Dynamics in Z_Soil 2010. Current stage and future developments

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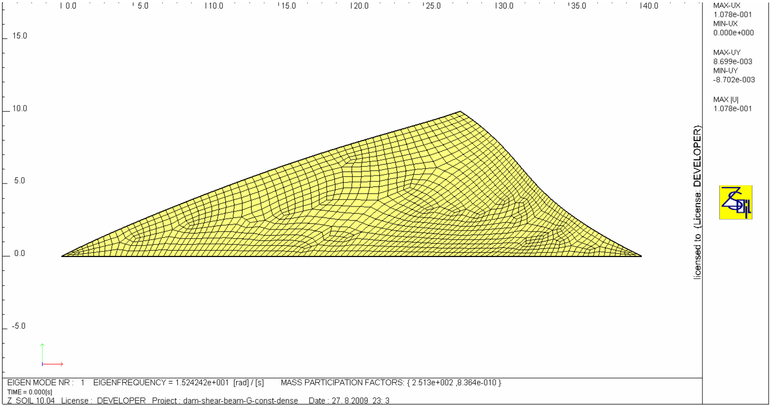
Dynamics in Z_Soil: history

- First developments appeared in Z_Soil v2007
- Time history analysis for structures
  - beam elements
  - truss elements
  - membrane/shell elements
- Time integration schemes
  - Implicit Newmark
  - Implicit HHT
- Seismic input
  - Defined through imposed BC: displacements/velocities/accelerations
  - Shaking table approach (relative motion with respect to the rigid base)
  - No baseline correction for given acceleration records

Dynamics in Z_Soil: main goals

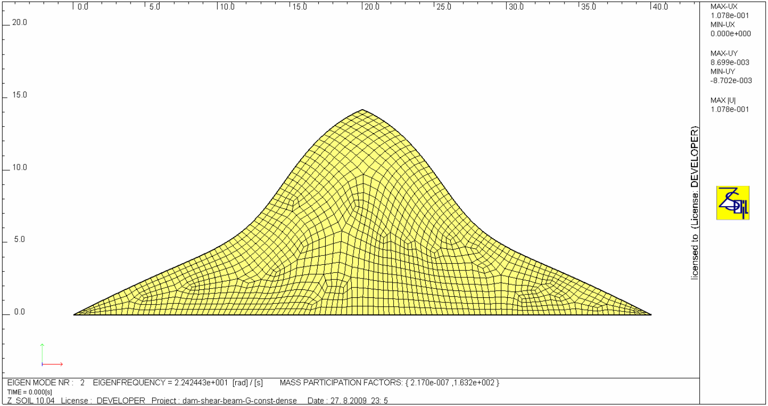
- Extraction of eigenmodes
  - Solving large eigenvalue problems with block Lanczos method
  - Till now only first mode was extracted by running pushover driver
- Solve dynamic problems for soil-structure interaction problems
  - Reduction of the computational model via DRM method
  - Main problem: boundary conditions (viscous boundaries...)
- Solve materially/geometrically nonlinear problems in general
  - Structures and subsoil may exhibit nonlinear behavior
- Single-phase or two-phase partially saturated media
- Solving dynamic problems in time domain
  - For continuum only HHT scheme
  - Explicit schemes are needed (mainly for 3D)

Eigenmodes and eigenfrequencies: example of an embankment



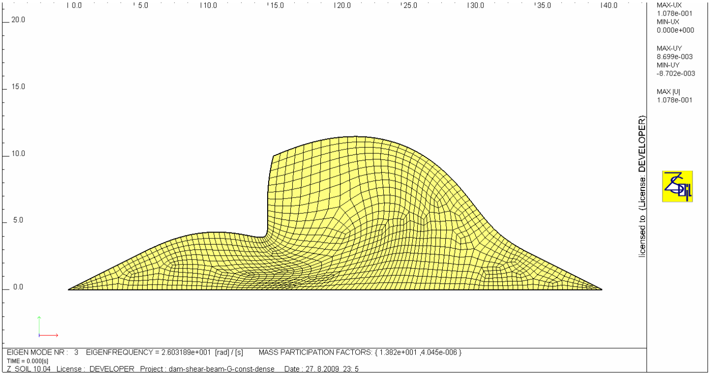
Mode: 1

Eigenmodes and eigenfrequencies: example of an embankment



Mode: 2

Eigenmodes and eigenfrequencies: example of an embankment



Viscous boundaries: single phase media

- Goal: cancel reflected waves from artificial boundaries
- Lysmer type viscous dampers are used

$$\boldsymbol{\sigma} = -\left\{ \frac{1}{c_p} \left(\lambda_s + 2\mu_s \right) \mathbf{n}\mathbf{n}^T + \frac{\mu_s}{c_s} \left(\mathbf{t_1}\mathbf{t_1}^T + \mathbf{t_2}\mathbf{t_2}^T \right) \right\} \mathbf{v}^s \quad (1)$$

$$c_s = \sqrt{\frac{\mu_s}{\rho}} \quad (2)$$

$$c_p = \sqrt{\frac{\lambda + 2\ \mu_s}{\rho}} \quad (3)$$

Remark: For two-phase media problem is definitely more complicated (paraxial boundary of 0- or 1-st order approximation will be used (after Modaressi) ???)

