ELASTIC-PLASTIC DAMAGE MODEL FOR CONCRETE

ZSoil®.PC 160102 report

Andrzej Truty
Contents

1 Introduction 3

2 Reference plastic damage model for concrete (by Lee and Fenves) and its modifications 5
   2.1 Theory ........................................................................................................... 5

3 Estimating model parameters 15
   3.1 Calibrating model parameters corresponding to the compressive stress domain 15
   3.2 Calibrating model parameters corresponding to the tensile stress domain . . . 17

4 Numerical implementation 19
   4.1 Stress strain integration scheme .............................................................. 19

5 Creep and aging 21

6 User interface 25

7 Benchmarks 29
   7.1 Gopalaratnam and Shah monotonic and cyclic uniaxial tensile tests (1985) . 29
   7.2 Karsan and Jirsa monotonic and cyclic uniaxial compression tests (1969) . . 32
   7.3 Kupfer’s tests ............................................................................................ 35
   7.4 Three point bending test .......................................................................... 40
   7.5 RC slab under point loading ..................................................................... 41
   7.6 Creep in monotonic tensile test ................................................................ 44
   7.7 Creep under variable loading conditions ................................................... 46
Chapter 1

Introduction

Current implementation of plastic damage model for concrete (CPDM) is based on the formulation proposed by Lee and Fenves [4, 5] including modifications of plastic flow potential introduced then by Omidi and Lotfi [7] and important modifications proposed by the author of this report. This model couples single surface elasto-plasticity with hardening and an enhanced scalar damage allowing for description of stiffness degradation and stiffness recovery in cyclic tension-compression tests.
Chapter 2

Reference plastic damage model for concrete (by Lee and Fenves) and its modifications

2.1 Theory

In classical elastoplasticity, assuming that the linear elastic Hooke's law is used for the irreversible part of the deformation, the relation between stress and strain can be described in the following total form

\[ \sigma = E : (\varepsilon - \varepsilon^p) \]  

(2.1)

Figure 2.1: Damaged configuration (damaged cross section of a bar is shown below; dark dots represent here voids)

In the continuum damage theory notion of nominal and effective stresses is introduced (here effective stresses have nothing to do with the classical effective stresses known from mechanics of porous media) (see Fig. 2.1). If we consider a 1D tensile test case then the corresponding nominal and effective axial stress can be defined as follows
\[ \sigma = \frac{F}{A_o} \]  
\[ \tilde{\sigma} = \frac{F}{A_o - A_{voids}} \]  

In the general 2D or 3D problems the following mapping rule from nominal to the effective stress is used

\[ \tilde{\sigma} = D : \sigma \]  

where \( D \) is the 4-th order mapping tensor (in general), and ":" symbol represents tensor product \( (\tilde{\sigma}_{ij} = D_{ijkl} \sigma_{kl}) \). Effective stresses are always overlined (e.g. \( \tilde{\sigma} \) in matrix notation, or \( \tilde{\sigma}_{ij} \) in the tensorial one) in the report, while principal stresses (no matter if nominal or effective) are distinguished by an extra hat symbol (e.g. \( \hat{\sigma}_i \), if nominal, or \( \tilde{\sigma}_i \), if the effective principal stress is considered).

All major components of the CPDM model, basing on the assumption of strain equivalence in terms of the damage formulation, are summarized in Win.(2-1)
2.1. THEORY

Window 2-1: Major components of CPDM model

- Mapping tensor $D$ in isotropic damage ($I$ is an unit tensor here while $D$ is a damage parameter ($0 \leq D \leq 1$))
  \[ D = \frac{1}{1 - D} I \]

- Relation between nominal and effective stresses
  \[ \sigma = (1 - D) \bar{\sigma} \]

- Constitutive equation written in terms of nominal stresses
  \[ \sigma = (1 - D) E : (\varepsilon - \varepsilon^p) \]

- Set of internal damage variables: $\kappa = \{\kappa_c, \kappa_t\}^T$

- Evolution law for damage variables ($H$ is a diagonal matrix of hardening functions specified later on)
  \[ \dot{\kappa} = \lambda H (\bar{\sigma}, \kappa) \]

- Enhanced damage parameter (combination of two damage parameters corresponding to the compression $D_c(\kappa_c)$ (see Win. (2-2)) and tension $D_t(\kappa_t)$) (see Win. (2-4))
  \[ D = 1 - (1 - D_c(\kappa_c))(1 - s D_t(\kappa_t)) \]

- Stiffness recovery function
  \[ s(\bar{\sigma}) = s_o + (1 - s_o) r(\bar{\sigma}) \]

