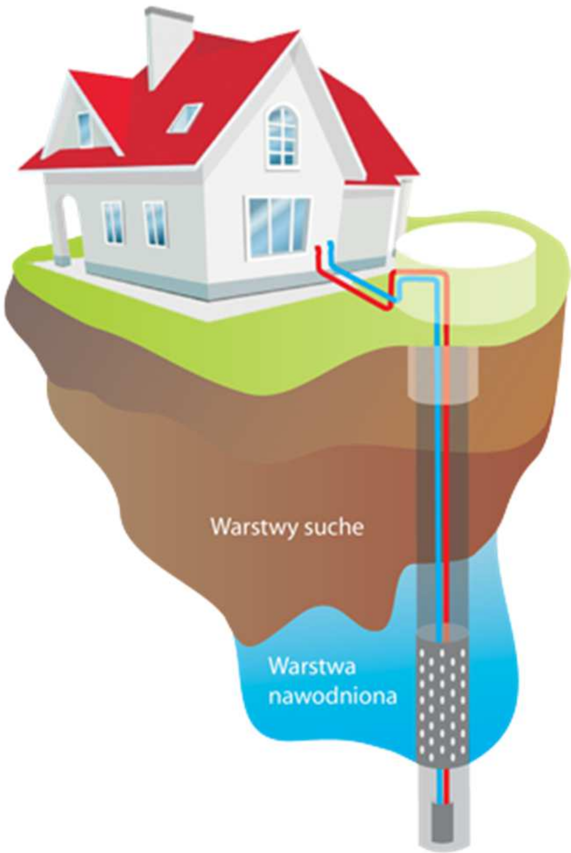
ZSoil Days '2016

Aleksander Urbański

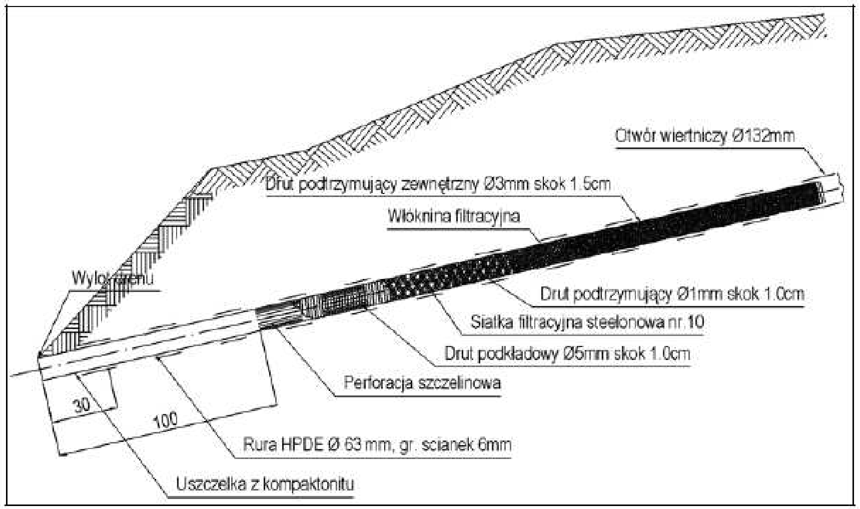
Multi-scale analysis of a flow to drainage tubes

Application area

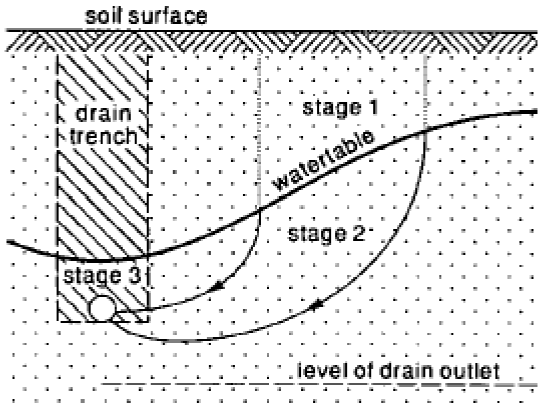
drilled well (water supply)



drilled drain (slope stabilization)

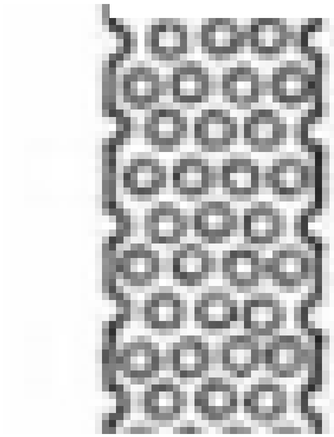


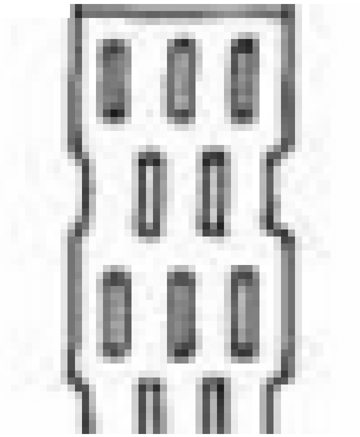
French drain (dewatering)



