Constitutive aspects in time history analysis. ZSoil 2012

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What do we expect from constitutive models?

- **cohesionless soils**
  - stiffness stress/strain dependency
  - dissipation of energy for cyclic loads
  - for loose sands densification mechanism plays an important role (in the two-phase applications it may produce liquefaction phenomenon)
  - for dense sands cyclic mobility effect is observed

- **cohesive soils**
  - stiffness stress/strain dependency
  - dissipation of energy in cyclic range
  - pore pressure build-up/stabilization in two-phase applications

NB. Complex cyclic constitutive models for soils are still in the stage of research rather than practical use.
NB. For different densification ($D_r$) we can get a different behavior. For loose states we have a continuous soil tendency to densify and this leads to pore pressure build-up while for dense ones strong dilatancy effect may turn the effective stress path to follow critical state line.
Fuji river sand

Once the stress path is near the transition (instability) line we can observe characteristic "butterfly loops"
Constitutive models that can reproduce soil behavior for cyclic loadings...to some extent.

1. **HSs** model $\rightarrow$ overconsolidated cohesive soils and dense sands
2. **Densification** model by Zienkiewicz or Sawicki $\rightarrow$ loose sands
1. Only small strain version can be used
2. It has built in 2 plastic shear mechanisms for domain of medium/large plastic strains
3. The external plastic surface is the M-C one
4. The internal shear plastic surface allows to describe nonlinear triaxial $q - \varepsilon_1$ curves
5. Within the internal shear mechanism a hypoelastic Hardin-Drnevich law is used
6. The loading/unloading stress reversal points are traced through strain history matrix $H_{ij}$ as in the classical HSs model
HSs features: $G - \varepsilon$ reduction curve

Triaxial tests of dense Hostun sand

$\sigma_3 = 100$ kPa  \quad  \sigma_3 = 300
The small strain effects are visible for $\gamma = 0 \div \gamma_c$ ($G = G_{ur}$)

For $\gamma > \gamma_c$ Hardin-Drnevich law does not reduce stiffness anymore
- All stress cycles of same amplitude will produce plastic straining and variation of position of the internal shear mechanism only in the first cycle.
- Densification/dilation is produced only by shear plastic mechanisms.
- But.....cyclic unloading/reloading will produce hysteresis loops via Hardin-Drnevich law (!)
Notion of damping coefficient $D$

Fig. 3 Definition of the secant shear modulus, $G$, and damping ratio, $D$

plot from paper by Ishihara. Evaluation of soil properties... (Dungar et al. 1982)
Hardin-Drneveich law

- **Primary loading:**
  \[ G_s = G_o \frac{1}{1 + a \frac{\gamma_{hist}}{\gamma_{0.7}}} \quad a = 0.385 \]

- **For unloading/reloading:**
  \[ G_s = G_o \frac{1}{1 + a \frac{\gamma_{hist}}{2 \gamma_{0.7}}} \]

![Graph showing the relationship between Gamma and Tau]
Note that for \( \frac{G_{ur}}{G_o} < 0.3 \) we will obtain a different damping values from HSs because of plasticity.
HSs in undrained limit: how do we get excess pore pressure?

- Fluid mass balance equation: $\dot{\varepsilon}_v + \text{div} (v^F) - c \dot{p} = 0$
- In the undrained limit $\rightarrow \dot{\varepsilon}_v = 0$ and $\text{div} (v^F) = 0$
- To satisfy incompressibility constraint we may use penalty form: $\dot{\varepsilon}_v - c \dot{p} = 0$ where $c$ is a large value ($c = \frac{n}{K_F}$)
- Note that: $\dot{\varepsilon}_v = \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^p = 0$
- Hence for $\dot{\varepsilon}_v^p = 0 \rightarrow \dot{\sigma}_m = 0$ because $\dot{\varepsilon}_v^e = 0$
- Conclusion $\rightarrow$ to generate excess pore water pressure as observed in the experiment we need to generate plastic volumetric strains
HSs in undrained limit: how do we get excess pore pressure?

