Dynamic capabilities of ZSoil 2012

- Transient linear/nonlinear time history analysis for
  1. structures
  2. single-phase soil-structure interaction
  3. two-phase fully/partially saturated soil-structure interaction

- Eigenvalues and eigenvectors detection via Lanczos method
Geodynamic tools

- Implicit time integration schemes (Newmark, HHT\(_{\alpha}\)) for single phase and two-phase problems
- Viscous boundaries for single and two-phase media
- Domain reduction for single and two-phase media
- Modeling of damping
  - Rayleigh damping
  - Hysteretic damping: creep, dissipative stress-strain laws
  - Numerical damping (via time integration schemes)
- Mass discretization (lumped/consistent)
- Dynamic analyses run in the absolute or relative format
- Boundary conditions
  - Imposed displacements, velocities and accelerations
- Correcting acceleration time histories via Butterworth filtering and baseline correction procedures
- Application of HSs model for hard soils and Densification model for loose deposits
Nonlinear Newmark scheme

- Nonlinear balance equation
  \[ Ma_{n+1} + Cv_{n+1} + N(d_{n+1}) = F_{n+1} \]

- Integration scheme in time
  \[
  d_{n+1} = d_n + \Delta t v_n + \frac{\Delta t^2}{2} [(1 - 2\beta) a_n + 2\beta a_{n+1}]
  \\
  v_{n+1} = v_n + \Delta t [(1 - \gamma) a_n + \gamma a_{n+1}]
  \]

- Integration coefficients for zero numerical damping:
  \[ \beta = 0.25, \gamma = 0.5 \]
Nonlinear HHT\(\alpha\) scheme

- Linear form of the algorithm is written:

\[
M a_{n+1} + (1 + \alpha) C v_{n+1} - \alpha C v_n + (1 + \alpha) K d_{n+1} - \alpha K d_n = F_{n+\alpha}
\]

\[
[*]_{n+\alpha} = (1 + \alpha)[*]_{n+1} - \alpha[*]_n
\]

\[
F_{n+\alpha} = (1 + \alpha) F_{n+1} - \alpha F_n
\]

- Nonlinear version

\[
M a_{n+1} + C v_{n+\alpha} + N(d_{n+a}) = F_{n+\alpha}
\]

- Integration scheme

\[
d_{n+1} = d_n + \Delta t v_n + \frac{\Delta t^2}{2} [(1 - 2\beta) a_n + 2\beta a_{n+1}]
\]

\[
v_{n+1} = v_n + \Delta t [(1 - \gamma) a_n + \gamma a_{n+1}]
\]

- Hilber proposes: \(-0.3 < \alpha < 0\), \(\gamma = \frac{(1 - 2\alpha)}{2}\), \(\beta = \frac{(1 - \alpha)^2}{4}\)

\(\alpha = 0\) corresponds to Newmark’s algorithm
Goal: analyze computational model that concerns the structure and only a small adjacent part of subsoil

Single and two-phase formulations are supported
This model with a large subsoil zone and source of load $P_e(t)$ is decomposed into two models:

- background model
- reduced model
1. In the background model the structure is removed and free field motion is analyzed.

2. Displacements, velocities and accelerations induced by $P_e(t)$ are denoted by $u^0(t)$, $\dot{u}^0(t)$, $\ddot{u}^0(t)$.
1. Viscous dampers are added to $\tilde{\Gamma}^+$ to cancel wave reflections
2. Displ. decomposition in the exterior domain: $u_e = u^0_e + w_e$
Frame dimensions: \( L = 20\text{m}, \ H = 12\text{m} \) (spacing 6m)

Columns: 0.6m × 0.8m

Horizontal beams: 0.4m × 0.75m

Distributed mass: 750 kg/m

Concrete: \( E = 30000000 \text{kPa}, \ \nu = 0.2 \)

Subsoil: \( E = 100000 \text{kPa}, \ \nu = 0.35, \ \rho = 2000 \text{kg/m}^3 \)

Subsoil depth: 30m

Excitation: 5 Hz harmonic imposed base displacements

Damping: \( \delta_{\text{con}} = 0.0/0.15, \ \delta_{\text{soil}} = 0.0/0.30 \)
To verify results a large model 3600m long was analyzed.

Periodic BC were assumed on left and right vertical walls.

In the considered case $v_s = 136.1 \text{ m/s}$, $v_R = 127.1 \text{ m/s}$ and $\lambda_R = 25.4 \text{ m}$.
Reduced model lengths for undamped vibrations $L_i$: 200m, 150m, 100m

$L_i$ for damped vibrations: 200m, 150m, 100m, 70m oraz 50m
Comparizon of $u_x$ time histories at point P for reference and 100m long DRM model.