Common feature

diameter D ~0.1 m
perforation of the pipe in mm





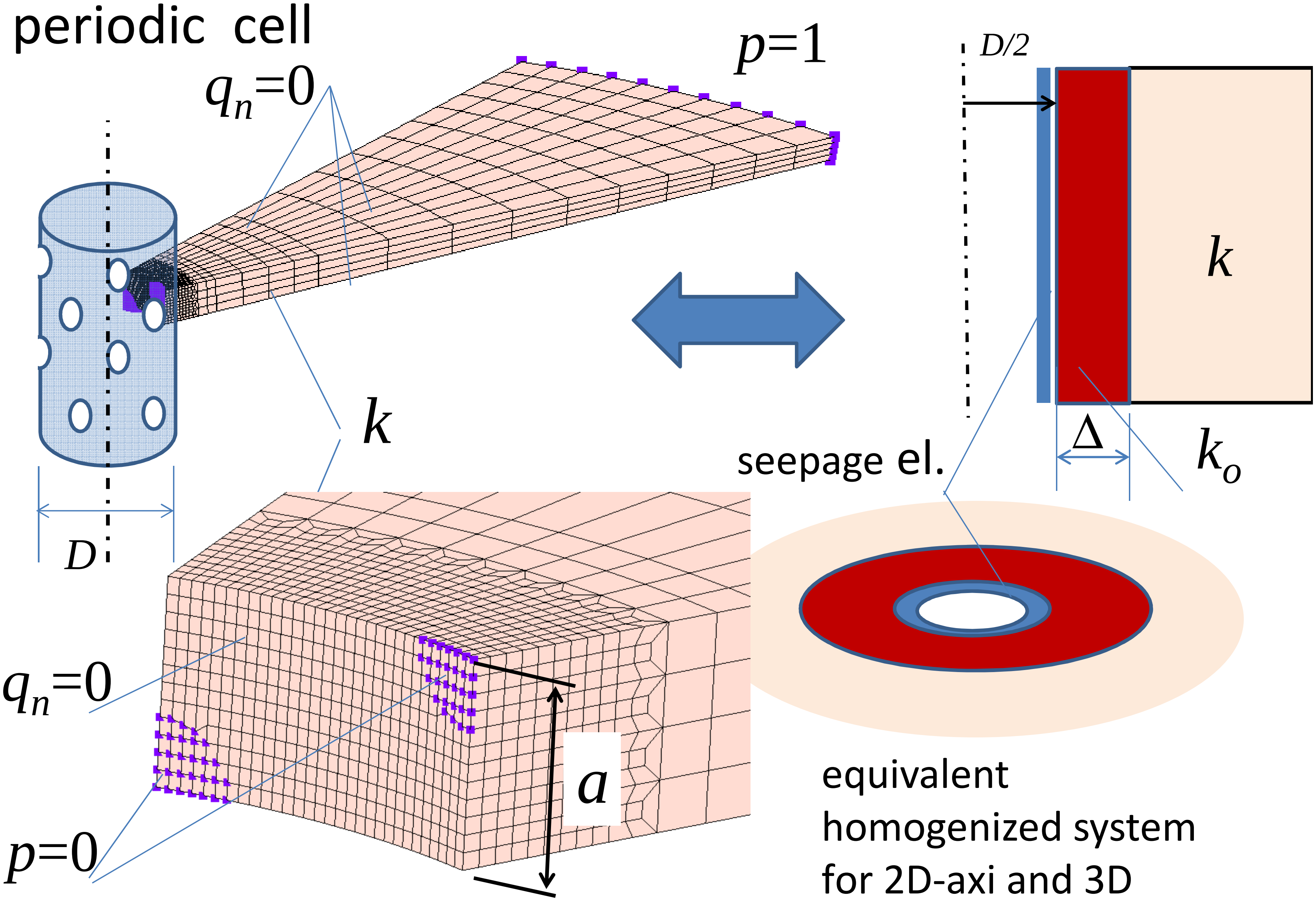
determine outflow q

which results in filtration flow field
(pressure p, flow $\boldsymbol{q}$)
in the area ~10m

It is difficult to build single model dealing with filtration nearby drain tube (scale mm) and in the distance from it (m).

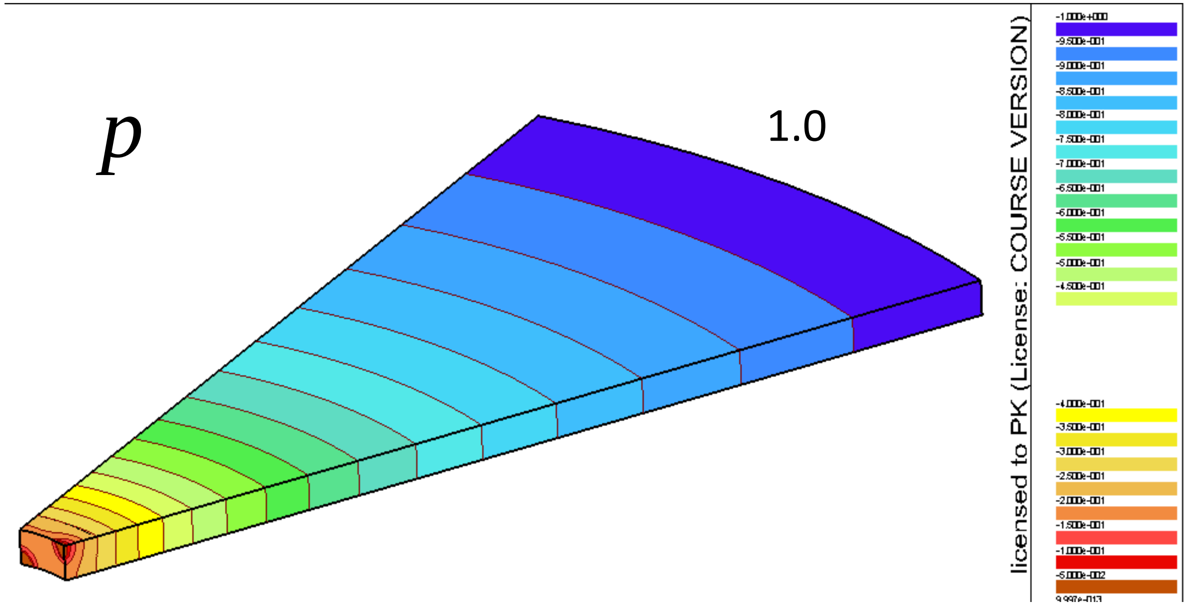
Scale separation leads to multi-scale problem
micro – 3D periodic cell in surrounding of the drain
macro – whole system (2D or 3D)

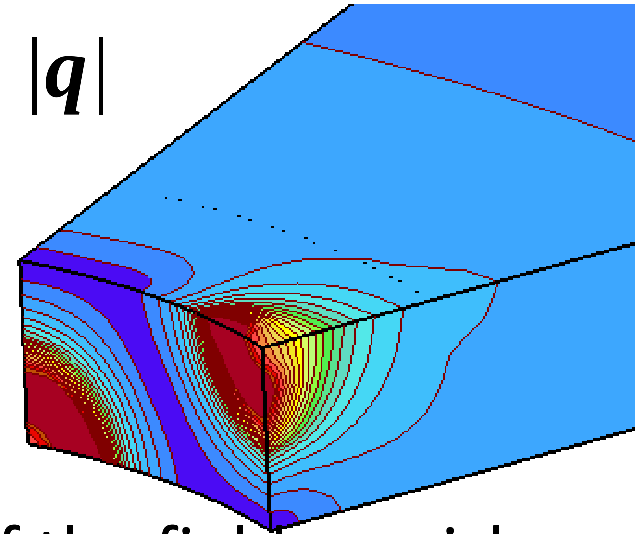
Numerical analysis basing on homogenization of the flow