Viscous boundaries: single phase media

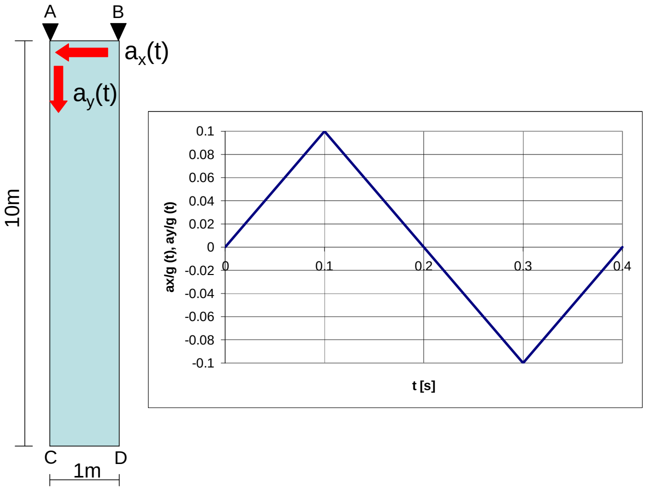


Figure: Soil layer subject to excitation applied to the boundary A-B

Remark: Viscous dashpots are created along C-D boundary

Viscous boundaries: single phase media

Material	Model	Data group	Properties	Unit	Value
1	Elastic	Elastic	E	[kN/m^2]	100000
		Poisson ratio	ν	[-]	0.25
		Density	γ	[kN/m^3]	9.80665
			ρ	[kg/m^3]	1000

- $G = 40000$ kPa, $\lambda = 40000$ kPa
- Shear wave velocity $c_s = 200$ m/s
- Dilatational wave velocity $c_p = 346.41$ m/s
- The horizontal velocity at points C,D should vanish after time $t = 0.4 + \dfrac{10}{200} = 0.45\text{s}$
- The vertical velocity after time $t = 0.4 + \dfrac{10}{346.41} = 0.429\text{s}$

Viscous boundaries: single phase media

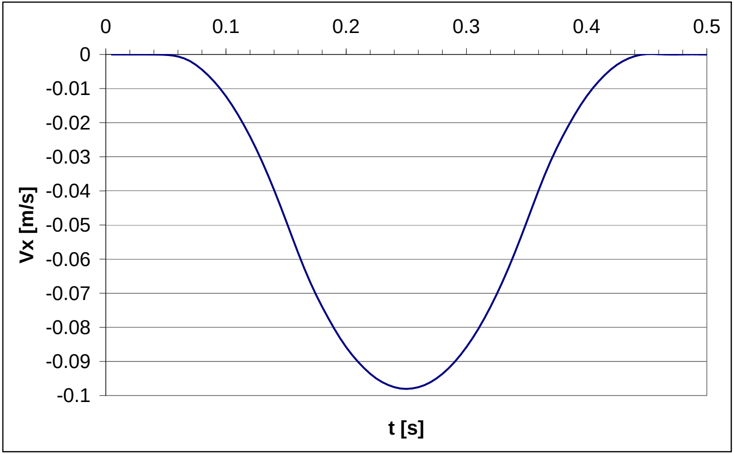


Figure: Solid velocity $v_x(t)$ (zoom) at point C

Viscous boundaries: single phase media

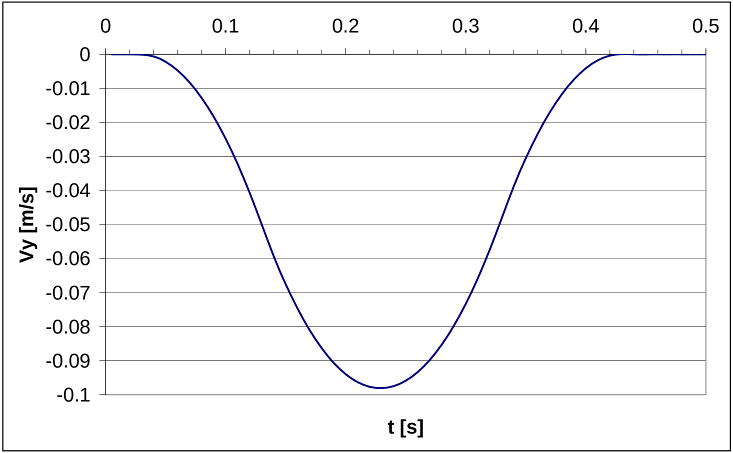
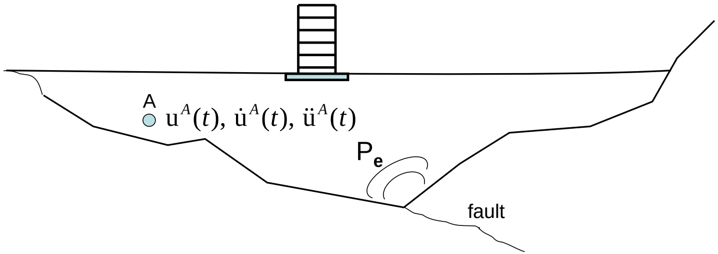


Figure: Solid velocity $v_y(t)$ (zoom) at point C

Domain reduction method (DRM)

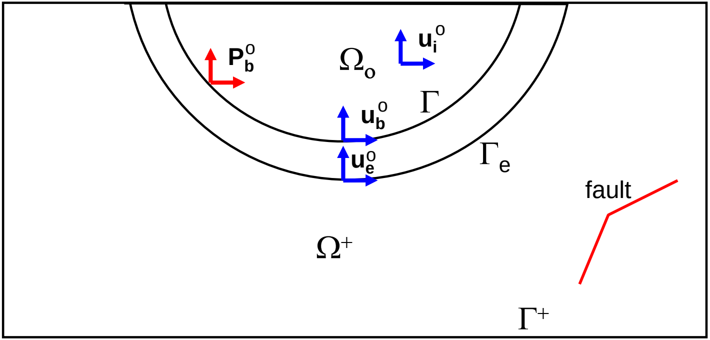
- The main goal: analyze computational model that concerns the structure and only a small adjacent part of subsoil
- The Domain Reduction Method (DRM) was proposed by J. Bielak et al. (2001)
- This way the size of the problem to be solved is substantially reduced