- Effective stress domain function
  \[ r(\bar{\sigma}) = \frac{\sum_{i=1}^{3} <\hat{\sigma}_i>}{\sum_{i=1}^{3} |\hat{\sigma}_i|} \]

- Plastic yield condition (see Win. (2-5))
  \[ F(\bar{\sigma}, \kappa) = 0 \]

- Plastic flow rule ($G$ is a modified Drucker-Prager type potential) (see Win. (2-6))
  \[ \dot{\varepsilon}^p = \lambda \frac{\partial G}{\partial \sigma} \]
CHAPTER 2. REFERENCE PLASTIC DAMAGE MODEL FOR CONCRETE (BY LEE AND FENVES) AND ITS MODIFICATIONS

Window 2-2: Internal damage variable $\kappa_c$ in compression and damage factor $D_c$

- Nominal compressive stress: $\sigma_c = f_{co} \left[ (1 + a_c) \exp(-b_c \varepsilon_c^p) - a_c \exp(-2 b_c \varepsilon_c^p) \right]$  
- Normalized fracture energy: $g_c = \frac{G_c}{f_c} = \int_0^\infty \sigma_c(\varepsilon_c^p) \, d\varepsilon_c^p$  
- Evolution law for hardening variable $\varepsilon_c^p$: $\dot{\varepsilon}_c^p = -(1 - r(\hat{\sigma})) \dot{\varepsilon}_{\min}$  
- Internal damage variable: $\kappa_c = \frac{1}{g_c} \int_0^{\varepsilon_c} \sigma_c(\varepsilon_c^p) \, d\varepsilon_c^p$  
- Evolution law for damage variable: $\dot{\kappa}_c = \frac{1}{g_c} \sigma_c(\kappa_c) \dot{\varepsilon}_c^p$  
$\kappa_c = 1 - \frac{f_{co}}{g_c b_c} \left[ (1 + a_c) \exp(-b_c \varepsilon_c^p) - \frac{1}{2} a_c \exp(-2 b_c \varepsilon_c^p) \right]$  

- Notion of normalized fracture energy $g_c$ and uniaxial $\sigma - \varepsilon$ diagram

- Damage parameter in compression: $D_c = 1 - \exp(-d_c < \varepsilon_c^p - \varepsilon_{c,D}^p >)$  
- Replacing hardening parameter $\varepsilon_c^p$ by damage variable $\kappa_c$ leads to the following expressions for $\sigma_c$ and $D_c$:

$$\sigma_c(\kappa_c) = \frac{f_{co}}{a_c} \left( 1 + a_c - \sqrt{\phi_c} \right) \sqrt{\phi_c}$$

$$D_c = \begin{cases} 0 & \text{for } \kappa_c \leq \kappa_{c,D} \\ 1 - \left( \frac{1 + a_c - \sqrt{\phi_c}}{a_c x_1} \right)^{d_c/b_c} & \text{for } \kappa_c > \kappa_{c,D} \end{cases}$$

$$x_1 = \frac{1 + a_c - \sqrt{\phi_c(\kappa_{c,D})}}{\frac{a_c}{b_c} \frac{g_c(\kappa_c - 1)}{f_{co}} + (1 + a_c)^2}$$

Notion of normalized fracture energy $g_c$ and uniaxial $\sigma - \varepsilon$ diagram

- Damage parameter in compression: $D_c = 1 - \exp(-d_c < \varepsilon_c^p - \varepsilon_{c,D}^p >)$
2.1. THEORY

- Effective compressive stress:

\[
\bar{\sigma}_c(\kappa_c) = \begin{cases} 
\frac{f_{co}}{a_c} \left(1 + a_c - \sqrt{\phi_c} \right) \sqrt{\phi_c} & \text{for } \kappa_c \leq \kappa_{c,D} \\
\frac{f_{co}}{a_c} \left(1 + a_c - \sqrt{\phi_c} \right)^{1-a_c/b_c} \sqrt{\phi_c} & \text{for } \kappa_c > \kappa_{c,D}
\end{cases}
\]

Remarks

1. \(f_{co}\) is the initial compressive strength (according to the EC2 \(f_{co}/f_c \approx 0.4\))

2. \(\sigma_{c,D}/f_c\) can be assumed as \(\sigma_{c,D}/f_c = 1 - \frac{f_t}{f_c}\) (at this stress level the transverse normal strain reaches value \(f_t/E\)); in the reference model \(\epsilon_{c,D}^p = 0\) (\(\sigma_{c,D}/f_c = f_{co}/f_c\))

