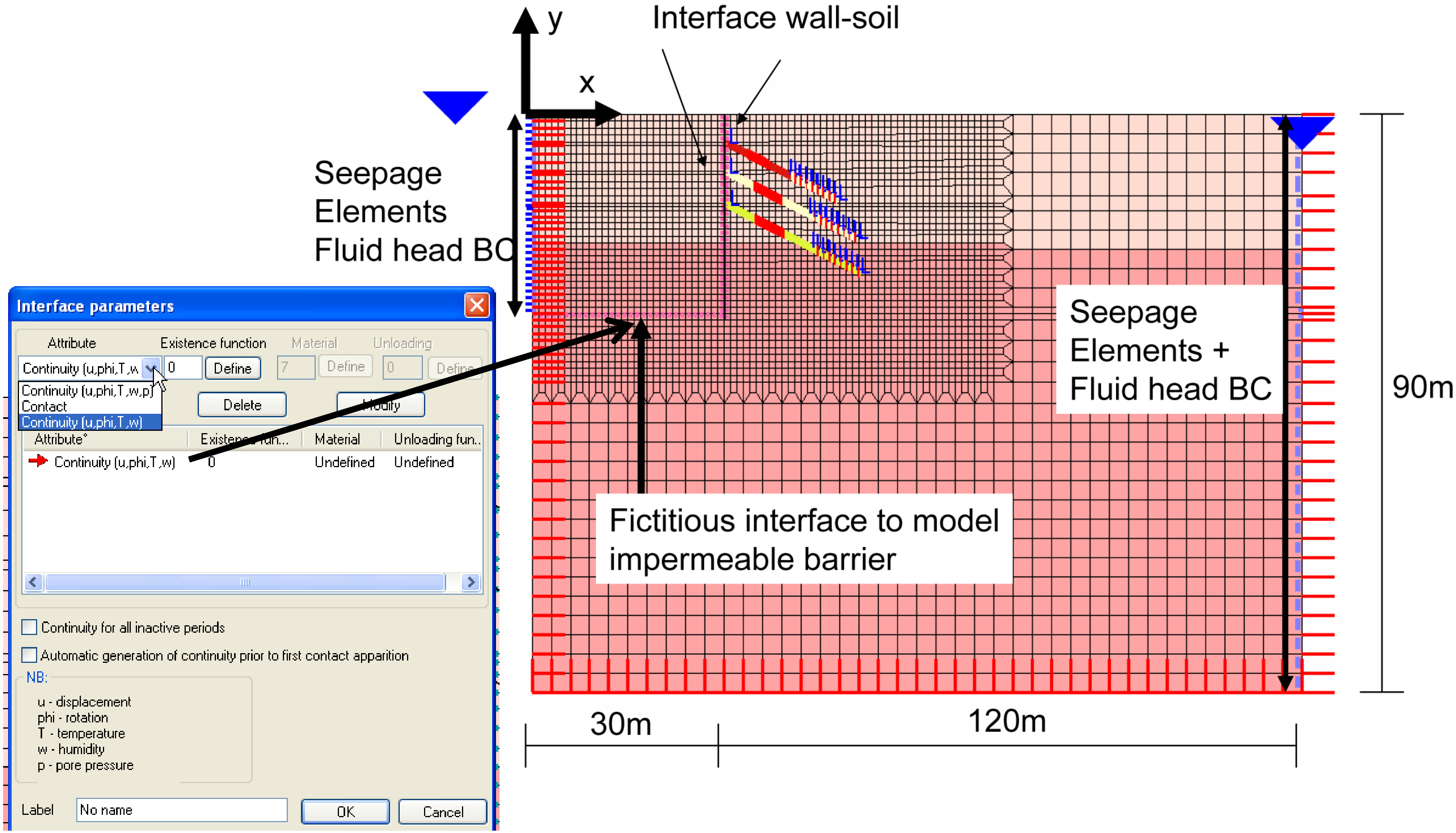
Engineering draft and the sequence of excavation



Input files:

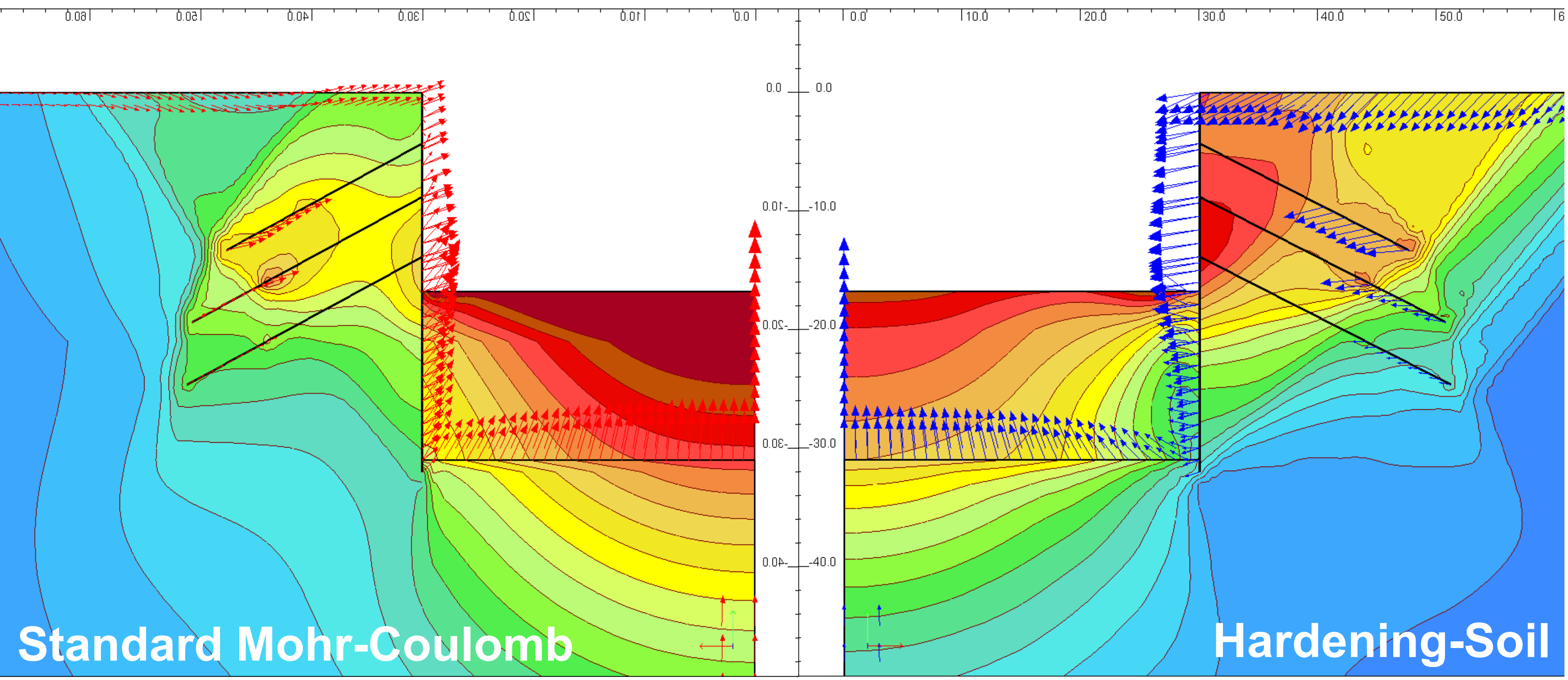
```
HS-small-Exc-Berlin-Sand-2phase.inp
HS-std-Exc-Berlin-Sand-2phase.inp
MC-Exc-Berlin-Sand-2phase.inp
```