DRM: general idea

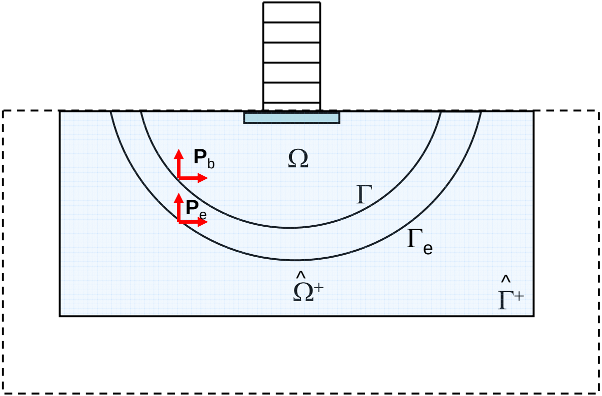


Full model of subsoil and structure, and source of the loading $\mathbf{P}_e(t)$

1. At any point displacements, velocities and accelerations induced by $\mathbf{P}_e(t)$ are denoted by $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, $\ddot{\mathbf{u}}(t)$
2. This model with a large subsoil zone and source of load $\mathbf{P}_e(t)$ is decomposed into two models:
   - background model
   - reduced model



1. In the background model the structure is removed and free field motion is analyzed
2. Displacements, velocities and accelerations induced by $\mathbf{P}_e(t)$ are denoted by $\mathbf{u}^0(t)$, $\dot{\mathbf{u}}^0(t)$, $\ddot{\mathbf{u}}^0(t)$



1. $\hat{\Gamma}^+$ is a boundary where viscous damping elements are to be put to cancel wave reflections
2. Displacement decomposition in the exterior domain: $\mathbf{u}_e = \mathbf{u}_e^0 + \mathbf{w}_e$

DRM: governing equations

Equations of motion after partitioning of the whole domain into Ω and Ω^+

$$\begin{bmatrix} \mathbf{M}^{\Omega}_{ii} & \mathbf{M}^{\Omega}_{ib} \\ \mathbf{M}^{\Omega}_{bi} & \mathbf{M}^{\Omega}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^{\Omega}_{ii} & \mathbf{K}^{\Omega}_{ib} \\ \mathbf{K}^{\Omega}_{bi} & \mathbf{K}^{\Omega}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{P_b} \end{Bmatrix} \tag{4}$$

$$\begin{bmatrix} \mathbf{M}^{\Omega^+}_{bb} & \mathbf{M}^{\Omega^+}_{be} \\ \mathbf{M}^{\Omega^+}_{eb} & \mathbf{M}^{\Omega^+}_{ee} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_e \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^{\Omega^+}_{bb} & \mathbf{K}^{\Omega^+}_{be} \\ \mathbf{K}^{\Omega^+}_{eb} & \mathbf{K}^{\Omega^+}_{ee} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_b \\ \mathbf{u}_e \end{Bmatrix} = \begin{Bmatrix} -\mathbf{P}_b \\ \mathbf{P}_e \end{Bmatrix} \tag{5}$$

The two above sets of equations can be written in the global form as follows:

$$\begin{bmatrix} \mathbf{M}^{\Omega}_{ii} & \mathbf{M}^{\Omega}_{ib} & \mathbf{0} \\ \mathbf{M}^{\Omega}_{bi} & \mathbf{M}^{\Omega}_{bb} + \mathbf{M}^{\Omega^+}_{bb} & \mathbf{M}^{\Omega^+}_{be} \\ \mathbf{0} & \mathbf{M}^{\Omega^+}_{eb} & \mathbf{M}^{\Omega^+}_{ee} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_e \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^{\Omega}_{ii} & \mathbf{K}^{\Omega}_{ib} & \mathbf{0} \\ \mathbf{K}^{\Omega}_{bi} & \mathbf{K}^{\Omega}_{bb} + \mathbf{K}^{\Omega^+}_{bb} & \mathbf{K}^{\Omega^+}_{be} \\ \mathbf{0} & \mathbf{K}^{\Omega^+}_{eb} & \mathbf{K}^{\Omega^+}_{ee} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \\ \mathbf{u}_e \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{P_e} \end{Bmatrix} \tag{6}$$

Let us decompose displacement vector in the exterior domain $\mathbf{u}_e$ into free field displacement $\mathbf{u}_e^0$ and residual one $\mathbf{w}_e$

$$\mathbf{u}_e = \mathbf{u}_e^0 + \mathbf{w}_e \tag{7}$$

Substituting eq.(7) into eq.(6) modifies eq.(6) to the following form:

$$\begin{bmatrix} \mathbf{M}_{ii}^{\Omega} & \mathbf{M}_{ib}^{\Omega} & \mathbf{0} \\ \mathbf{M}_{bi}^{\Omega} & \mathbf{M}_{bb}^{\Omega} + \mathbf{M}_{bb}^{\Omega^+} & \mathbf{M}_{be}^{\Omega^+} \\ \mathbf{0} & \mathbf{M}_{eb}^{\Omega^+} & \mathbf{M}_{ee}^{\Omega^+} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{w}}_e \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ii}^{\Omega} & \mathbf{K}_{ib}^{\Omega} & \mathbf{0} \\ \mathbf{K}_{bi}^{\Omega} & \mathbf{K}_{bb}^{\Omega} + \mathbf{K}_{bb}^{\Omega^+} & \mathbf{K}_{be}^{\Omega^+} \\ \mathbf{0} & \mathbf{K}_{eb}^{\Omega^+} & \mathbf{K}_{ee}^{\Omega^+} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \\ \mathbf{w}_e \end{Bmatrix} =$$

$$\begin{Bmatrix} \mathbf{0} \\ -\mathbf{M}_{be}^{\Omega^+} \ddot{\mathbf{u}}_e^0 - \mathbf{K}_{be}^{\Omega^+} \mathbf{u}_e^0 \\ \mathbf{P_e} - \mathbf{M}_{ee}^{\Omega^+} \ddot{\mathbf{u}}_e^0 - \mathbf{K}_{ee}^{\Omega^+} \mathbf{u}_e^0 \end{Bmatrix} \tag{8}$$