3. \(\kappa_{c,D}\) is the damage variable that corresponds to the stress level \(\sigma_{c,D}/f_c\) in the uniaxial compression test

4. \(a_c, b_c\) and \(d_c\) are material properties (see Win.(3-1) and Win.(3-2) for details concerning their calibration)

5. the characteristic length \(l_c\) is equal to the size of the finite element \(h_c\)

6. \(\epsilon_c^p\) corresponds is the largest compressive plastic strain

Window 2-2: Determining value of damage variable \(\kappa_c\) for given stress level

- To determine value of damage variable \(\kappa_c\) for a given stress level in the uniaxial compression test we may use the equation

\[
\bar{\sigma}_c(\kappa_c) = \frac{f_{co}}{a_c} \left(1 + a_c - \sqrt{\phi_c} \right) \sqrt{\phi_c}
\]

- Its nondimensional form is as follows

\[
\frac{\sigma_c}{f_c} = \frac{f_{co}}{f_c a_c} \left(1 + a_c - \sqrt{\phi_c} \right) \sqrt{\phi_c}
\]

- In the pre-peak zone one can compute \(\sqrt{\phi_c}\) value for a given stress level \(\frac{\sigma_c}{f_c}\) as follows

\[
\sqrt{\phi_c} = \frac{\Delta_f}{2f_{co} \frac{a_c}{f_{co}}} (1 + a_c - \sqrt{\phi_c})
\]
• In the post-peak zone

\[
\sqrt{\phi_c} = \frac{f_{co}(1 + a_c) + \sqrt{\Delta} f_c a_c}{2 f_{co}}
\]

• \( \Delta = \left( \frac{f_{co}(1 + a_c)}{f_c a_c} \right)^2 - 4 \frac{f_{co} \sigma_c}{f_c a_c f_c} \)

• Then for given \( \sqrt{\phi_c} \) one may compute \( \kappa_c \) value using the following formula

\[
\kappa_c = \frac{f_{co} (\phi_c - (1 + a_c)^2)}{2 a_c b_c g_c} + 1
\]
2.1. THEORY

Window 2-4: Internal damage variable $\kappa_t$ in tension and damage factor $D_t$

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- Nominal tensile stress: $\sigma_t = f_{to} [(1 + a_t) \exp(-b_t \varepsilon^p_t) - a_t \exp(-2 b_t \varepsilon^p_t)]$
- Here we assume: $a_t = 0$
- Normalized fracture energy: $g_t = \frac{G_t}{l_c} = \int_0^\infty \sigma_t(\varepsilon^p_t) \, d\varepsilon^p_t$
- Internal damage variable: $\kappa_t = \frac{1}{g_t} \int_0^\varepsilon^p_t \sigma_t(\varepsilon^p_t) \, d\varepsilon^p_t = \frac{f_{to}}{g_t b_t} (1 - \exp(-b_t \varepsilon^p_t))$
- Evolution law for the hardening variable $\varepsilon^p_t$: $\dot{\varepsilon}^p_t = r(\hat{\sigma}) \dot{\varepsilon}^p_{max}$
- Evolution law for damage variable: $\dot{\kappa}_t = \frac{1}{g_t(\kappa_t)} \dot{\varepsilon}^p_t$

Notion of normalized fracture energy $g_t$ and uniaxial $\sigma - \varepsilon$ diagram

- Damage factor in tension: $D_t = 1 - \exp(-d_t \varepsilon^p_t)$
- Replacing hardening parameter $\varepsilon^p_t$ by $\kappa_t$ yields the following expressions for $\sigma_t$ and $D_t$
  $\sigma_t(\kappa_t) = f_{to}(1 - \kappa_t)$
  $D_t = 1 - (1 - \kappa_t)^{d_t/b_t}$
- Effective tensile stress: $\bar{\sigma}_t(\kappa_t) = f_{to} (1 - \kappa_t)^{1-d_t/b_t}$

Remarks

1. $\varepsilon^p_t$ is the largest positive principal plastic strain
CHAPTER 2. REFERENCE PLASTIC DAMAGE MODEL FOR CONCRETE (BY LEE AND FENVES) AND ITS MODIFICATIONS

