New damage-plastic model for concrete in ZSoil 2016

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Models for concrete in ZSoil till 2015

- Menetrey-Willam model for continuum 2D/3D and shells (plane stress formulation)
- Damage type user defined 1D models for beam fibers (unloading takes place always towards the origin of $\sigma - \varepsilon$ diagram)
- Major drawbacks of Menetrey-Willam model
  1. Elasto-plastic model $\rightarrow$ no stiffness degradation
  2. Cyclic behavior is not represented
  3. Lack of nonlinearity in compression
  4. Proportional loss of tensile/compressive strength during plastic straining

Motivation for the new development

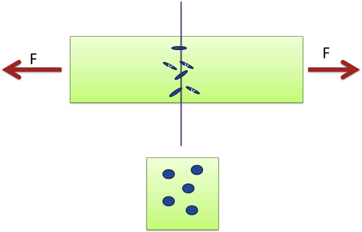
- To arm ZSoil with a model that could be used to solve fire problems according to the EC2 provisions
- Reduce the model from 3D $\rightarrow$ 2D $\rightarrow$ 1D
- Include significant effects for concrete like:
  1. Damage effects $\rightarrow$ stiffness degradation
  2. Non-proportional strength degradation in compression and tension
  3. Partial stiffness recovery for stress paths switching from tension to compression
  4. Variable dilatancy $\rightarrow$ switch from plastic contractancy to dilatancy
  5. Aging and creep (according to the EC2 provisions)
  6. Thermal degradation of strength and stiffness (in longer perspective)

Reference damage-plastic model for concrete

- Jeeho Lee and Gregory L. Fenves.
  Plastic-damage model for cyclic loading of concrete structures.
  Journal of Engineering Mechanics ASCE.
- Original version of the model is implemented in Abaqus code
- Model requires few important modifications (to be disscussed later on)

Damage coupled with plasticity: why both together ?

- In elasto-plasticity: $\boldsymbol{\sigma} = \mathbf{E} : (\varepsilon - \varepsilon^p)$
- In continuum damage theory we introduce notion of the effective stress



- Mapping from nominal to effective stress: $\bar{\boldsymbol{\sigma}} = \mathbf{D} : \boldsymbol{\sigma}$
- $\boldsymbol{\sigma} = \dfrac{F}{A_o}$
- $\bar{\boldsymbol{\sigma}} = \dfrac{F}{A_o - A_{voids}}$

Damage coupled with plasticity: why both together ?

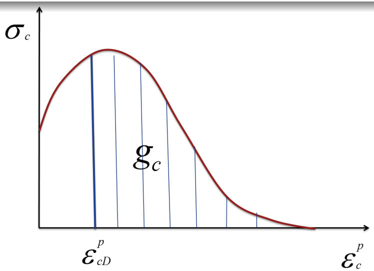
- In isotropic damage mapping tensor: $\mathbf{D} = \dfrac{1}{1-D}\mathbf{I}$
- Damage variable $0 \leq D \leq 1$
- Hence: $\boldsymbol{\sigma} = (1-D)\,\bar{\boldsymbol{\sigma}}$
- $\boldsymbol{\sigma} = (1-D)\,\mathbf{E} : (\varepsilon - \varepsilon^p)$
- Plastic flow rule: $\dot{\varepsilon^p} = \dot{\lambda}\dfrac{\partial G}{\partial \bar{\boldsymbol{\sigma}}}$
- In the reference model $D = 1 - (1 - D_c)(1 - s\ D_t)$
- $s(\hat{\bar{\boldsymbol{\sigma}}}) = s_o + (1 - s_o)\ r(\hat{\bar{\boldsymbol{\sigma}}})$
- $r(\hat{\bar{\boldsymbol{\sigma}}}) = \dfrac{\sum < \hat{\bar{\boldsymbol{\sigma}}}_i >}{\sum |\hat{\bar{\boldsymbol{\sigma}}}_i|}$