The $\mathbf{P}_e$ term can now be derived from eq.(5) assuming that it is solved for a background model

$$\mathbf{P}_e = \mathbf{M}_{eb}^{\Omega^+}\ddot{\mathbf{u}}_b^0 + \mathbf{M}_{ee}^{\Omega^+}\ddot{\mathbf{u}}_e^0 + \mathbf{K}_{eb}^{\Omega^+}\mathbf{u}_b^0 + \mathbf{K}_{ee}^{\Omega^+}\mathbf{u}_e^0 \tag{9}$$

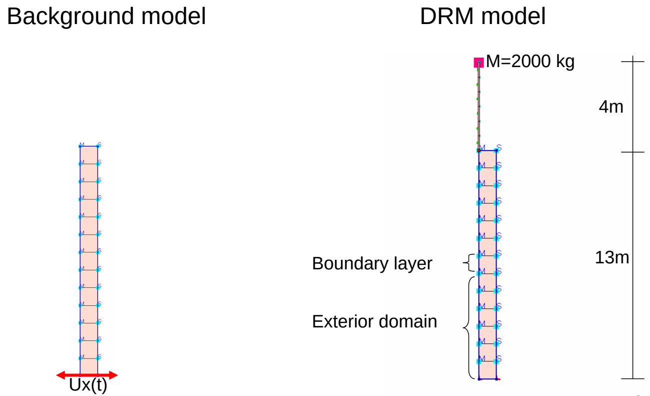
By substituting the above $\mathbf{P}_e$ term to the eq.(8) the following form of the right hand side term is obtained:

$$\mathbf{P}^{eff} = \left\{ \begin{array}{c} \mathbf{0} \\ -\mathbf{M}_{be}^{\Omega^+}\ \ddot{\mathbf{u}}_e^0 - \mathbf{K}_{be}^{\Omega^+}\ \mathbf{u}_e^0 \\ \mathbf{M}_{eb}^{\Omega^+}\ \ddot{\mathbf{u}}_b^0 + \mathbf{K}_{eb}^{\Omega^+}\ \mathbf{u}_b^0 \end{array} \right\} \tag{10}$$

Remarks

1. **For lumped mass matrix terms $\mathbf{M}_{be}^{\Omega^+}\ \ddot{\mathbf{u}}_e^0$, $\mathbf{M}_{eb}^{\Omega^+}\ \ddot{\mathbf{u}}_b^0$ dissappear**
2. **$\mathbf{P}_{eff}$ is computed only in elements of the boundary layer**

Example 1: 1D soil-column interaction

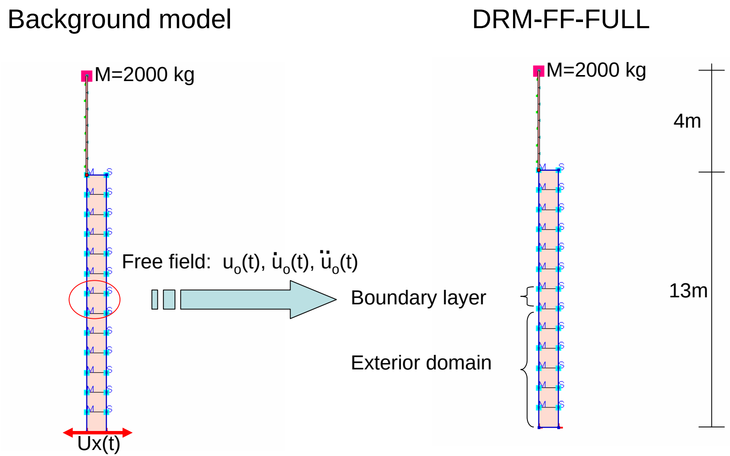


- Continuum material data: $E = 48600$ kPa, $\nu = 0.35$, $\rho = 1800$ kg/m^3
- Beam material data: $E = 20000000$ kPa, $\nu = 0.2$, $\rho = 0$ kg/m^3
- Added mass on top of the beam: $M = 2000$ kg
- Imposed base displacements: $u_x(t) = sin(2\ \pi\ t) \times 1m$ in time range $t = 0..6$s
- Periodic BC are enforced for both walls of the layer (for all displacement components)
- Time stepping: $\Delta t = 0.01$s
- Time duration: $t_{end} = 0..12$s

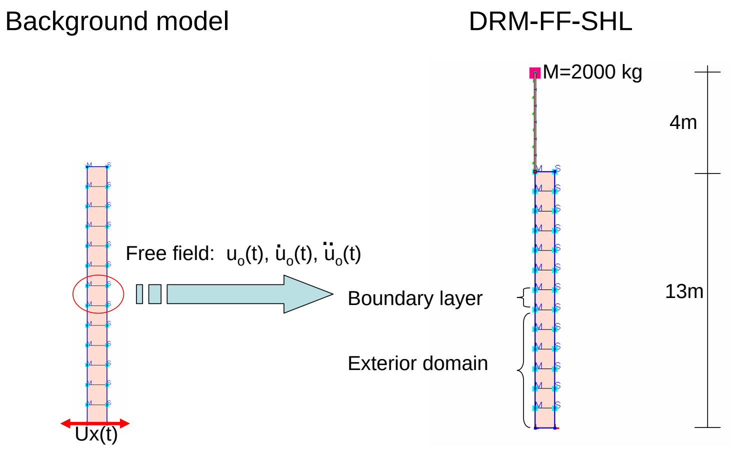
Example 1: sequence of analyzes

- Solve background model (SHL) $\Longrightarrow$ free field motion
- Solve full model subsoil-beam (FULL) $\Longrightarrow$ reference solution
- Solve DRM model (DRM-FF-FULL) with free field motion from full model
- Solve DRM model (DRM-FF-SHL) with free field motion from background model