Window 2-5: Plastic yield condition

- Yield condition (reference model ($\rho = 0$)):
  \[
  F(\bar{\sigma}, \kappa_t, \kappa_c) = \frac{1}{1 - \alpha} \left( \alpha I_1 + \sqrt{3} J_2 + \beta(\kappa_t, \kappa_c) < \hat{\sigma}_{max} > - c_c(\kappa_c) \right) - c_t(\kappa_t)
  \]
  \[
  \beta = \frac{c_c(\kappa_c)}{c_t(\kappa_t)} (1 - \alpha) - (1 + \alpha)
  \]
  \[
  c_c = \bar{\sigma}_c(\kappa_c) \quad c_t = \bar{\sigma}_t(\kappa_t)
  \]
- Yield condition (modified version ($\rho = 0.6$)):
  \[
  F(\bar{\sigma}, \kappa_t, \kappa_c) = \frac{1}{1 - \alpha} \left( \alpha I_1 + \sqrt{3} J_2 + \frac{\beta(\kappa_t, \kappa_c)}{1 - \rho} < \hat{\sigma}_{max} - \rho c_t > \right) - c_c(\kappa_c)
  \]

Initial strength envelopes for reference and modified models

Remarks

1. In the mixed tension-compression tests (Kupfer tests) reference model may undershoot the ultimate compressive stress by more than 40%; the introduced modification (via $\rho = 0.6$ parameter) cancels this parasitic effect
2.1. THEORY

Window 2-6: Plastic flow rule and evolution laws for plastic hardening variables $\varepsilon_p^t$, $\varepsilon_p^c$

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Smoothed plastic flow potential in $I_1 - \sqrt{J_2}$ plane

- Plastic flow potential: $G(\bar{\sigma}) = \sqrt{2} J_2 + \beta_H^2 + \alpha_p^* I_1$
- Constant dilatancy: $\alpha_p^* = \alpha_p$
- Variable dilatancy: $\alpha_p^* = \alpha_{po} f_{\kappa_t}(\kappa_t) + (1 - f_{\kappa_t}(\kappa_t)) f_{\kappa_c}(\kappa_c) \alpha_p$
  
  \[ f_{\kappa_t}(\kappa_t) = \begin{cases} 
  \frac{3}{2} \kappa_t - \frac{1}{2} \kappa_t^3 & \text{for } \kappa_t \leq 1 \\
  1 & \text{for } \kappa_t > 1 
\end{cases} \]
  
  \[ \kappa_t = \frac{\kappa_t}{\kappa_{t,ref}} \quad (\kappa_{t,ref} = 0.1) \]
  
  \[ f_{\kappa_c}(\kappa_c) = \begin{cases} 
  0 & \text{for } \kappa_c \leq 0 \\
  3\kappa_c^2 - 2\kappa_c^3 & \text{for } 0 \leq \kappa_c \leq 1 \\
  1 & \text{for } \kappa_c > 1 
\end{cases} \]
  
  \[ \kappa_c = \frac{\kappa_c - \kappa_{c,dil}}{\kappa_{c,peak} - \kappa_{c,dil}} \]
- Smoothing factor that cancels apex effect: $\beta_H = \alpha_d f_t \alpha_{po}$

Remarks

1. Omidi and Lotfi [7] added the term $\beta_H$ to the plastic flow potential; this trick helps to avoid apex state that may appear in the standard Drucker-Prager plastic flow potential used by Lee and Fenves; $\alpha_d = 1.0$ seems to be a reasonable value of this smoothing parameter.

2. $\kappa_{c,dil}$ and $\kappa_{c,peak}$ are damage variables corresponding to the stress levels $\varepsilon_{c,dil}/f_c$ and 1, respectively, in the uniaxial compression test (see Win.(2-3)).

3. In the extended version dilatancy may vary with plastic straining; its value is kept zero till $\kappa_c = \kappa_{c,dil}$ and then it grows up till $\alpha_p$ value at $\kappa_c = \kappa_{c,peak}$ (here a spline function is used).
Chapter 3

Estimating model parameters

3.1 Calibrating model parameters corresponding to the compressive stress domain

Window 3-1: Estimation of $a_c$ parameter

- The general expression for the current compressive strength:
  $\sigma_c = f_{co} \left[(1 + a_c) \exp(-b_c \varepsilon^p_c) - a_c \exp(-2 b_c \varepsilon^p_c)\right]$  

- Let us assume that the peak compressive strength is equal to $\max(\sigma_c) = f_c$

- The extremum of $\sigma_c$ is achieved at $\varepsilon_{c,extr}^p = - \frac{\ln\left(\frac{11 + a_c}{2 a_c}\right)}{b_c}$

- If we substitute $\varepsilon_{c,extr}^p$ to the expression for $\sigma_c$, we will obtain the relation:
  $f_c = \frac{1}{4} f_{co} \left(1 + a_c\right)^2 / a_c$