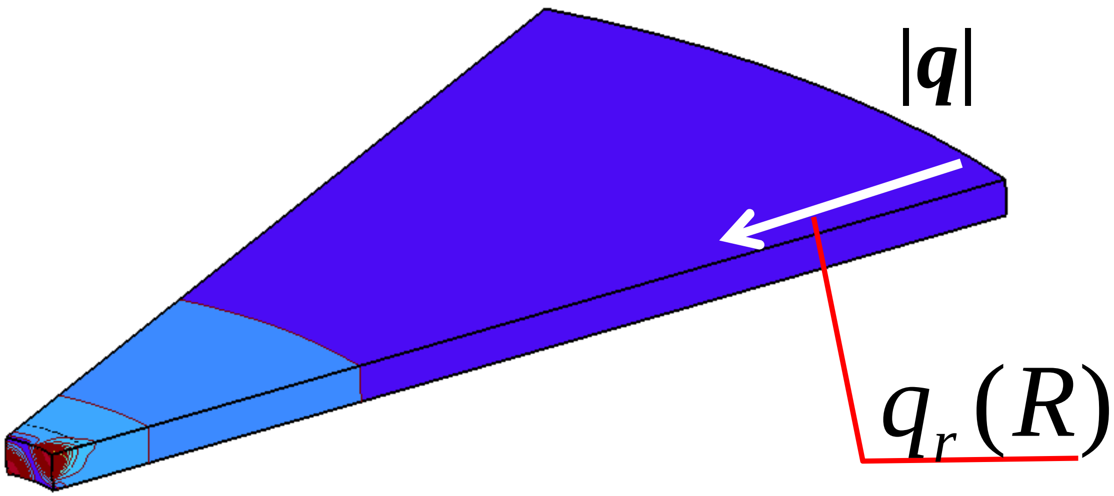
Results on periodic cell

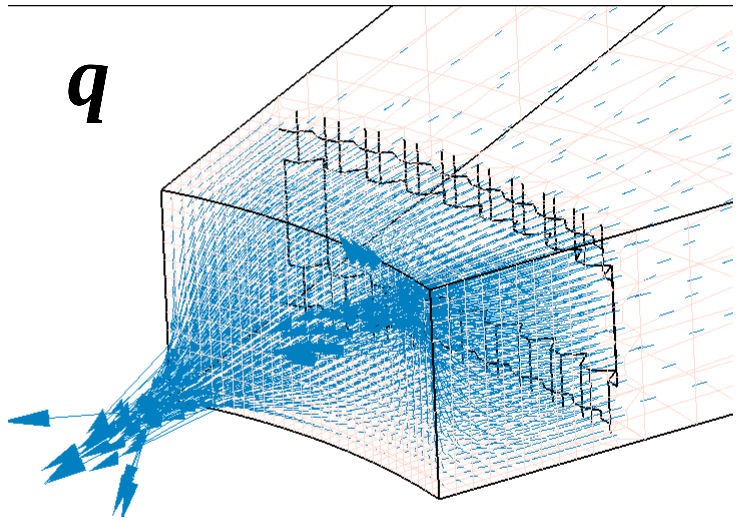
skipped gravity term
in Darcy law





radius R set up such that fluctuations of the fields vanishes





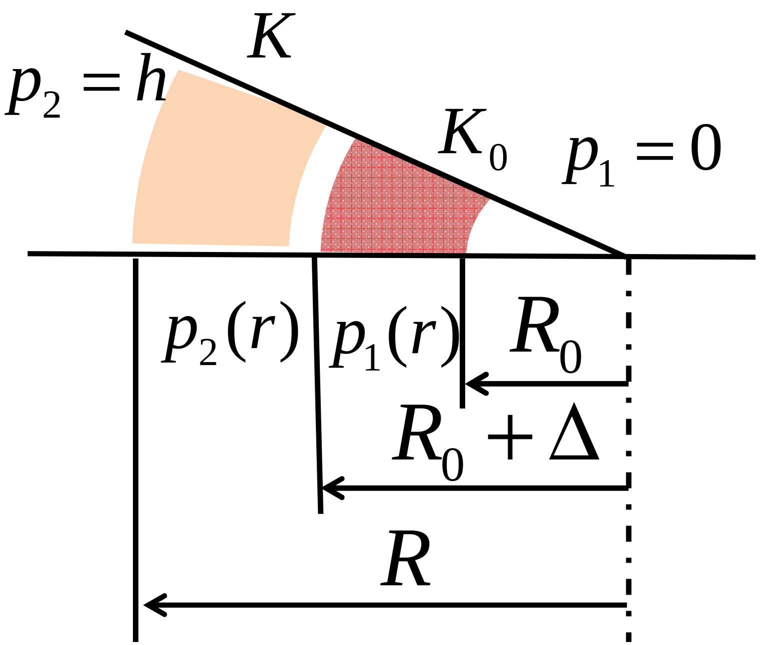
Procedure to find properties of equivalent system

Assumed: axial symmetry, z-indepent: $\frac{\partial p}{\partial \theta}=0, \quad \frac{\partial p}{\partial z}=0, \quad p=\frac{\hat{p}}{\gamma_F}\,[m]$

gravity potential =0

$$\nabla^2 p = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial z^2}=\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial z^2}=0$$

$$\Rightarrow \boxed{\frac{d^2 p}{dr^2}+\frac{1}{r}\frac{dp}{dr}=0 \Rightarrow p(r)=C\ln|r|+D}$$



boundary conditions: $p_1(R_0)=0$

$p_2(R)=h$

compatibility of:

flow $\quad K_0 \left.\frac{\partial p_1}{\partial r}\right|_{r=R_0+\Delta} = K \left.\frac{\partial p_2}{\partial r}\right|_{r=R_0+\Delta}$

pressures $\quad p_1(R_0+\Delta)=p_2(R_0+\Delta)$

*

3D analysis of flow through periodic cell: $q_2(R)$ → $Q = a \cdot 2\pi R \cdot q_2(R)$

from the analytical solution of *

$$Q = \frac{2\pi a h K_0}{\frac{K_0}{K}\ln\frac{R}{R_0+\Delta} - \ln\frac{R_0+\Delta}{R_0}}$$

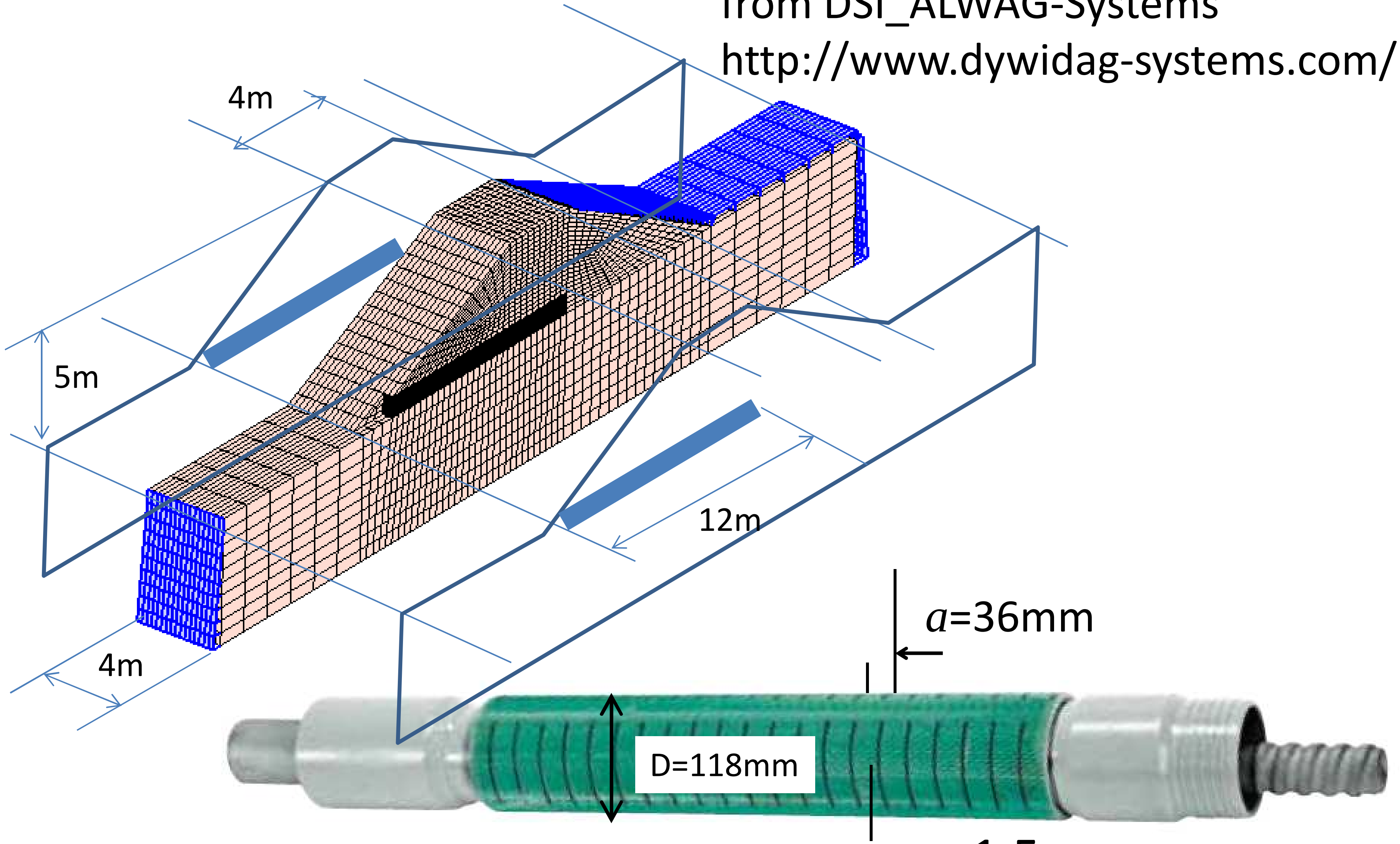
then, for given q, K, R, R_0, h and assumed Δ