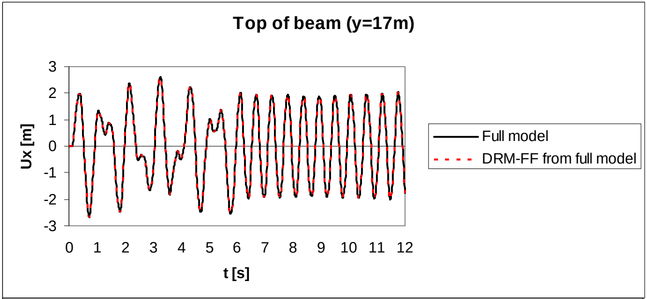
Example 1: DRM with free field from full model (DRM-FF-FULL)

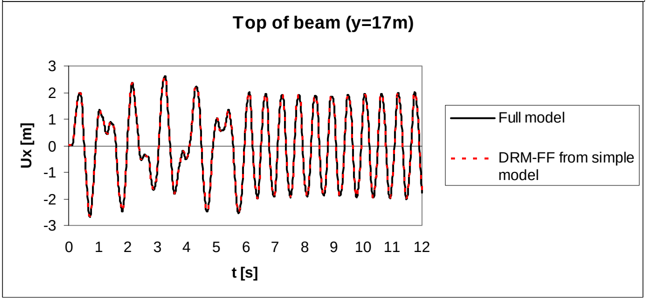


Example 1:
DRM with free field from backgr. model (DRM-FF-FULL)

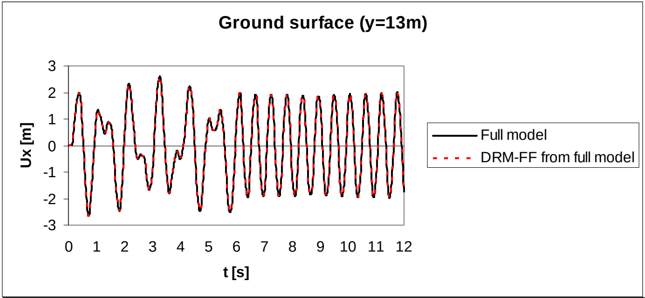


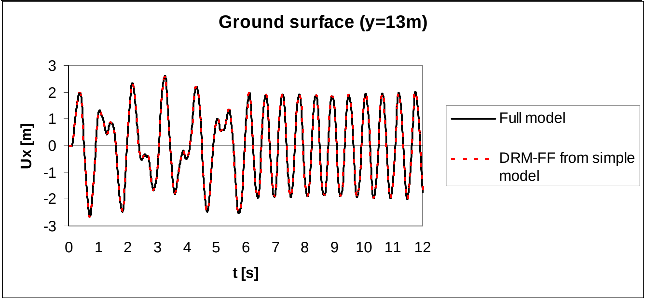
Example 1: Results



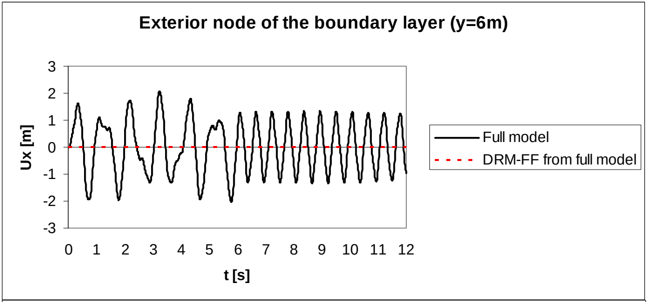


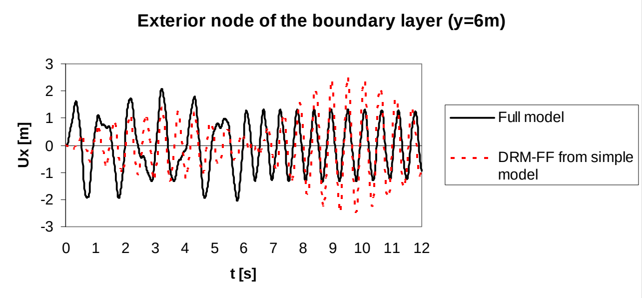
Example 1: Results



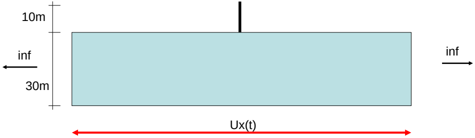


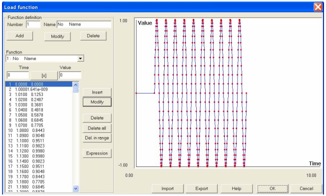
Example 1: Results





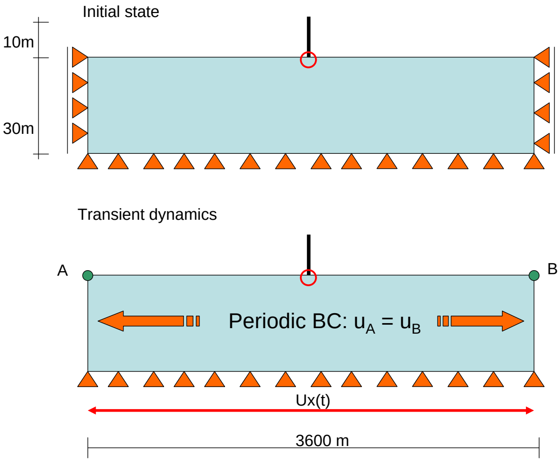
Example 2: 2D soil-column interaction





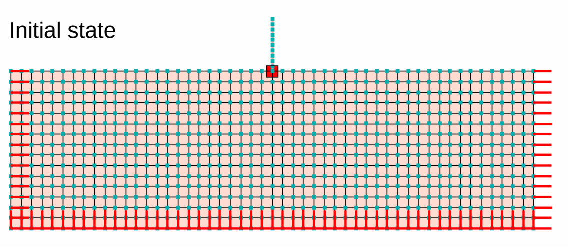
- Continuum material data: $E = 192000$ kPa, $\nu = 0.2$, $\rho = 2000$ kg/m^3
- Beam material data: $E = 20000000$ kPa, $\nu = 0.2$, $\rho = 2500$ kg/m^3, $A = 1$m^2
- Imposed base displacements: $u_x(t) = sin(2\,\pi\,(t-1)) \times 1m$ in time range $t = 1..7$s
- Periodic BC are enforced for both walls of the layer (for all displacement components)
- Time stepping: $\Delta t = 0.01$s
- Time duration: $t = 1..6$s
- Element size: $h^e = 2$m