Stress cycle in undrained triaxial test

Current position of internal shear mechanism

M-C

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HSs: Rowe’s dilatancy law

\[ \sin \psi_m = \frac{\sin \phi_m - \sin \phi_{cs}}{1 - \sin \phi_m \sin \phi_{cs}} \]

\[ \sin \phi_m = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2c \cot \phi} = \frac{\sigma_1 - \sigma_3}{-(\sigma_1 + \sigma_3) + 2c \cot \phi} \]

\[ \phi_m \rightarrow \text{mobilized friction angle} \]

NB. Contractancy for \( \sin \phi_m = 0 \div \sin \phi_{cs} \) is the source of plastic volumetric strain \( \rightarrow \) pore pressure build up

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Sophisticated models that describe progressive volumetric strain accumulation during cyclic loading are too complex.

Zienkiewicz (in 1978) proposed to use a simplified approach:

\[ \Delta \sigma = D^e (\Delta \varepsilon - \Delta \varepsilon^p - \Delta \varepsilon^{acc}) \]

The accumulated volumetric strain \( \Delta \varepsilon^{acc} \) is defined as a separate mechanism.

The plastic strain is produced by M-C mechanism with zero dilatancy.

This way all volumetric irreversible changes are generated by mechanism of accumulation.

The two versions of this model are considered:

1. original one by Zienkiewicz (1978)
2. second by Sawicki (1986)
Accumulation mechanism in Zienkiewicz’s model:

- Some norm of deviatoric strain: \( d\xi = \sqrt{de_{ij} \, de_{ij}} \)
- \( \Delta \varepsilon_{v}^{acc} = - \frac{A}{1 + B \, \kappa} \, d\kappa \)
- \( d\kappa = \exp(C \, \eta) \, d\xi : \) this formula amplifies accumulation for stress paths that are near the plastic state
- \( \eta = \frac{\sqrt{J_2}}{p_o} \)
- For \( C = 0 \) we can integrate accumulation law:
  \[ \varepsilon_{v}^{acc} = - \frac{A}{B} \ln (1 + B\xi) \]
- Mapping of accumulated strain on spatial directions
  \[ \begin{align*}
  \Delta \varepsilon_x^{acc} &= \frac{1}{3} \Delta \varepsilon_v^{acc} \\
  \Delta \varepsilon_y^{acc} &= \frac{1}{3} \Delta \varepsilon_v^{acc} \\
  \Delta \varepsilon_z^{acc} &= \frac{1}{3} \Delta \varepsilon_v^{acc}
  \end{align*} \]
DNS model by Zienkiewicz (1978): simple shear test

\[ \gamma_o \sin(\omega t) \]

\[ d\xi = \frac{\sqrt{2}}{2} \parallel d\gamma_{xy} \parallel \]

\[ \xi(N) = 2\sqrt{2} \ N \ \gamma_o \]

\[ \varepsilon^{acc}_v(N) = -\frac{A}{B} \ln(1 + 2\sqrt{2} \ N \ \gamma_o) \]
Sawicki shown that for simple shear test there exists so-called common compaction curve

\[ \varepsilon_{\text{acc}}^v = -e_o \ C_1 \ ln (1 + C_2 \ z) \quad \text{and} \quad z = \frac{1}{4} \gamma_o^2 \]
How to convert Sawicki’s model to Zienkiewicz’s one

- **Sawicki:** \( \varepsilon^\text{acc}_v(N) = -e_o \ C_1 \ ln \left(1 + C_2 \frac{1}{4} \gamma_o^2\right) \)

- **Zienkiewicz:** \( \varepsilon^\text{acc}_v(N) = -\frac{A}{B} \ ln(1 + 2\sqrt{2} \ N \ \gamma_o) \)

1. Conclusion: Zienkiewicz’s model does not reproduce common compaction curve

2. Question: is there a way to correct Zienkiewicz’s model to reproduce common compaction curve? **YES**
Verification of Sawicki’s model in time domain

\[ p_o = 20\text{kPa} \quad \quad \quad p_o = 50\text{kPa} \]

NB. here z axis is scaled by factor $10^6$
Verification of Sawicki’s model in time domain

\[ p_0 = 150\text{kPa} \quad \text{and} \quad p_0 = 200\text{kPa} \]

NB. here z axis is scaled by factor \(10^6\)
Soil layer subject to horizontal shaking (N-S El Centro scaled to 0.1g)