![Graph showing comparison of $u_x$ time histories](image-url)
2D example: error in solution for undamped vibrations

200m

150m

100m

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Geodynamics in ZSoil 2012
DRM: 2D example: error in solution for damped vibrations

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Problem: Protection of the 4km long piped gas borehole and restrictions put on road construction technology
Pipe may be subject to excitation caused by road construction vibratory rollers

\[ F|\sin(\theta t)| \]
Protection of the pipe

Mata wibroizolacyjna
Nasyp

4m

MIN 5m

Podłoże

Ściany żelbetowe 40cm

12m

Ściany szczelinowe 60cm

Zarurowany odwiert

Mata wibroizolacyjna
Nasyp
DRM is a robust tool for this example

The free field (background model) is solved only once as an axisymmetric problem

The reduced model including soil, pipes and protection walls is generated once as a 3D

Analysis of influence of the distance $L$ can be made by shifting the 3D model along x-axis
Setting Rayleigh damping parameters to match attenuation curve

\[ y = 166.71e^{-0.0417x} \]

\[ R^2 = 0.9279 \]
3D mesh: 270 000 DOFS
1 layer of elements in exterior domain and 1 layer of elements in the interior domain
Diaphragm wall and piped gas borehole
Computations on 64-bit Windows Vista Professional system

With the limited output computation time was 6h on Pentium Quad Core Extreme with 8 Gb of RAM

With the full output for each time step computation time was about 12h

The animation will be shown for force 237 kN with 35 Hz frequency

Show animation

video
Balance of the momentum: \( \frac{\partial \sigma_{ij}}{\partial x_j} + \gamma b_i = \rho \ddot{u}_i \)

Balance of the mass for the fluid: \( S \dot{\varepsilon}_{kk} - v_{i,i}^F - c \dot{p} = 0 \)

Effective stress principle by Bishop: \( \sigma_{ij} = \sigma'_{ij} + S \delta_{ij} p \)

Darcy law: \( v_i^F = k_{ij} k_r(S) \left( \frac{1}{\gamma_F} p_{,j} + b_j - \frac{1}{g} \ddot{u}_j \right) \)

Simplified \( u - p \) formulation is adopted (valid for earthquake analysis)
Integration schemes in time

Expressions for solid displacements and velocities

\[ u_{n+1} = u_n + \dot{u}_n \Delta t + \frac{\Delta t^2}{2} [(1 - 2\beta) \ddot{u}_n + 2\beta \ddot{u}_{n+1}] \]
\[ \dot{u}_{n+1} = \dot{u}_n + \Delta t [(1 - \gamma) \ddot{u}_n + \gamma \ddot{u}_{n+1}] \]

Expression for pore pressure:

\[ p_{n+1} = p_n + (1 - \theta) \dot{p}_n \Delta t + \theta \Delta t \dot{p}_{n+1} \]

In the HHT scheme

\[ p_{n+\alpha} = (1 + \alpha)p_{n+1} - \alpha p_n \]
\[ \dot{p}_{n+\alpha} = (1 + \alpha)\dot{p}_{n+1} - \alpha \dot{p}_n \]
\[ u_{n+\alpha} = (1 + \alpha)u_{n+1} - \alpha u_n \]
\[ \dot{u}_{n+\alpha} = (1 + \alpha)\dot{u}_{n+1} - \alpha \dot{u}_n \]
\[ \ddot{u}_{n+\alpha} = (1 + \alpha)\ddot{u}_{n+1} - \alpha \ddot{u}_n \]
Two phase: matrix form of balance equations

- Balance of momentum (HHT scheme)

\[ M \ddot{u}_{n+1} + C \dot{u}_{n+\alpha} + F'_{INT}(u_{n+\alpha}) + C^F p_{n+\alpha} = F_{EXT n+\alpha} \]

- Balance of the mass for fluid phase written at time step \( n + \alpha \)

\[
\left( C^F \right)^T \dot{u}_{n+\alpha} - \frac{1}{\gamma_F^F} H^F p_{n+\alpha} + R^F \ddot{u}_{n+\alpha} - h^F - M^F \dot{p}_{n+\alpha} + Q^F = 0
\]

- After linearization of this system we solve for unknown \( \delta \ddot{u} \) and \( \delta \dot{p} \)
Dynamic consolidation of a soil layer benchmark

- To achieve steady state harmonic solution more than 20 cycles of the loading are applied
- Solution of this problem depends on some parameters $\Pi_1$ and $\Pi_2$

\[
\Pi_1 = \frac{2}{\beta \pi} \frac{k T_o}{g \hat{T}^2}
\]

\[
\Pi_2 = \pi^2 \left( \frac{\hat{T}}{T_o} \right)^2
\]

- Natural period of vibration $\hat{T}$ and dilatational wave velocity $v_c$ are defined as

\[
\hat{T} = \frac{2H}{v_c}
\]

\[
v_c = \sqrt{\frac{E_{oed} + K_F/n}{\rho}}
\]
The considered test cases

<table>
<thead>
<tr>
<th>No</th>
<th>$\Pi_1$</th>
<th>$\Pi_2$</th>
<th>$k$ [m/s]</th>
<th>$\omega$ [rad/s]</th>
<th>$\Delta t$</th>
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<td>31.62277660</td>
<td>0.00496729</td>
</tr>
</tbody>
</table>

Table: $k$ and $\omega$ values
\[ \Pi_1 = 0.1 \text{ and } \Pi_2 = 0.1 \]

\[ \Pi_1 = 0.1 \text{ and } \Pi_2 = 1.0 \]
\( \Pi_1 = 1.0 \) and \( \Pi_2 = 0.1 \)

\( \Pi_1 = 10 \) and \( \Pi_2 = 0.1 \)
Why do we need stabilization?