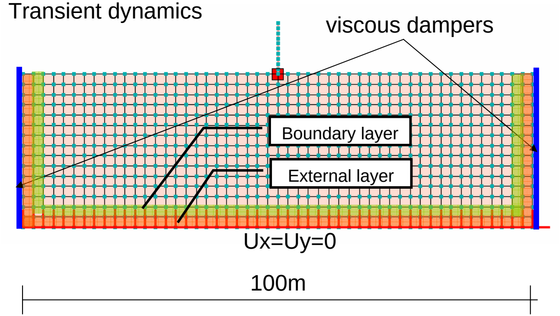
Example 2: Full model

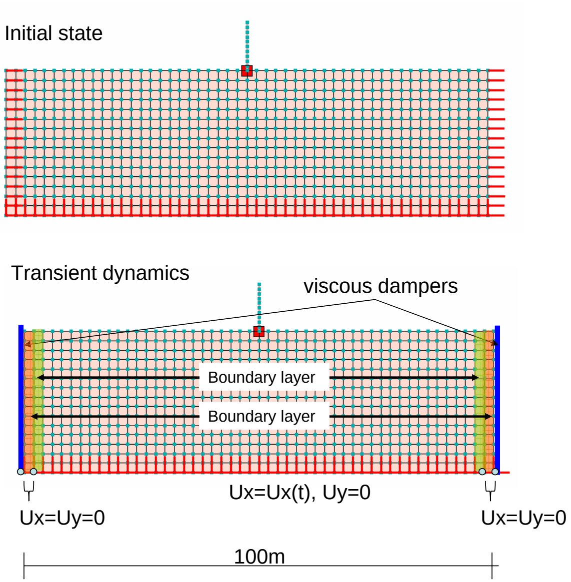


- 27000 Q4 elements are used + 10 beam elements

Example 2: DRM model 1



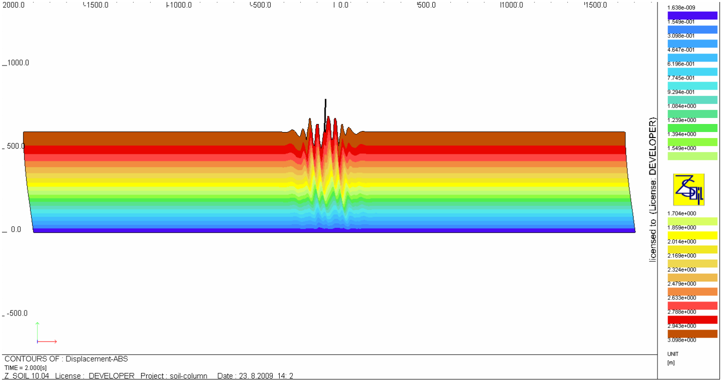



Initial state
Transient dynamics
viscous dampers
Boundary layer
Boundary layer
Ux=Ux(t), Uy=0
Ux=Uy=0
Ux=Uy=0
100m

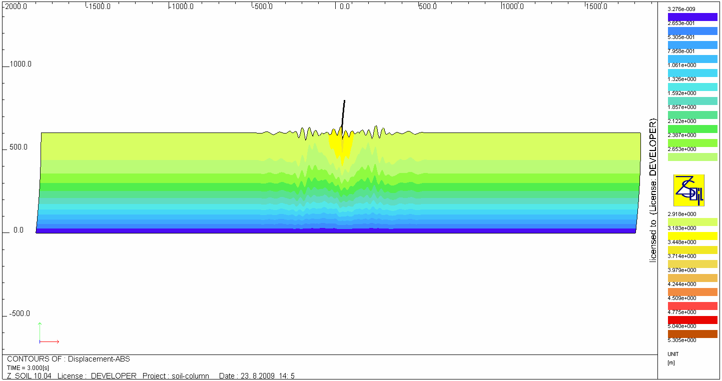
Example 2: sequence of analyzes

- Solve full model subsoil-beam (FULL) $\Longrightarrow$ reference solution (3600m long, 1800$\times$ 15=27000 Q4 elements+10 beam elements)
- Solve background model (SHL) $\Longrightarrow$ free field motion (3600m long, 1$\times$ 15=15 Q4 elements)
- Solve DRM-1 model (DRM-1-FF-FULL) with free field motion from full model
- Solve DRM-1 model (DRM-1-FF-SHL) with free field motion from background model
- Solve DRM-2 model (DRM-2-FF-FULL) with free field motion from background model
- Solve DRM-2 model (DRM-2-FF-SHL) with free field motion from background model

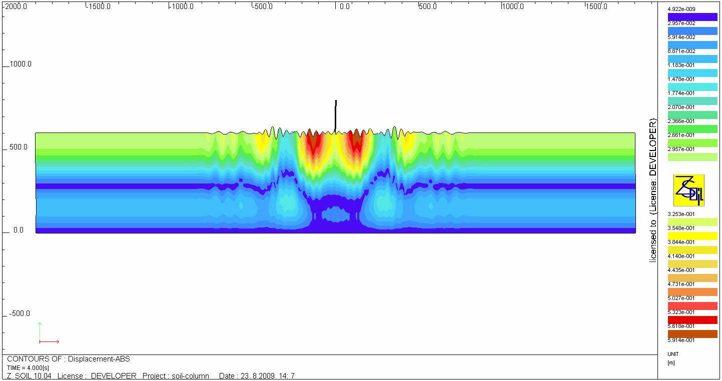
Results for full model: deformation after 1s of excitation