- From the above expression one may derive $a_c$ as:
  $a_c = 2 f_c / f_{co} - 1 + 2 \sqrt{(f_c / f_{co})^2 - f_c / f_{co}}$
Window 3-2: Estimation of $b_c$ and $d_c$: approach that preserves $G_c$ and $\tilde{D}_c$

- In this procedure we want to preserve
  * normalized fracture energy $g_c = G_c/l_c$ value
  * given damage factor $\tilde{D}_c$ at a given stress $\tilde{\sigma}_c$

- Using the expression for normalized fracture energy in compression one may easily derive value of $b_c$ parameter
  \[ b_c = \frac{f_{co}(1 + a_c/2)}{g_c} \]

- Then the $d_c$ parameter is computed as
  \[ d_c = b_c \frac{\ln(1 - \tilde{D}_c)}{\ln \left( \frac{1 + a_c - \sqrt{\phi^*_c}}{x_1 a_c} \right)} \]
  the $\phi^*_c$ value corresponds to the stress level value $\tilde{\sigma}_c/f_c$ (use solution for the post-peak zone from Win.(2-3))

Remarks

1. Strain at peak $\varepsilon_{c1}$ is not under control
3.2 Calibrating model parameters corresponding to the tensile stress domain

Window 3-3: Estimation of $b_t$ and $d_t$

- Once the normalized fracture energy $g_t$ is known one may derive $b_t$ parameter:
  \[ b_t = \frac{f_{to}}{g_t} \]

- Here for given stress value $\tilde{\sigma}_t$ damage factor in tension is known and equal to $\tilde{D}_t$

- Plastic tensile strain corresponding to the stress value $\tilde{\sigma}_t$ is equal to
  \[ \tilde{\varepsilon}_p = -\frac{\ln(\tilde{\sigma}_t/f_{to})}{b_t} \]

- $d_t$ parameter is equal to
  \[ d_t = -\frac{\ln (1 - \tilde{D}_t)}{\tilde{\varepsilon}_p} \]
• Directional element size is defined as follows:
  \[ h_e = r(\bar{\sigma}_i) h_e^{\varepsilon_1} + (1 - r(\bar{\sigma}_i)) h_e^{\varepsilon_3} \]

• \( h_e^{\varepsilon_1} \) is the element size measured along direction of \( \varepsilon_1 \)
• \( h_e^{\varepsilon_3} \) is the element size measured along direction of \( \varepsilon_3 \)

• Element size in the direction of a given unit vector \( v \) is defined as follows

  \[ h_e = \sum_{i=1}^{Ndm} h_{\xi_i} \left( \frac{v_{\xi_i}}{\|v\|} \right)^2 \]

  \[ h_{\xi_i}^{(\xi)} = \| x \left( A_{\xi_i}^{(+)\xi} \right) - x \left( A_{\xi_i}^{(-)\xi} \right) \| \]

where

\[ \left( A_{\xi_i}^{(+)\xi} \right)_k = \delta_{ki}, \quad \left( A_{\xi_i}^{(-)\xi} \right)_k = -\delta_{ki}, \quad v_{\xi_i}^T = J^{-1} v^T, \]

\[ \xi_1 = \xi, \quad \xi_2 = \eta, \quad \xi_3 = \zeta \]

and \( J \) is the Jacobian matrix of the isoparametric mapping, \( v_{\xi_i} \) is the i-th component of a given vector in the local \( (\xi, \eta, \zeta) \) coordinate system, \( \delta_{ki} \) is the Kronecker’s symbol, \( A_{\xi_i}^{(+)\xi} \) and \( A_{\xi_i}^{(-)\xi} \) are points with \( Ndm \) coordinates defined in the local system. The geometrical interpretation of this definition is shown in Fig. 1.

![Figure 1: Definition (D2) for element Q4](image)

**Remarks**

1. In the current implementation element size is kept constant within the given time step and based on the total strain tensor set up at the previous converged step (explicit scheme)
Chapter 4

Numerical implementation

The stress-strain integration procedure of the modified Lee-Fenves model was developed in the principal stress space following the scheme proposed by Runnesson and Larson. This scheme can be adopted here as the principal effective stress directions become unchanged during plastic correction. This is caused by the conical form of the plastic potential.