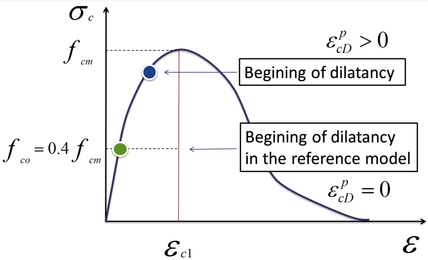
Internal damage variable κ_c in compression and damage factor D_c

- Compressive stress
  $\sigma_c = f_{co}\left[(1 + a_c)\exp(-b_c \varepsilon_c^p) - a_c \exp(-2\, b_c \varepsilon_c^p)\right]$
- Fracture energy in compression: G_c
- Normalized fracture energy: $g_c = \dfrac{G_c}{l_c}$
- Internal damage variable: $\kappa_c = \dfrac{1}{g_c} \int_{\varepsilon_{cD}^p}^{\varepsilon_c^p} \sigma_c(\varepsilon_c^p)\, d\varepsilon_c^p$
- Normalized fracture energy: $g_c = \int_{\varepsilon_{cD}^p}^{\infty} \sigma_c(\varepsilon_c^p)\, d\varepsilon_c^p$
- Evolution law for $\dot{\kappa}_c = \dfrac{1}{g_c} \sigma_c(\kappa_c) \dot{\varepsilon}_c^p$

NB. In the reference model $\varepsilon_{cD}^p = 0$ (!)

Internal damage variable κ_c in compression



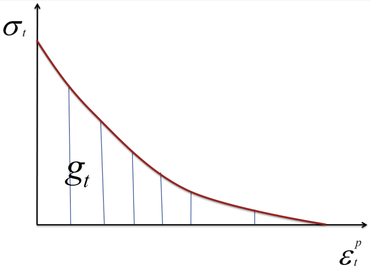


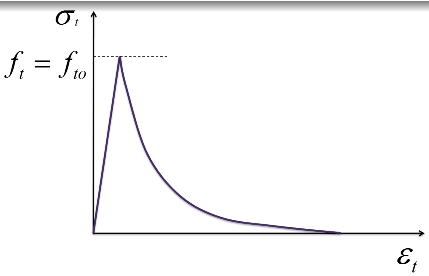
- $D_c = 1 - exp(-d_c \ < \varepsilon^p_c - \varepsilon^p_{cD}) >)$
- $\kappa_c =$
  $$\kappa_{cD} - \frac{f_{co}}{g_c \ b_c}\left[(1 + a_c)exp(-b_c \ \varepsilon^p_c) - \frac{1}{2} a_c \exp(-2 \ b_c \ \varepsilon^p_c)\right]$$
- If $\varepsilon^p_c \geq \varepsilon^p_{cD}$ then we can express ε^p_c in terms of κ_c
- Also σ_c and D_c can be expressed via κ_c
- Effective compressive stress $\bar{\sigma}_c(\kappa_c) = \dfrac{\sigma_c(\kappa_c)}{1 - D_c(\kappa_c)}$

Internal damage variable κ_t in tension and damage factor D_t

- Tensile stress
  $\sigma_t = f_{to}\left[(1 + a_t)\exp(-b_t\varepsilon_t^p) - a_t\exp(-2\ b_t\varepsilon_t^p)\right]$
- Here we may assume $a_t = 0$
- Fracture energy in tension: G_t
- Normalized fracture energy: $g_t = \dfrac{G_t}{l_c}$
- Internal damage variable: $\kappa_t = \dfrac{1}{g_t}\int\limits_0^{\varepsilon_t^p} \sigma_t(\varepsilon_t^p)\ d\varepsilon_t^p$
- Normalized fracture energy: $g_t = \int\limits_0^{\infty} \sigma_t(\varepsilon_t^p)\ d\varepsilon_t^p$
- Evolution law for $\dot{\kappa}_t = \dfrac{1}{g_t}\sigma_t(\kappa_t)\dot{\varepsilon}_t^p$