Practical applications – Excavation in Berlin sand



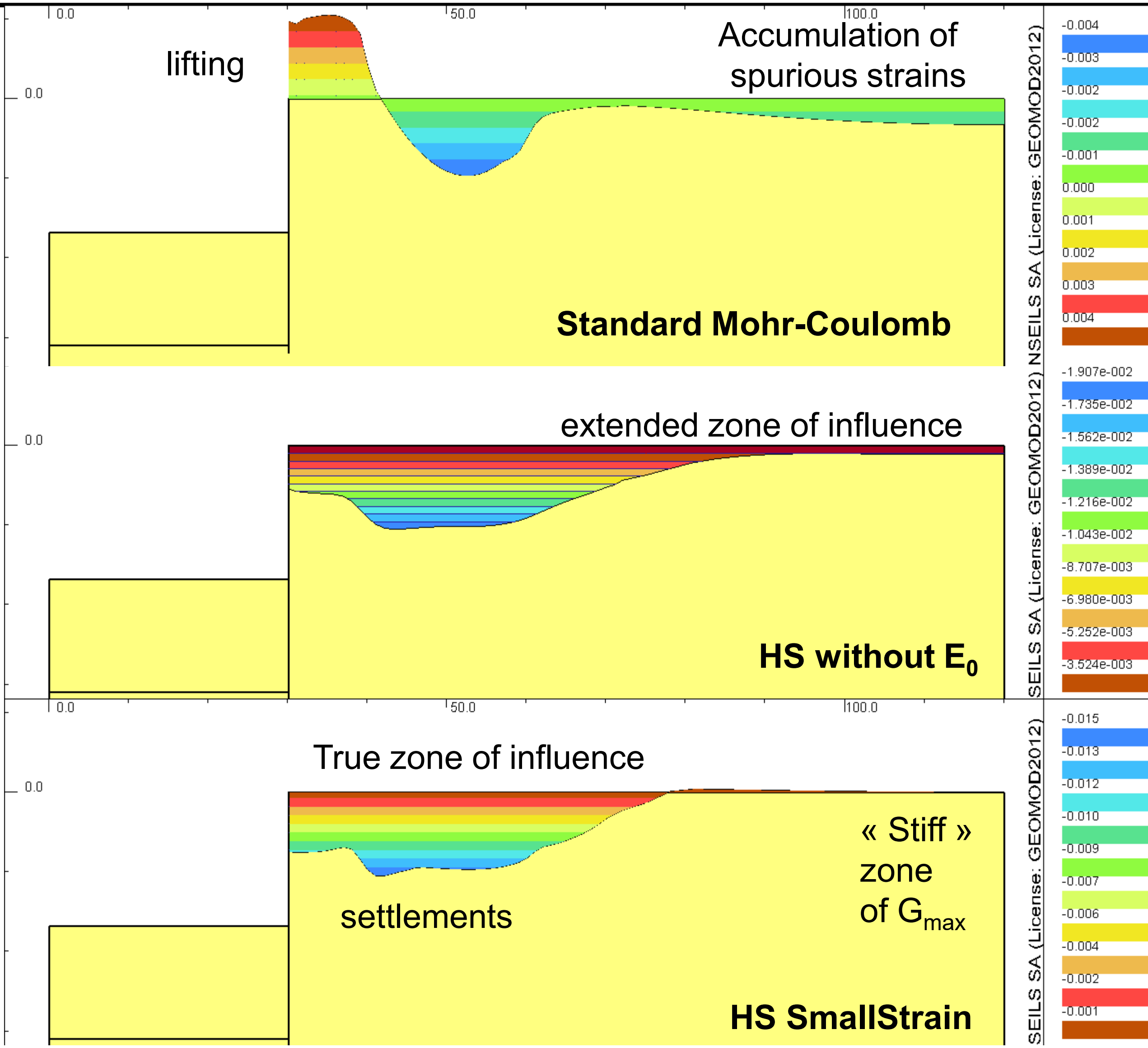
Practical applications – Excavation in Berlin sand