permeability of homogenized zone:

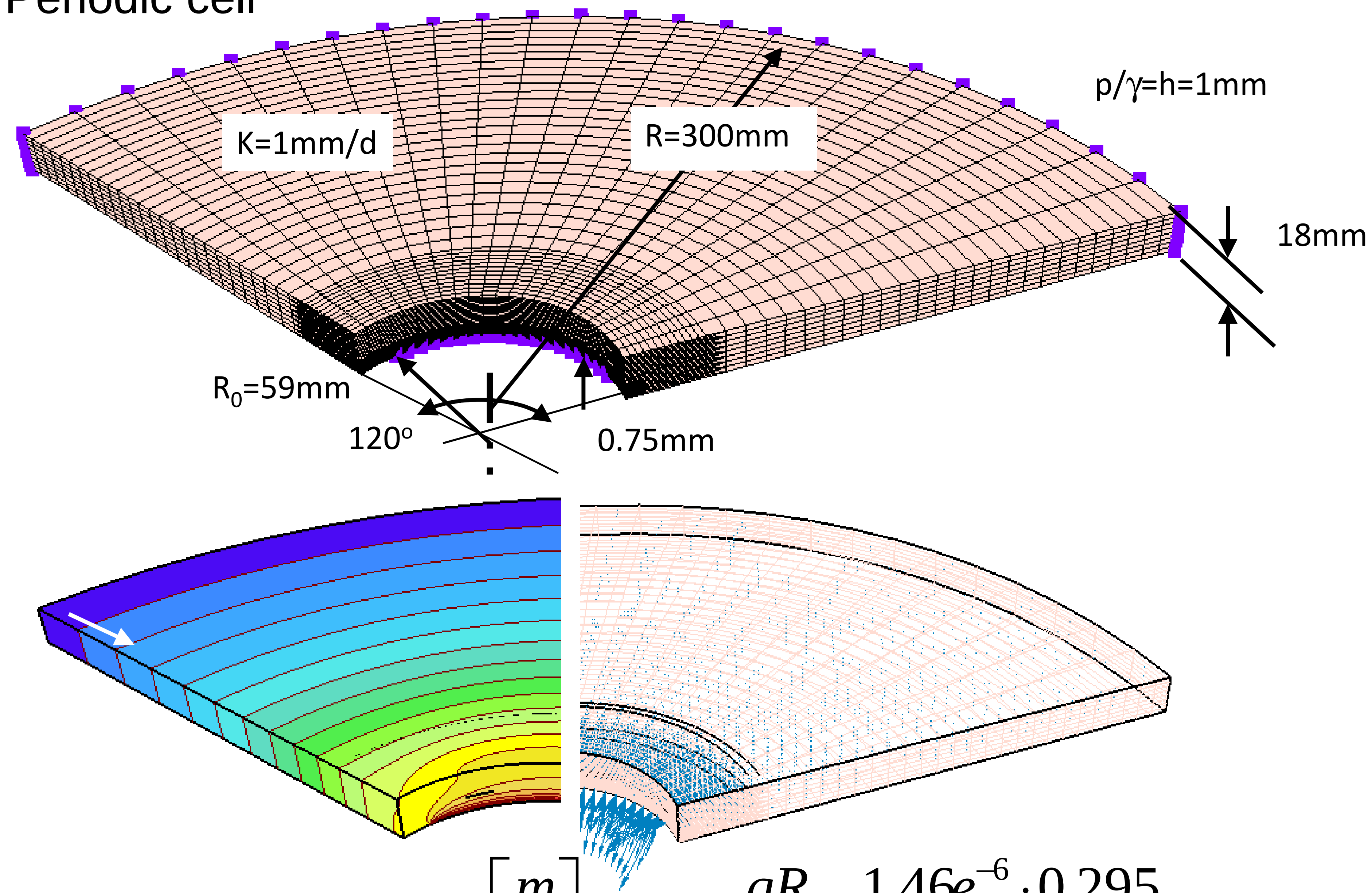
$$K_0 = \frac{\beta \ln\frac{R_0+\Delta}{R_0}}{1-\beta\ln\frac{R}{R_0+\Delta}} K, \quad \beta = \frac{qR}{Kh}$$

Application 1: Drilled drain in an earth dam

example: AT - Drainage System
from DSI_ALWAG-Systems
http://www.dywidag-systems.com/