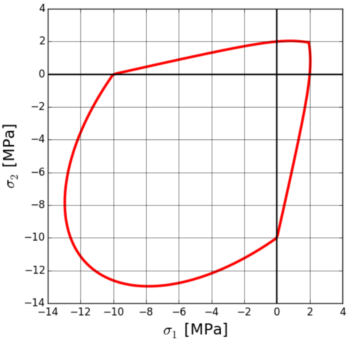
Internal damage variable κ_t in tension





- $D_t = 1 - exp(-d_t \ \varepsilon_t^p)$
- $\kappa_t = \dfrac{f_{to}}{g_t \ b_t}(1 - \exp(-b_t \ \varepsilon_t^p))$
- Now we can express ε_t^p in terms of κ_t
- Also σ_t and D_t can be expressed via κ_t
- $\sigma_t(\kappa_t) = f_{to}(1 - \kappa_t)$
- $D_t = 1 - (1 - \kappa_t)^{d_t/b_t}$
- Effective tensile stress $\bar{\sigma}_t(\kappa_t) = \dfrac{\sigma_t(\kappa_t)}{1 - D_t(\kappa_t)}$

Initial strength envelope in plane stress



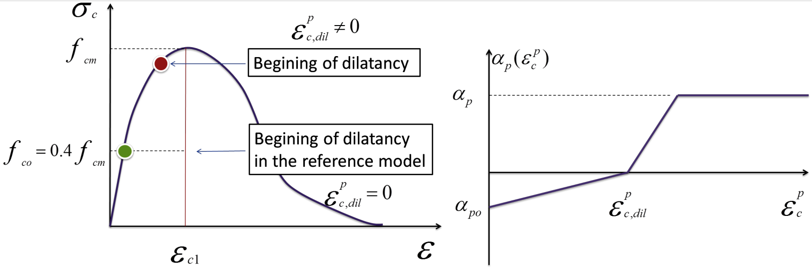
$$F(\bar{\boldsymbol{\sigma}}, \kappa_t, \varepsilon_c^p) =$$

$$\frac{1}{1-\alpha}\left(\alpha\ I_1 + \sqrt{3\ J_2} + \beta(\kappa_t, \varepsilon_c^p) < \hat{\bar{\sigma}}_{max} >\right) - c_c(\varepsilon_c^p)$$

$$\beta = \frac{c_c(\varepsilon_c^p)}{c_t(\kappa_t)}(1-\alpha) - (1+\alpha)$$

$$c_c = \bar{\sigma_c} = \sigma_c(\varepsilon_c^p) \qquad c_t = \bar{\sigma}_t(\kappa_t)$$

Plastic flow rule and internal plastic hardening variables ε_t^p, ε_t^c

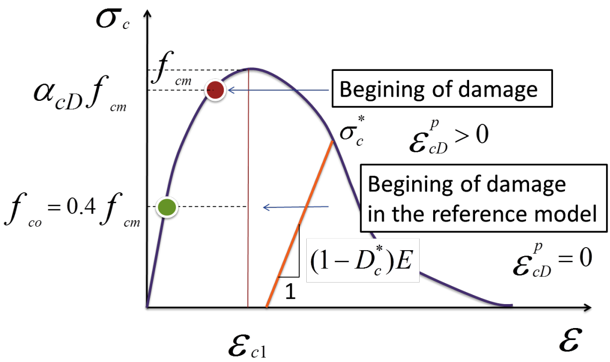


- $G(\hat{\boldsymbol{\sigma}}) = \sqrt{2\ J_2} + \alpha_p\ I_1$
- In the reference model α_p is a constant (?) $\Rightarrow$ dilatancy occurs at relatively low compressive stress (?)
- $\dot{\varepsilon}_t^p = r(\hat{\bar{\sigma}})\ \dot{\varepsilon}_{max}^p$
- $\dot{\varepsilon}_c^p = -(1 - r(\hat{\bar{\sigma}}))\ \dot{\varepsilon}_{min}^p$
- $r(\hat{\bar{\boldsymbol{\sigma}}}) = \dfrac{\sum < \hat{\bar{\boldsymbol{\sigma}}}_i >}{\sum |\hat{\bar{\boldsymbol{\sigma}}}_i|}$