Berlin sand excavation



Absolute displacements

Practical applications – Excavation in Berlin sand

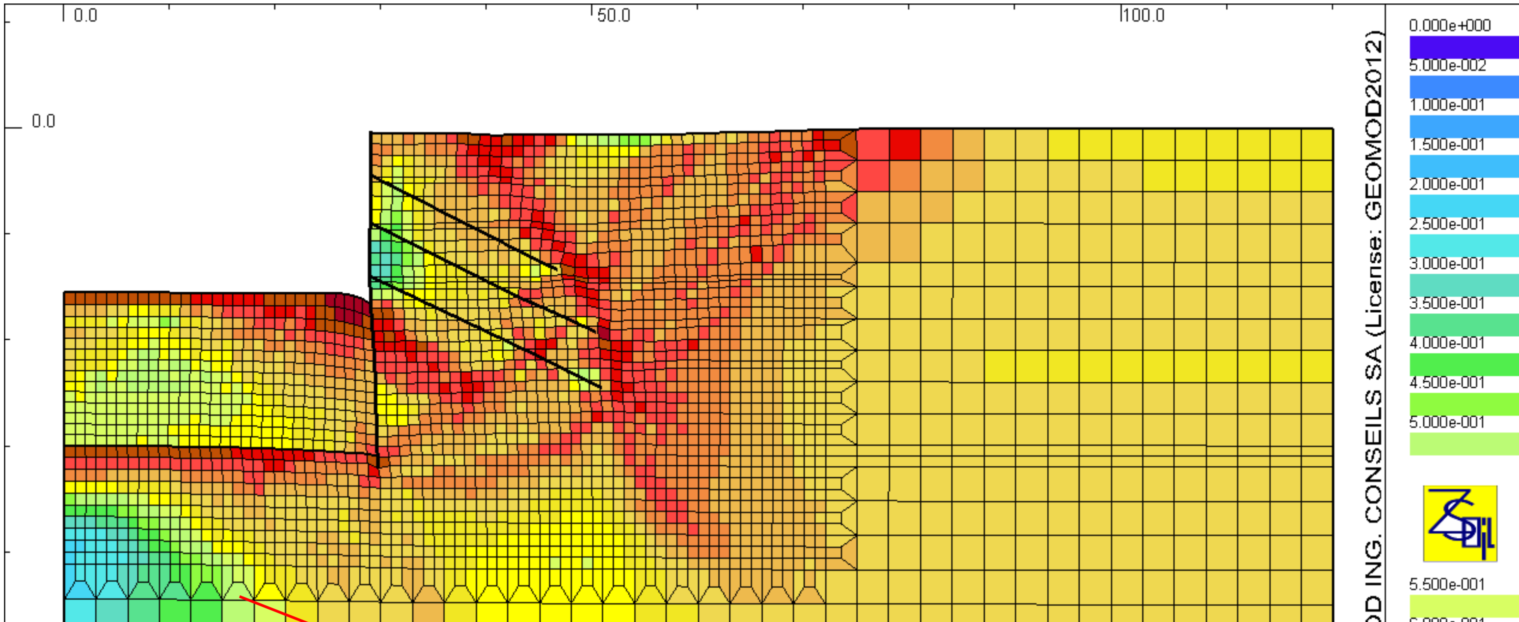


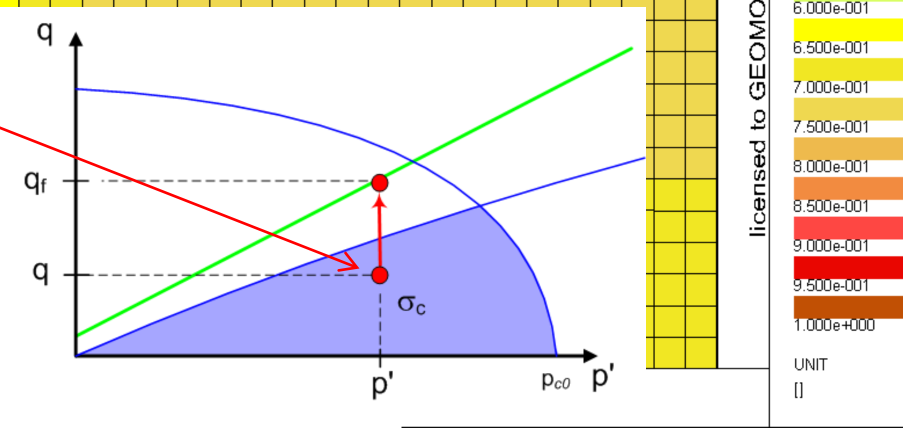
Vertical displacements at soil surface

NB. HS SmallStrain makes it possible to reduce the extension of the FE mesh

Practical applications – Excavation in Berlin sand

Meaning of stress level in HS model





Displayed stress levels are computed as:

$SL = q / q_f$

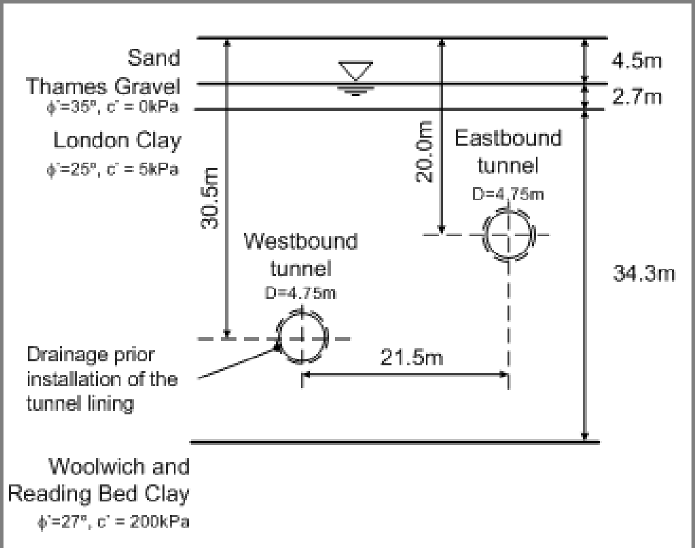
q – current deviatoric stress at given p'

q_f – failure stress corresponding to p'

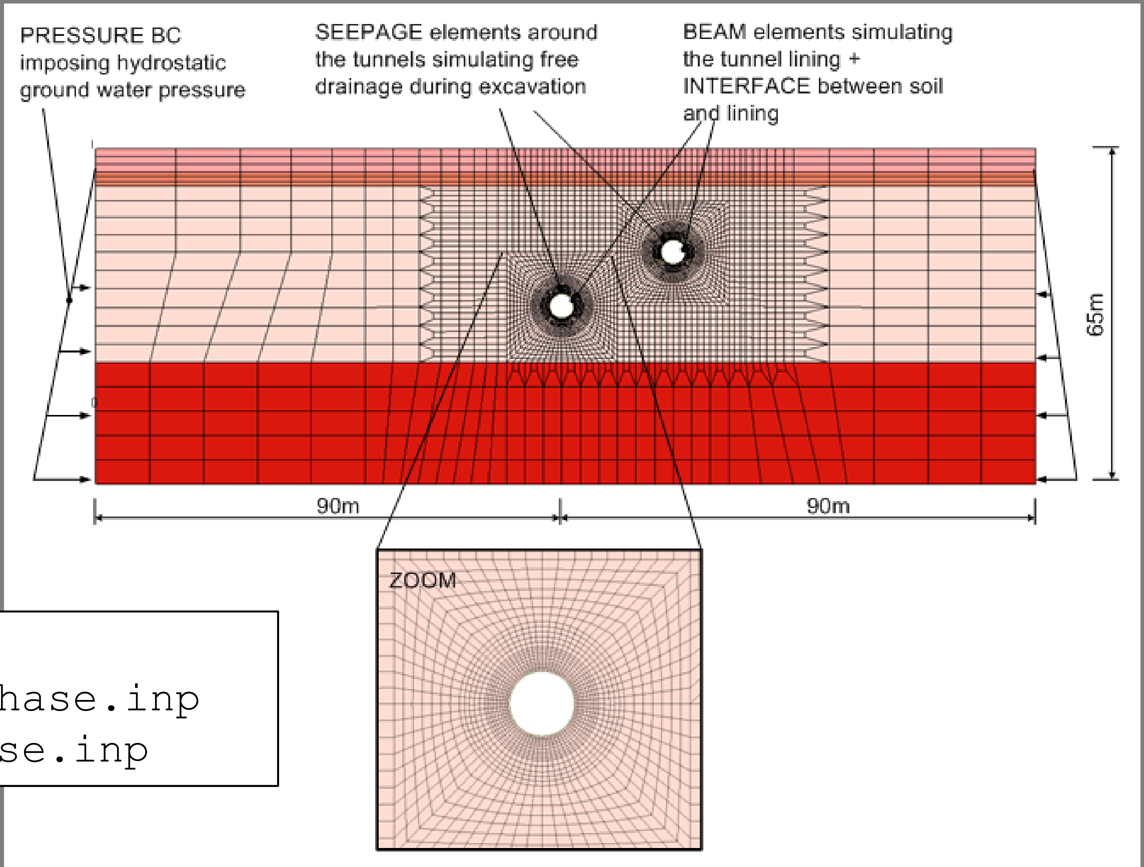
Practical applications – Tunnel excavation