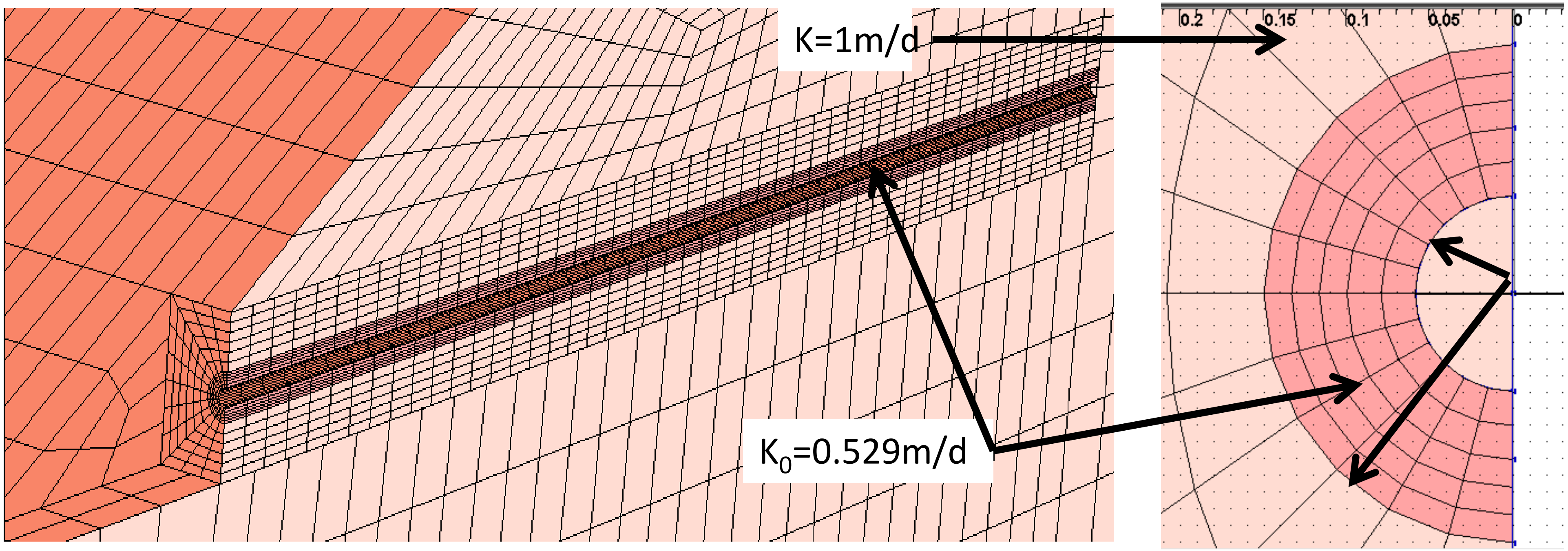
Periodic cell



$$q(R=0.295)=1.46\cdot e^{-6}\left[\frac{m}{d}\right]\Rightarrow\beta=\frac{qR}{Kh}=\frac{1.46e^{-6}\cdot 0.295}{1e^{-3}\cdot 1e^{-3}}=0.407[-]$$

3D model of the drain

$$R_0 = 0.059\,m$$



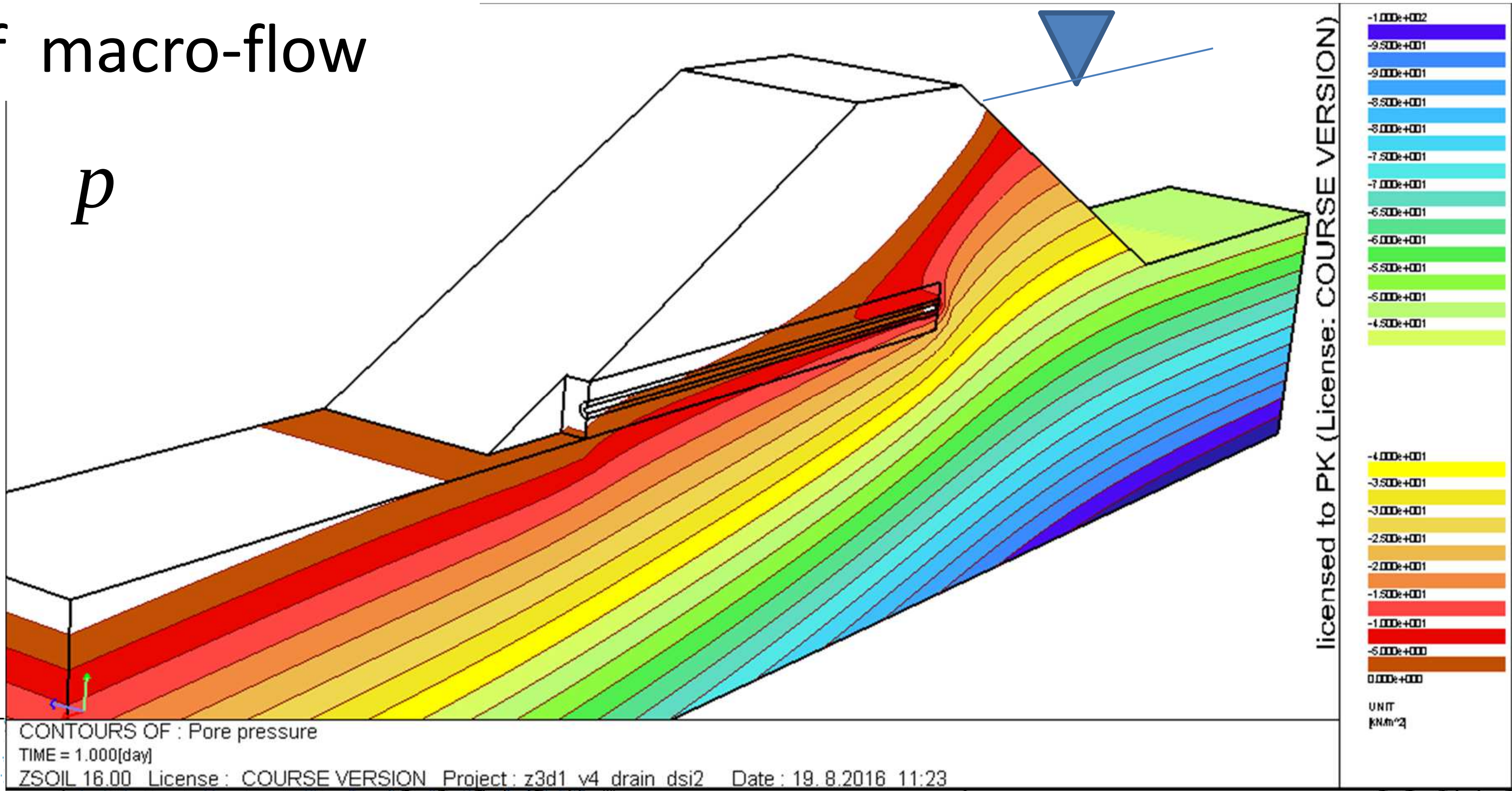
$$R_0 + \Delta = 0.15\,m$$

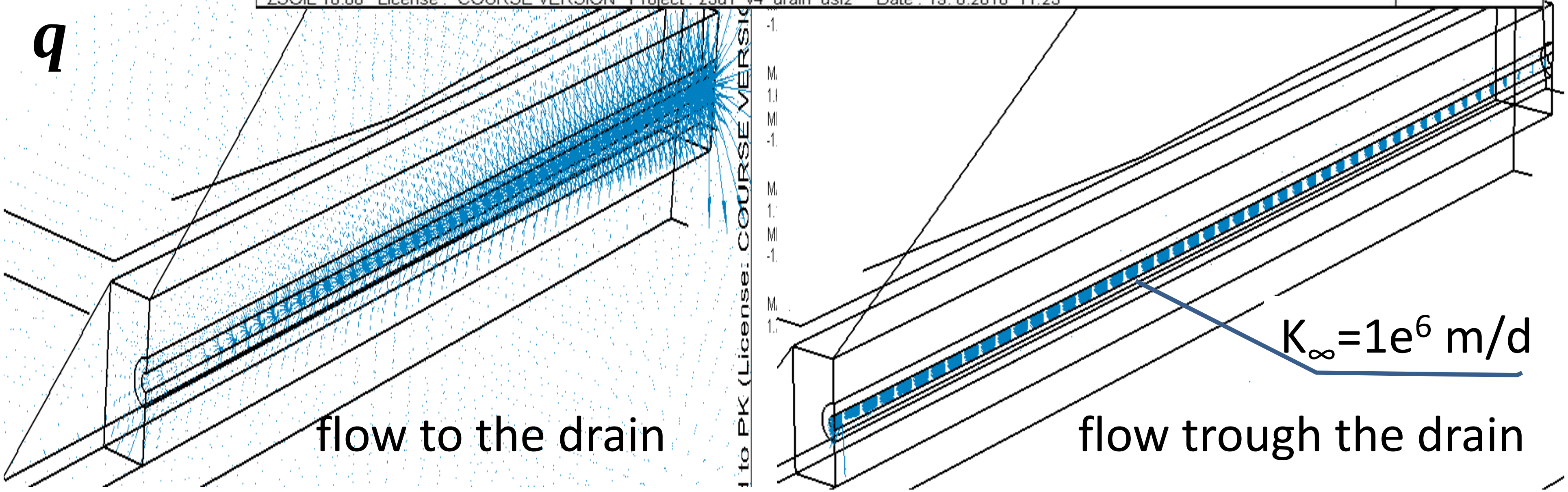
permeability of homogenized zone:

$$\beta = \frac{qR}{Kh} = 0.407[-], \quad R = 0.30\,m$$

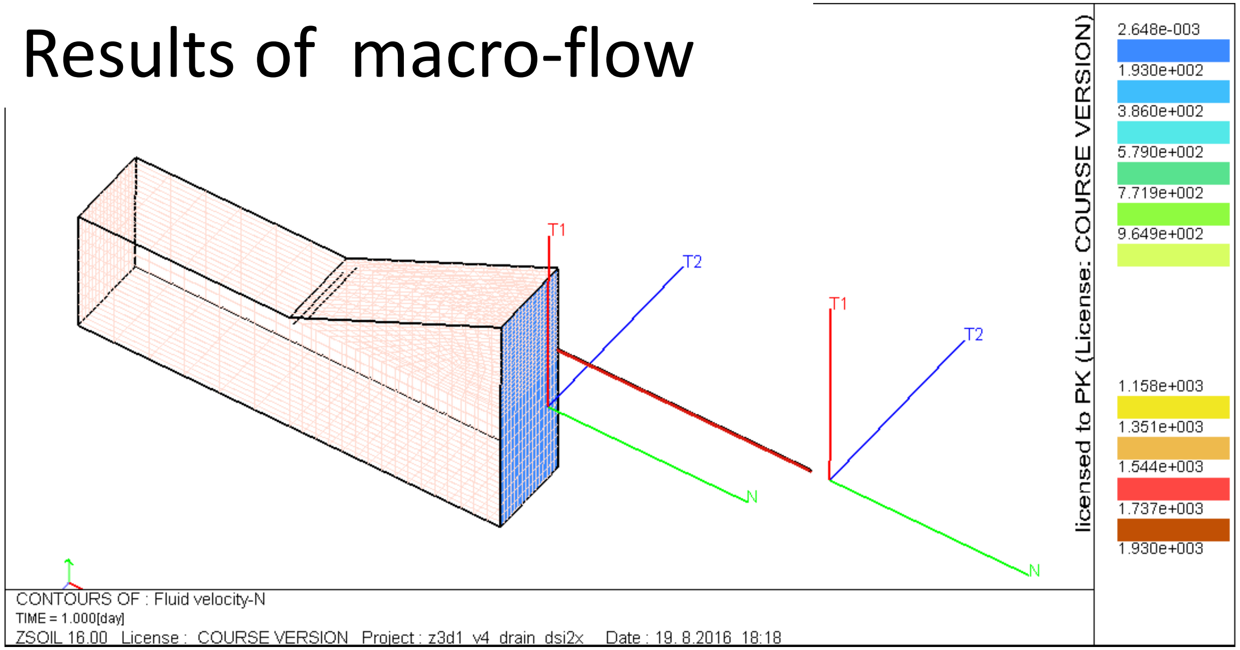
$$K_0 = \frac{\beta \ln \dfrac{R_0 + \Delta}{R_0}}{1 - \beta \ln \dfrac{R}{R_0 + \Delta}} K = \frac{0.407 \ln 2.542}{1 - 0.407 \ln 2} K = 0.529\,K,$$