Nonstabilized solution video
Stabilized solution video
Viscous boundaries for single phase

- Lysmer dashpots are the simplest to cancel wave reflections outgoing from the domain (exact only in 1D)
- Damping force vector

\[ F^v = - \int_{\Gamma} N^T \sigma^s d\Gamma \]

- \( \sigma^s \) is a viscous stress defined as

\[ \sigma = - \left\{ \frac{1}{c_p} \left( \lambda_s + 2\mu_s \right) \mathbf{n}\mathbf{n}^T + \frac{\mu_s}{c_s} \left( \mathbf{t}_1\mathbf{t}_1^T + \mathbf{t}_2\mathbf{t}_2^T \right) \right\} \mathbf{v}^s \]

- The corresponding shear and dilatational wave velocities are denoted by

\[ c_s = \sqrt{\frac{G}{\rho}} \quad c_p = \sqrt{\frac{\lambda + 2G}{\rho}} \]

NB. The normalized normal and tangential vectors are denoted by \( \mathbf{n} \) and \( \mathbf{t}_1, \mathbf{t}_2 \)
Viscous boundaries: example

Viscous damper is added at C-D boundary

NB. Viscous dampers may inherit their properties from adjacent continuum elements (to be set at material level)

Material data:

- $E = 200000$
- $\nu = 0.25$
- $\gamma = 9.81$ [kN/m$^3$]
- $\rho = \frac{\gamma}{g} = 1000$ [kg/m$^3$]
- $c_s = \sqrt{\frac{G}{\rho}} = 200$ m/s
- $c_p = \sqrt{\frac{\lambda + 2G}{\rho}} = 346.41$ m/s
Viscous boundaries: example

- horizontal velocity at points C,D should vanish after time
  \[
  t = 0.4 + \frac{10}{200} = 0.45 \text{ s}
  \]
Viscous boundaries: example

- vertical velocity should vanish after time

\[ t = 0.4 + \frac{10}{346.41} = 0.429 \text{ s} \]
\[ \mathbf{F}_u^v = - \int_{\Gamma} \mathbf{N}^T \mathbf{s} \, d\Gamma \]

\[ \mathbf{F}_p^v = - \int_{\Gamma} \mathbf{N}^T \Phi \, d\Gamma \]

\[ \mathbf{s}^s = - \left\{ \rho \frac{c_p^2}{V_{p1}} (\lambda_s + 2\mu_s) \mathbf{n} \mathbf{n}^T + \rho \, c_s \left( \mathbf{t}_1 \mathbf{t}_1^T + \mathbf{t}_2 \mathbf{t}_2^T \right) \right\} \mathbf{v}^s + \mathbf{n} (S \rho - S_0 \rho_o) \]

\[ \Phi = k \left[ \rho \left( 1 - \frac{c_p^2}{V_{p1}^2} \right) - \rho^F \right] \mathbf{n}^T \mathbf{a}^s \]

The approximate first dilatational wave velocity for saturated medium is defined as

\[ V_{p1} = c_p \sqrt{ \left( 1 + \frac{Q}{\lambda + 2G} \right) } \]

\[ \frac{1}{Q} = n \frac{S}{K^F} + n \frac{dS}{dp} \]
Two-phase viscous boundaries: benchmark

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Solution for 300m model
Solution for 100m model without paraxial elements
Solution for 100m model with paraxial elements
Viscous boundaries in practice

How to handle transient dynamics preceded by an initial state or other static drivers?

(t=0)

Initial state

Relaxed fixities

Viscous dampers

Dynamic driver

\( \text{LTF} \)

\( t \)

\( a_x(t) \)

1

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Import earthquake records in 3 formats:
http://nsmp.wr.usgs.gov/data.html (US)
http://www.isesd.hi.is/ESD_Local/frameset.htm (EU)
free format (ti,ai)
Baseline correction and filtering seismic input

- Butterworth filtering (low-pass, high-pass, band-pass, band-stop)
- Baseline correction
For both corrected/uncorrected acceleration time history

- Export to excel Fourier spectra
- Export to excel elastic response spectra for different damping coefficients starting from $\xi = 0.05$ up to $\xi = 0.25$
HSs model for hard soils with strain dependent stiffness moduli

Densification model for loose deposits to describe liquefaction phenomenon (2 versions: by Zienkiewicz and by Sawicki)
Modeling liquefaction with DNS model

Soil layer subject to horizontal shaking (N-S El Centro scaled to 0.1g)