4.1 Stress strain integration scheme

Window 4-1: General scheme

- Transform nominal stress state at the last converged step $N$ to the effective stress state
  \[ \bar{\sigma}_N = \frac{1}{1 - D_N} \sigma_N \]
- Compute trial effective stress state:
  \[ \bar{\sigma}^t_{N+1} = \sigma_N + D^t \Delta e_{N+1} \]
- Compute principal effective stress state: \( \hat{\sigma}_{N+1} \)
- Set initial values of damage variables:
  \[ \kappa_{l,N+1} = \kappa_{l,N} \]
  \[ \kappa_{c,N+1} = \kappa_{c,N} \]
- If \( F(\hat{\sigma}_{N+1}, \kappa_{l,N}, \kappa_{c,N}) \geq 0 \): perform plastic corrector algorithm (Win.(4-2))
- If \( F(\hat{\sigma}_{N+1}, \kappa_{l,N}, \kappa_{c,N}) < 0 \): update current stress state: \( \bar{\sigma}_{N+1} = \bar{\sigma}^t_{N+1} \)
- Update damage parameter: \( D_{N+1} \)
- Map current effective stress to the nominal stress state:
  \[ \bar{\sigma}_{N+1} = (1 - D_{N+1}) \sigma_{N+1} \]
CHAPTER 4. NUMERICAL IMPLEMENTATION

Window 4-2: Plastic corrector algorithm

Set of independent variables: \( \{ \Delta \hat{\varepsilon}_N^{p+1}, \kappa_{t,N+1}, \kappa_{c,N+1}, \Delta \lambda_{N+1} \}^T \)

Stress-strain integration is carried out by solving the following set of nonlinear equations:

1. \( r_{\varepsilon} = \Delta \hat{\varepsilon}_N^{p+1} - \Delta \lambda_{N+1} \cdot b(\hat{\sigma}_N^{+1}, \kappa_{t,N+1}, \kappa_{c,N+1}) = 0 \)
2. \( r_{\kappa_t} = \kappa_{t,N+1} - \kappa_{t,N} - \frac{\sigma_t(\kappa_{t,N+1})}{g_t} \cdot r(\hat{\sigma}_N^{+1}) < \Delta \varepsilon_1^p > = 0 \)
3. \( r_{\kappa_c} = \kappa_{c,N+1} - \kappa_{c,N} - \frac{\sigma_c(\kappa_{c,N+1})}{g_c} \cdot (1 - r(\hat{\sigma}_N^{+1})) < -\Delta \varepsilon_3^p > = 0 \)
4. \( r_F = F(\hat{\sigma}_N^{+1}, \kappa_{t,N+1}, \kappa_{c,N+1}) = 0 \)

This system is solved using an iterative Newton-Raphson scheme.
Chapter 5

Creep and aging

The implemented model can also describe a visco-elastic aging creep phenomenon following the EC2 standard (EN 1992-1-1:2004+AC:2008). Nonlinear creep effects that may appear for larger compressive stresses are neglected and no distinction is made between creep in compression and tension.

Window 5-1: Description of creep phenomenon in EC2

- Time dependent creep coefficient: \( \phi(t, t_o) = \phi_o \beta_c(t, t_o) \)
- Basic creep coefficient: \( \phi_o = \phi_{RH} \beta(f_{cm}) \beta(t_o) \)

\[
\phi_{RH} = \left\{ \begin{array}{ll}
1 + \frac{1 - \frac{RH}{100}}{0.1 h_o^{1/3}} & \text{for } f_{cm} \leq 35 \text{ MPa} \\
\left(1 + \frac{1 - \frac{RH}{100}}{0.1 h_o^{1/3}}\right) \alpha_2 & \text{for } f_{cm} > 35 \text{ MPa}
\end{array} \right.
\]

- \( \beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} \)
- \( \beta(t_o) = \frac{1}{0.1 + t_o^{0.2}} \)
- \( \beta_c(t, t_o) = \left[\frac{t - t_o}{\beta_H + t - t_o}\right]^{0.3} \)
- \( \beta_H = \left\{ \begin{array}{ll}
\min (1.5 [1 + (0.012RH)^{18}] h_o + 250, 1500) & \text{for } f_{cm} \leq 35 \text{ MPa} \\
\min (1.5 [1 + (0.012RH)^{18}] h_o + 250\alpha_3, 1500\alpha_3) & \text{for } f_{cm} > 35 \text{ MPa}
\end{array} \right. \)

Remarks:

1. Creep coefficient is understood as a ratio between creep strain \( \varepsilon^{cr} \) and an instantaneous elastic strain computed for concrete loaded at \( t = 28 \) days
2. In the above expressions \( \alpha_1 = (35/f_{cm})^{0.7} \), \( \alpha_2 = (35/f_{cm})^{0.2} \), \( \alpha_3 = (35/f_{cm})^{0.5} \), relative humidity \( RH \) is expressed in [%], the equivalent member size \( h_o \) is expressed in [mm], an averaged compressive strength \( f_{cm} \) is expressed in [MPa] and loading time \( t_o \) is expressed in [days].