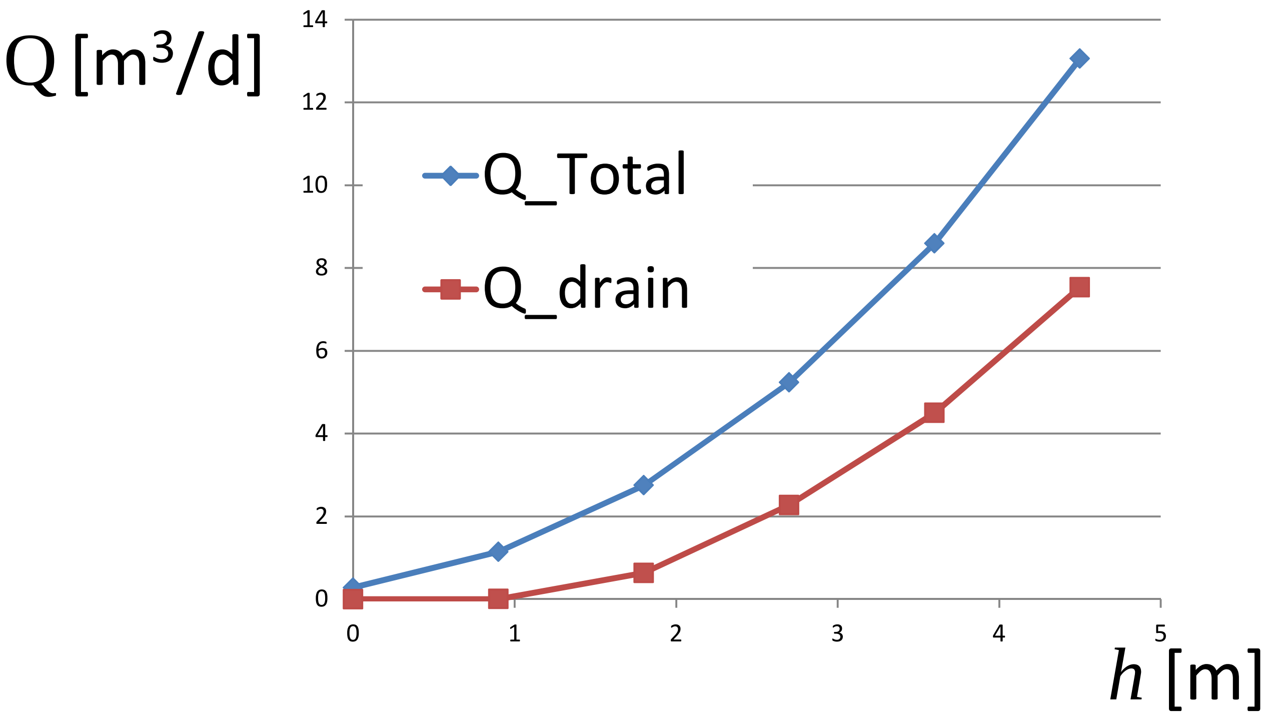
Results of macro-flow





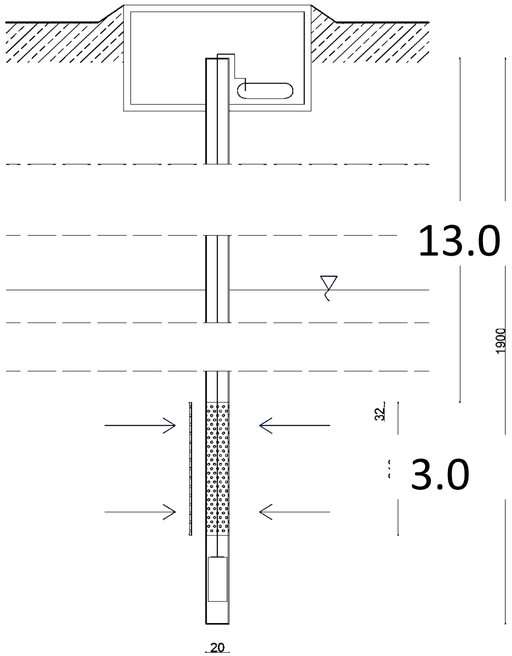
Results of macro-flow

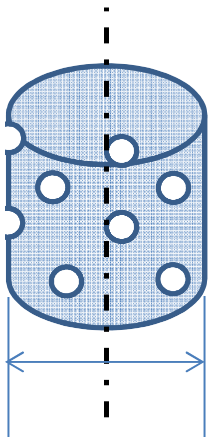


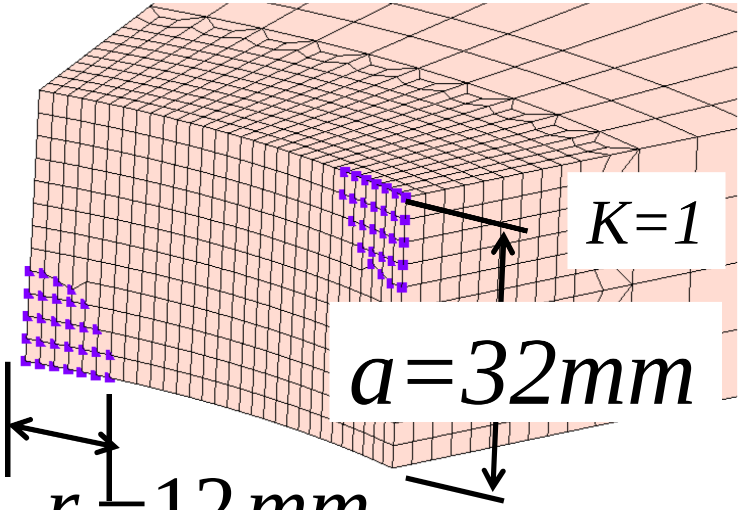


Application 2: **Drilled well**

in co-operation with phd. students: Karolina Kuc & Barbara Wilk







$$D=2R_0=200mm \qquad r=12\,mm$$

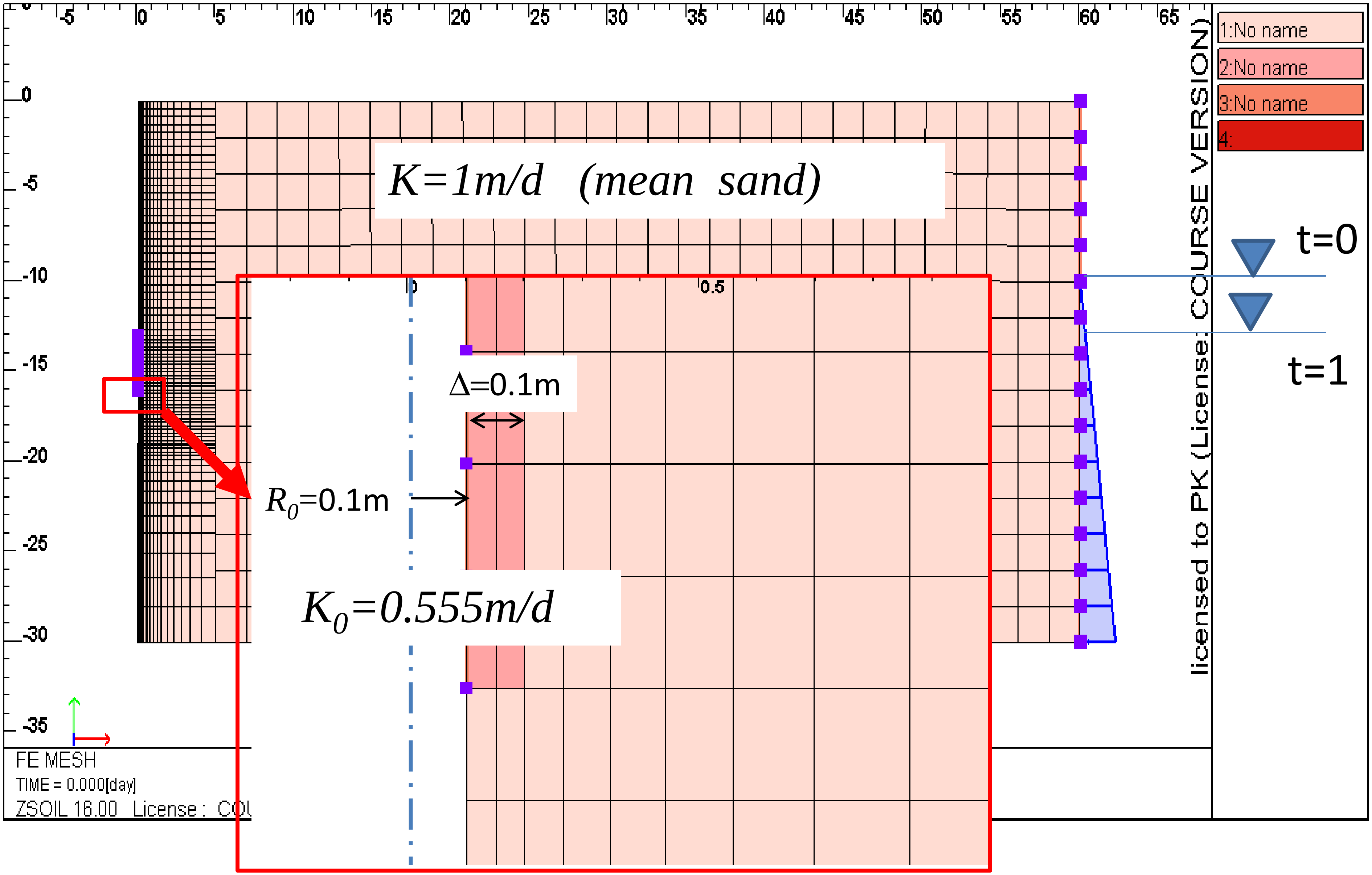
$$R=1.0, h=1.0[m] \quad \Rightarrow \quad q=0.35[m/d]$$

permeabiltity of the layer with assumed thickness $\Delta=0.1m$:

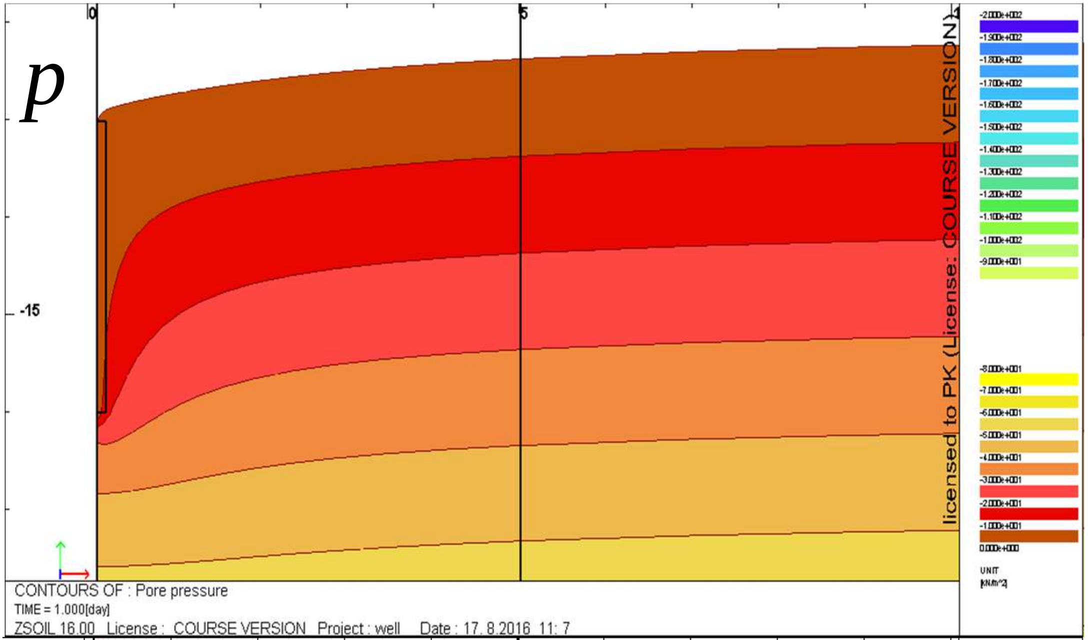
$$\beta=\frac{qR}{Kh}=0.35[-]$$

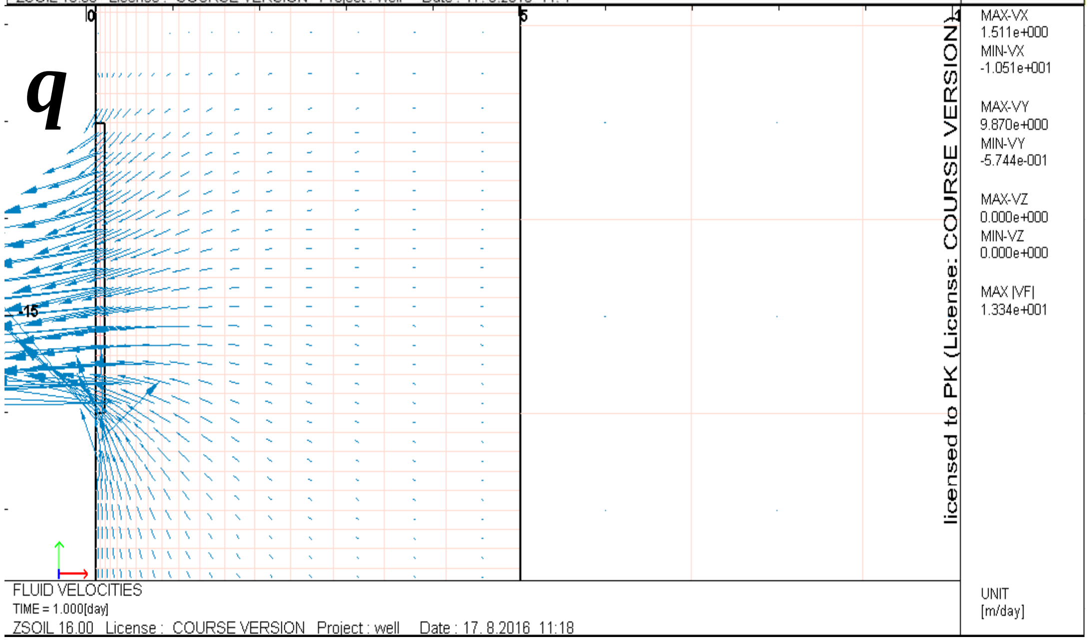
$$K_0=\frac{0.35\ln 2}{1-0.35\ln 5}K=\frac{0.35\cdot 0.693}{1-0.35\cdot 1.609}K=0.555K$$

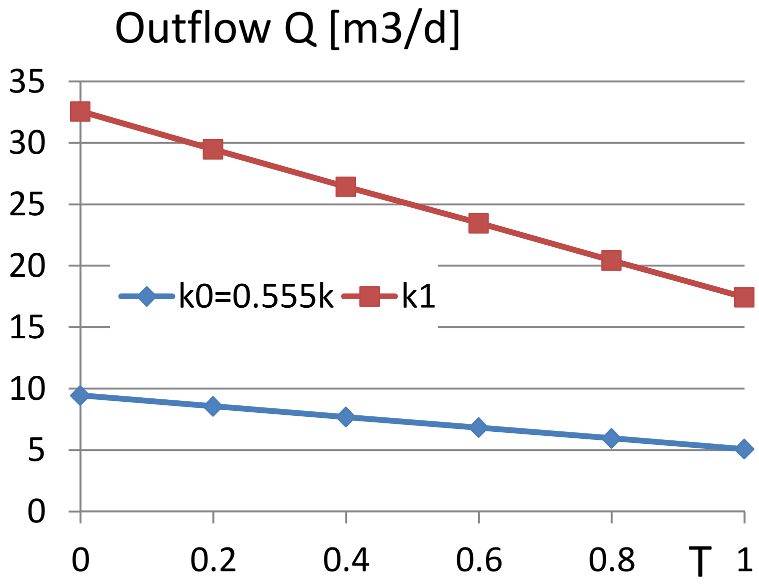
Axisymmetric macro-model of the well



Results

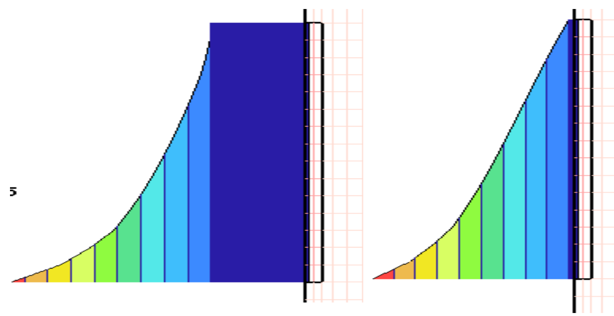






filtration stream (homogenized)

T=0 T=1



q(0)=19.5 q(1)=13.4 [m/d]

Thank you for your attention

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