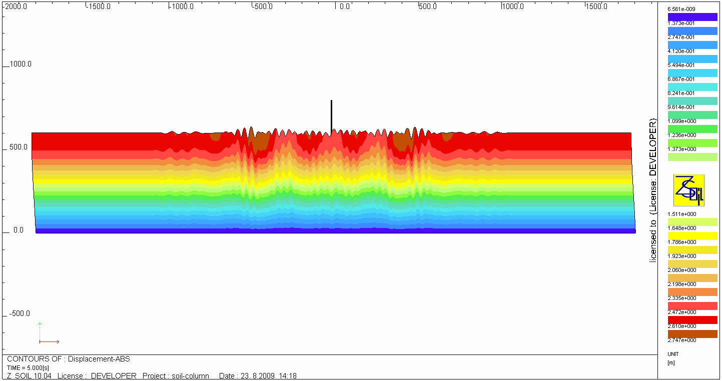
Results for full model: deformation after 2s of excitation



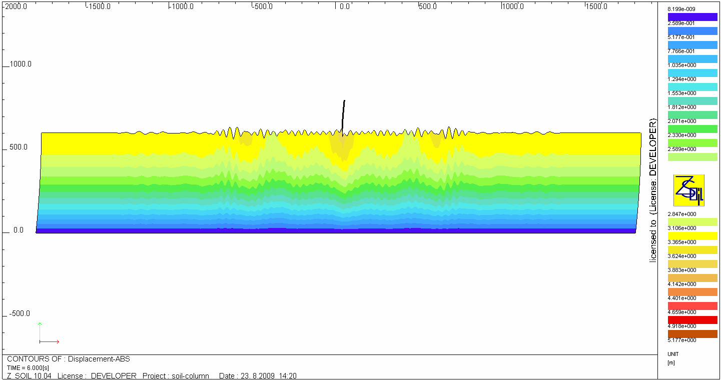
Results for full model: deformation after 3s of excitation



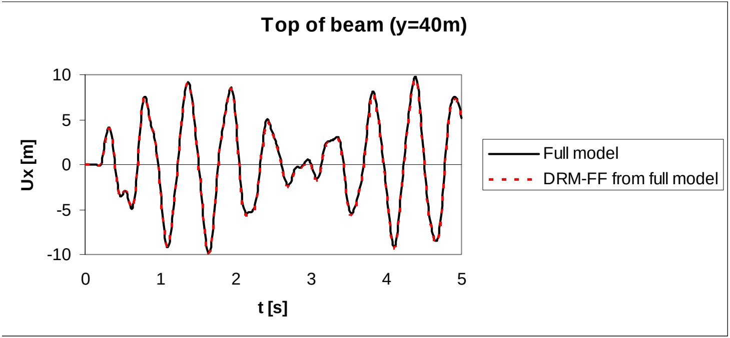
Results for full model: deformation after 4s of excitation



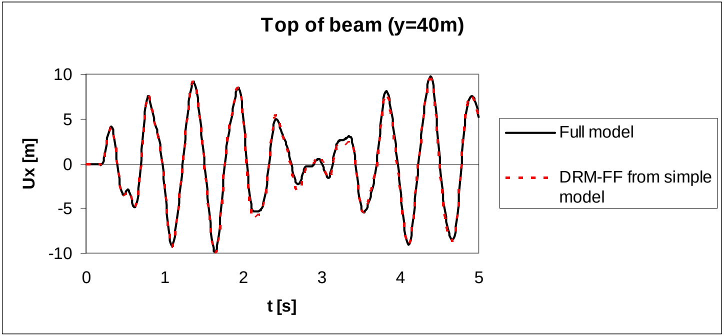
Results for full model: deformation after 5s of excitation



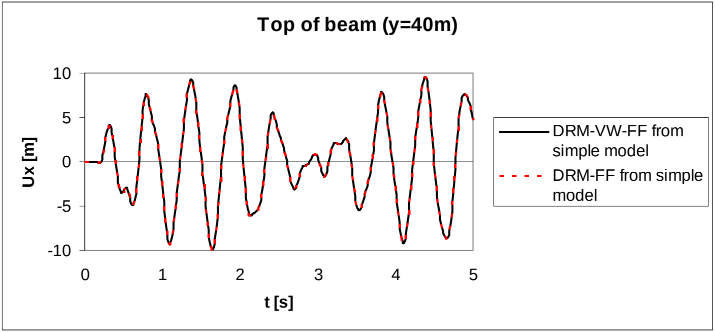
Results for DRM model-1 with free field from full model



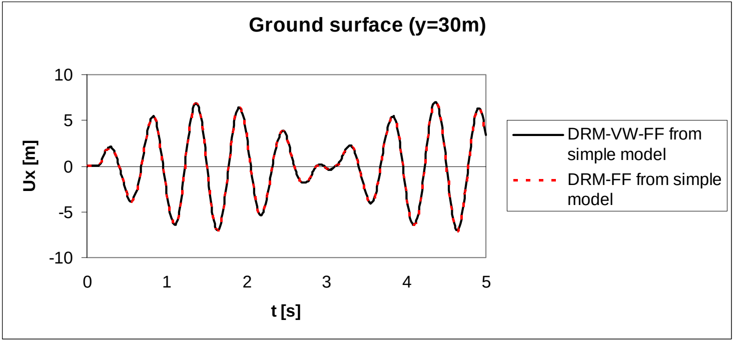
Results for DRM model-1 with free field from simple model



Comparizon of DRM model-1 and DRM model-2 (both with free field from simple model)