Calibrating a_c parameter

- $\sigma_c = f_{co}\left[(1 + a_c)\exp(-b_c \varepsilon_c^p) - a_c \exp(-2\, b_c \varepsilon_c^p)\right]$
- We want that the maximum compressive stress is equal to $f_{cm} = f_c$
- Extremum σ_c is achieved at $\varepsilon_{c,extr}^p = -\dfrac{\ln\left(\dfrac{1}{2}\dfrac{1 + a_c}{a_c}\right)}{b_c}$
- If we substitute $\varepsilon_{c,extr}^p$ to the expression for σ_c we will get: $f_{cm} = \dfrac{1}{4}\dfrac{f_{co}\,(1 + a_c)^2}{a_c} \Rightarrow a_c$ is known

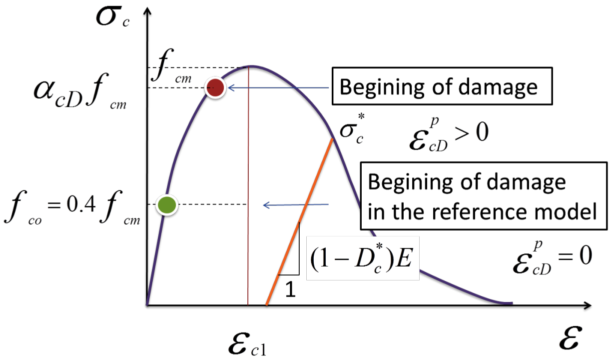
Calibrating b_c, d_c parameters (Approach A)



1. we assume that damage starts at $\sigma_c = \alpha_{cD}\ f_{cm}$
2. we want to preserve g_c value
3. for given σ_c^* we know D_c^*
4. in this approach parameters can be computed directly

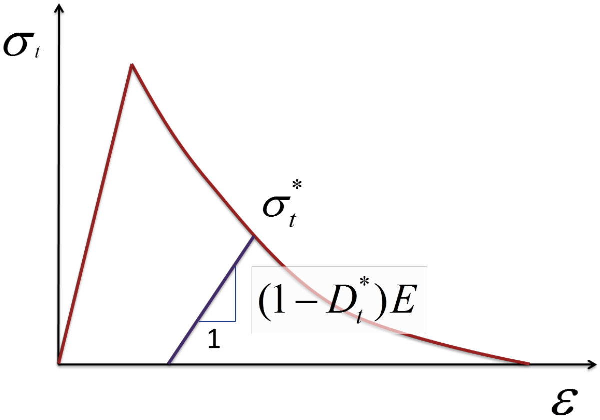
NB. $\alpha_{cD} \geq \alpha_{c,dil}$ and strain at peak ε_{c1} is not under control

Calibrating b_c, d_c parameters (Approach B)



1. we assume that damage starts at $\sigma_c = \alpha_{cD}\ f_{cm}$
2. g_c value is no more under control
3. we want to keep ε_{c1} at a given value
4. for given σ_c^* we know D_c^*
5. in this approach parameters are computed in the iterative manner (g_c is the optimized variable)

Calibrating b_t, d_t parameters

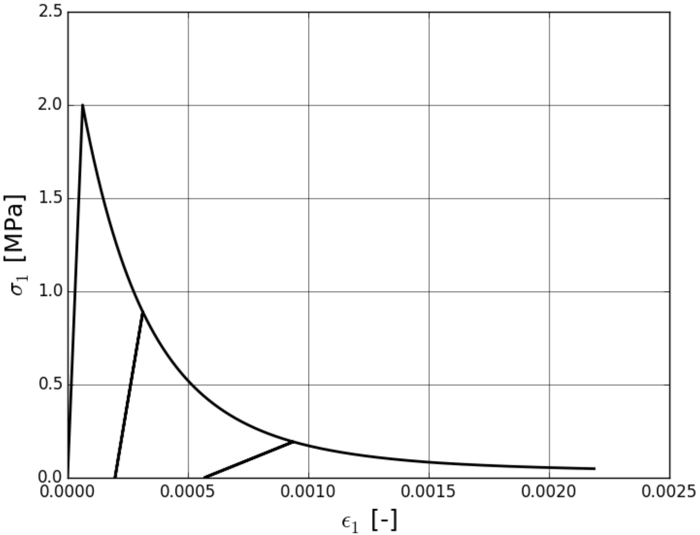


1. from known $g_t \Rightarrow b_t = \dfrac{f_{to}}{g_t}$
2. for given σ_t^* we know D_t^*
3. in this case b_t, d_t are derived in a closed form