London Clay - tunnel excavation (Addenbrooke et al., 1997)

Problem statement



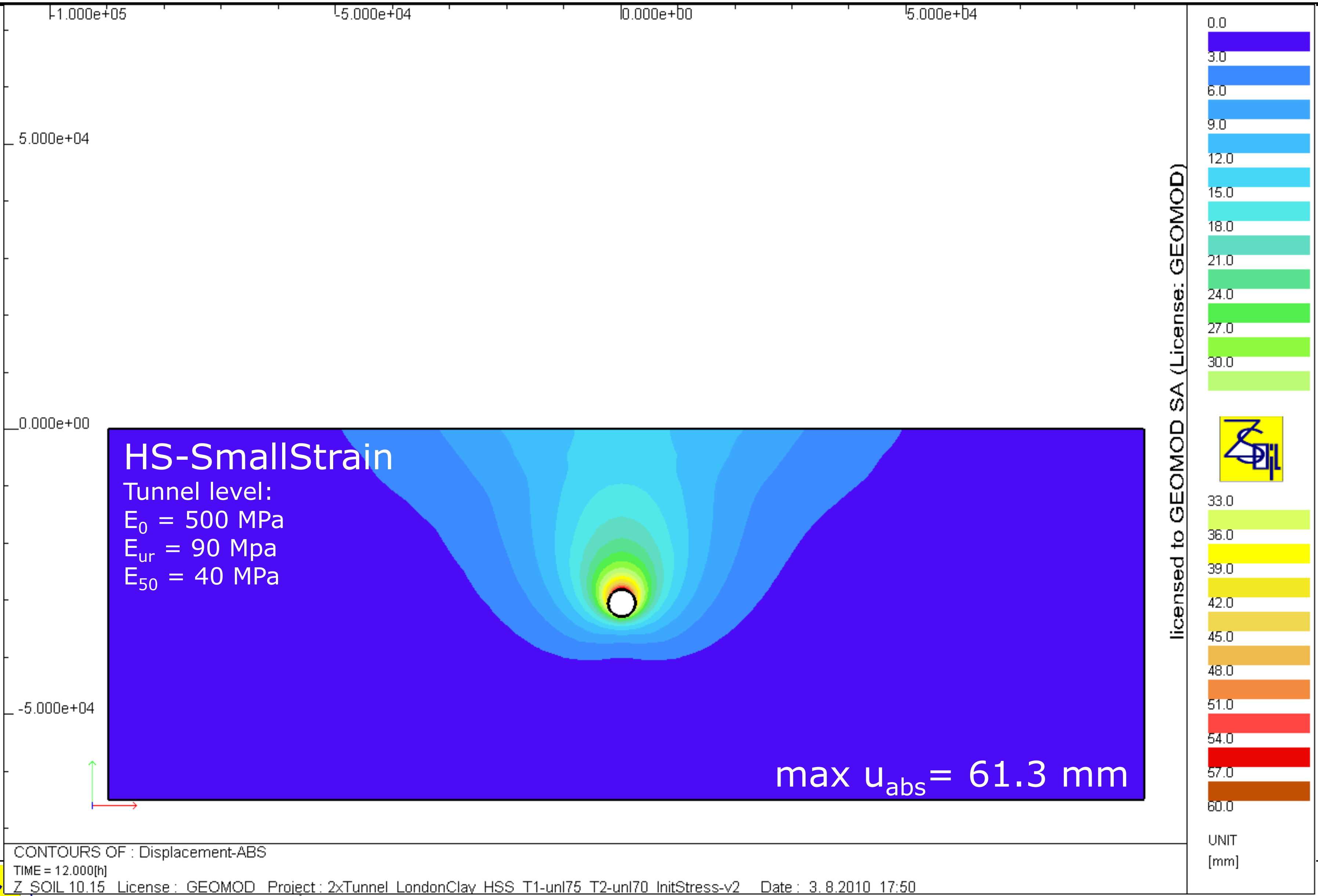
FE mesh



Input files:

```
HS-small-Exc-London-Clay-2phase.inp
HS-std-Exc-London-Clay-2phase.inp
```

Practical applications – Tunnel excavation



Practical applications – Tunnel excavation