3. Time parameter \( t \) and \( t_o \) can be replaced by a corresponding temperature adjusted value \( t_T \) defined as follows:

\[
t_T(t) = \int_t^{t_1} \exp\left(-\frac{Q}{R} \left( \frac{1}{273 + T(\tau)} - \frac{1}{273 + T_{ref}} \right) \right) d\tau
\]

\( T_{ref} = 20 [^\circ C] \)

Window 5-1

Window 5-2: Implementation of EC2 creep within visco-elastic framework

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Implementation scheme is partially based on algorithm proposed by Havlásek [1]. The main goal in the implementation scheme is to avoid time dependent values of \( A_\mu \) parameters. Therefore current creep strain increment is divided by the \( v^{cr} \) factor that amplifies (in early stages) or reduces (for old concrete) creep rate. In this approach the experimental creep curve is approximated by a chain of nonaging Kelvin units.

- Creep strain increment is computed using the following scheme:

\[
\Delta e^{cr}_{n+1} = D_o^{-1} \frac{1}{v^{cr}_{n+1/2}} \sum_{\mu=1}^{N} A_\mu \left( 1 - \beta_{\mu,n+1} \right) \sigma_{v\mu,n}
\]

where:

- \( D_o^{-1} \) is an elastic compliance matrix computed for unit Young’s modulus
- \( v^{cr}_{n+1/2} \) is an extra scaling factor amplifying creep rate due to aging phenomenon (here it is not equivalent to the fraction of solidified layers)
- \( \sigma_{v\mu,n+1} \) represents viscous effective stresses in \( \mu \)-th Kelvin unit (number of Kelvin units in chain is denoted by \( N \))
- \( \beta_{\mu,n+1} = \exp(-\Delta t_{n+1}/\tau_\mu) \)
- \( \tau_\mu \) is a retardation time of \( \mu \)-th Kelvin unit
- \( A_\mu \) is the ultimate creep strain value in \( \mu \)-th Kelvin unit

- Viscous stress update: \( \sigma_{v\mu,n+1} = \beta_{\mu,n+1} \sigma_{v\mu,n} + \lambda_{\mu,n+1} \Delta \sigma_{n+1} \)

- \( \lambda_{\mu,n+1} = (1 - \beta_{\mu,n+1}) \frac{\tau_\mu}{\Delta t_{n+1}} \)

- The algorithmic effective Young’s modulus is expressed as follows:

\[
\overline{E} = \frac{1}{v^E E_{28}} + \frac{1}{v^E} \sum_{\mu=1}^{N} (1 - \lambda_{\mu,n+1}) A_\mu
\]

- \( v^E = \sqrt{\beta_{cc}} \)
\[ \beta_{cr} = \begin{cases} \exp(s(1 - \sqrt{28/t})) & \text{for } t \leq 28 \text{ days} \\ 1 & \text{for } t > 28 \text{ days} \end{cases} \]

**Window 5-3: Derivation of \( v^{cr} \) function**

- Evolution of creep strain in time, according to EC2, can be expressed by the following equation

\[ \varepsilon^{cr} = A_1 \left( \frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o) \]

where \( A_1 = \phi_{RH}\beta(f_{cm}) \)

- Evolution of the reference creep strain for concrete loaded at \( t_o = 28 \) days (matured concrete) can be defined as

\[ \varepsilon_{ref}^{cr} = A_1 \left( \frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o = 28) \]

- The reference creep strain curve is taken here as a basis for optimization of \( A_\mu \) coefficients in chain of nonaging Kelvin units (retardation times \( \tau_\mu \) are predefined by considering duration of carried out analysis time)

- To derive \( v^{cr} \) we assume the following creep strain rates compatibility condition

\[ \dot{\varepsilon}^{cr} = \frac{1}{v^{cr}} \dot{\varepsilon}_{ref}^{cr} \]

- This yields the following definition of \( v^{cr} \) function

\[ v^{cr} = \frac{\beta(t_o = 28)}{\beta(t_o)} \]

where \( t_o \) is the age of concrete at the beginning of analysis
Chapter 6
User interface