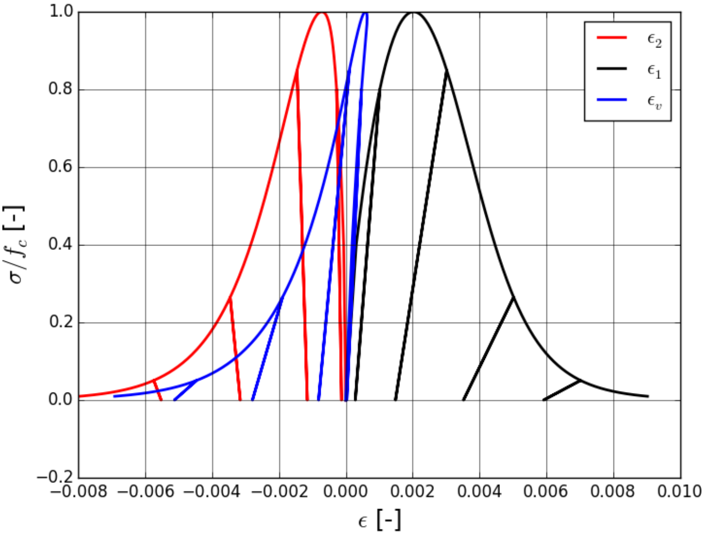
Implementation issues

- Stress state is memorized at Gauss points in damaged configuration
- The predictor step is computed in undamaged configuration $\Rightarrow$ nominal stresses are converted to the effective ones for frozen value of D parameter
- Plastic corrector is run on effective stresses (standard plasticity)
- Damage corrector maps effective stresses to the damaged configuration for the new updated D parameter

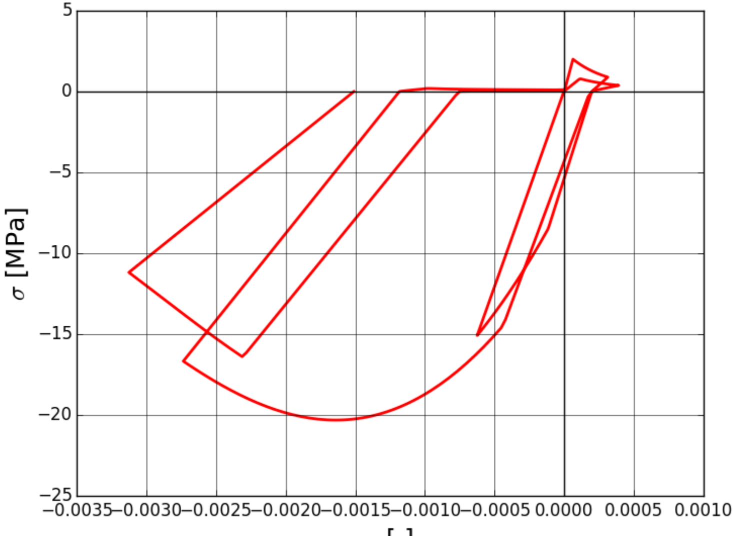
Uniaxial cyclic tensile test



Uniaxial cyclic compression test



Stiffness recovery in cyclic tension-compression test



Conclusions

1. Model should improve ZSoil predictions for concrete structures
2. Benchmarking is needed to calibrate or improve dilatancy description
3. Optimization routine will be run based on Kupfer's data
4. Aging and creep is the next step
5. Then thermal degradation of strength and stiffness