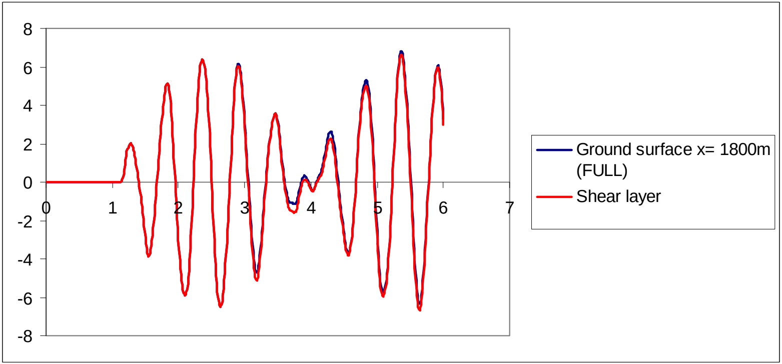


Comparizon of DRM model-1 and DRM model-2 (both with free field from simple model)

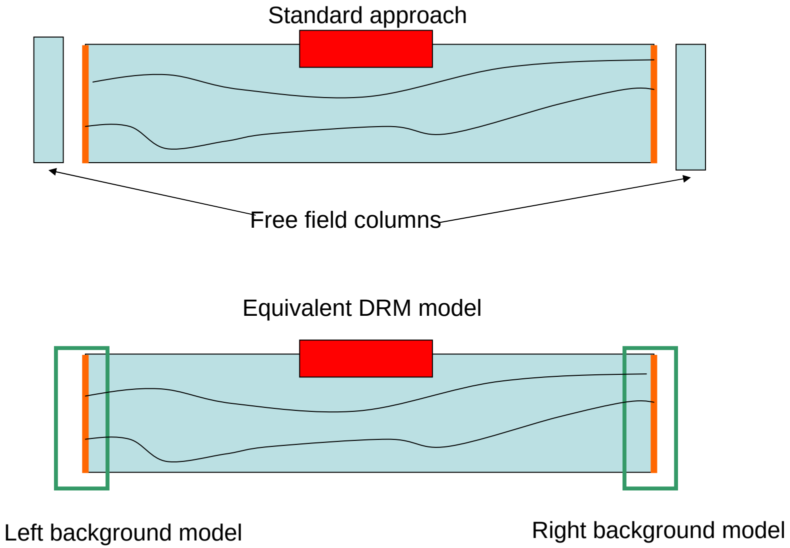


Source of the mismatch

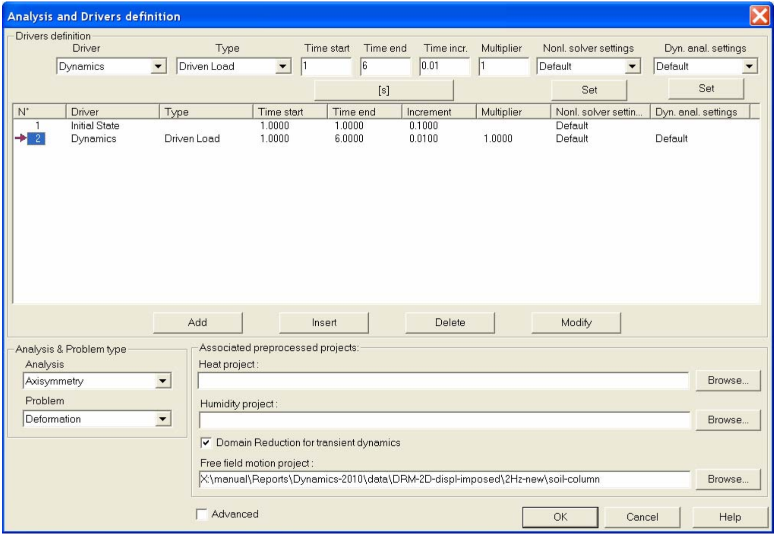
We check periodicity condition in the full model for two neighbouring nodes at the top at $x = -1800$ m



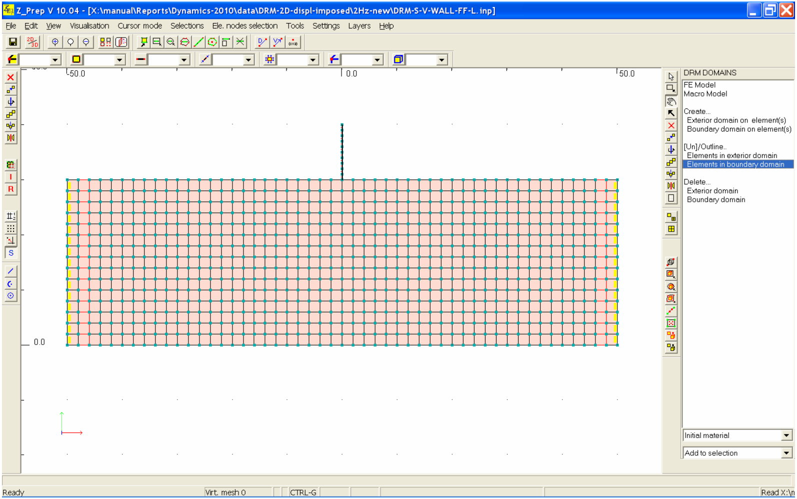
Different DRM setting



DRM user interface



DRM user interface



Future developments

- Partially/fully saturated two-phase media
  - For high frequency problems $u-U$, $u-W$ formulations must be used (6 DOFs per node)
  - For earthquakes $u-p$ formulation is sufficient (up to 10 Hz) (4 DOFs per node)
- Constitutive models for loose and dense deposits
  - Zienkiewicz-Pastor generalized plasticity model (liquefaction, cyclic mobility phenomena)
  - Densification model (after Zienkiewicz, Shiomi, Sawicki, Swidziñski) (liquefaction phenomenon)