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>Parameter</th>
<th>Unit</th>
<th>Range</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Strength in compression</td>
<td>$f_c$</td>
<td>[MPa]</td>
<td></td>
<td>Uniaxial compressive strength; positive in compression; use according to standards (EC2 for instance)</td>
</tr>
<tr>
<td></td>
<td>$f_{co}/f_c$</td>
<td>[-]</td>
<td>$0.4 \div 0.8$</td>
<td>Initial to peak compressive strength ratio</td>
</tr>
<tr>
<td></td>
<td>$f_{co}/f_c$</td>
<td>[-]</td>
<td>$1.1 \div 1.2$</td>
<td>Initial biaxial to uniaxial strength ratio</td>
</tr>
<tr>
<td>Damage in compression</td>
<td>$\sigma_{c,D}/f_c$</td>
<td>[-]</td>
<td>$0.4 \div 0.9$</td>
<td>Stress level at which damage starts to occur; $\sigma_{c,D}/f_c \geq f_{co}/f_c$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_c/f_c$</td>
<td>[-]</td>
<td>$\leq 1.0$</td>
<td>Stress level for damage calibration (post-peak)</td>
</tr>
<tr>
<td></td>
<td>$D_c$</td>
<td>[-]</td>
<td>$\geq 0.3$</td>
<td>Damage parameter in uniaxial compression at the assumed reference stress level; if this value is too small a warning will be generated</td>
</tr>
<tr>
<td></td>
<td>$G_c$</td>
<td>[MN/m]</td>
<td>$50 \ G_f \leq G_c \leq 100 \ G_f$</td>
<td>Fracture energy in compression;</td>
</tr>
<tr>
<td>Strength in tension</td>
<td>$f_t$</td>
<td>[MPa]</td>
<td></td>
<td>Uniaxial tensile strength; positive in tension; use according to standards (EC2 for instance)</td>
</tr>
<tr>
<td>Damage in tension</td>
<td>$\sigma_t/f_t$</td>
<td>[-]</td>
<td>$0.5 (&lt; 1)$</td>
<td>Stress level for damage calibration (post-peak)</td>
</tr>
<tr>
<td></td>
<td>$D_t$</td>
<td>[-]</td>
<td>$\geq 0.3$</td>
<td>Damage parameter in uniaxial tension at the assumed reference stress level; if this value is too small a warning will be generated</td>
</tr>
</tbody>
</table>


**CHAPTER 6. USER INTERFACE**

\[ G_t \quad [\text{MN/m}] \quad 50 \times 10^{-6} \div 150 \times 10^{-6} \quad \text{Fracture energy in tension; can be estimated as } 73 \cdot f_c^{0.18} \times 10^{-6} \quad \text{(use MPa as strength unit here)} \]

\[ s \quad [-] \quad 0.1 \div 0.3 \quad \text{Stiffness recovery factor due to crack closure for tension-compression cycles} \]

<table>
<thead>
<tr>
<th>Dilatancy type</th>
<th>Constant/Variable</th>
<th>Dilatancy type</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \alpha_p ]</td>
<td>[-]</td>
<td>0.1 \div 0.5</td>
</tr>
<tr>
<td>[ \alpha_{po} ]</td>
<td>[-]</td>
<td>0.1 \div 0.4</td>
</tr>
<tr>
<td>[ \sigma_{c,dil} / f_c ]</td>
<td>[-]</td>
<td>0.4 \div 1.0</td>
</tr>
<tr>
<td>[ \alpha_d ]</td>
<td>[-]</td>
<td>0.2 \div 2.0</td>
</tr>
</tbody>
</table>

**Char. length (RC)**

\[ t^{RC} \quad [\text{m}] \quad > 0 \quad \text{Characteristic length for reinforced concrete} \]

---

**Window 6-1**

**Window 6-2: Creep properties**

ZSoil®.PC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A ]</td>
<td>[1/MPa]</td>
<td>( A = A_1 = \phi_{RH} \beta(f_{cm}) / E ) (see EC2)</td>
<td></td>
</tr>
<tr>
<td>[ B ]</td>
<td>[day]</td>
<td>( B = \beta_H ) (see EC2)</td>
<td></td>
</tr>
<tr>
<td>Initial age</td>
<td>[day]</td>
<td>&gt; 0.5</td>
<td>Age of analyzed concrete</td>
</tr>
<tr>
<td>Equivalent time flag</td>
<td>( Q / R )</td>
<td>(&lt;\text{ON,OFF}&gt;)</td>
<td>Flag whether to use temperature adjusted time</td>
</tr>
<tr>
<td>[ T_{ref} ]</td>
<td>[K]</td>
<td>4000</td>
<td>Ratio between activation energy and universal gas constant</td>
</tr>
<tr>
<td>[ s ]</td>
<td>[-]</td>
<td>0.38</td>
<td>Strength evolution parameter (EC2)</td>
</tr>
<tr>
<td>[ t_{28} ]</td>
<td>[day]</td>
<td>28.0</td>
<td>Time of 28 days in formula for ( \beta_{cc} ) (EC2)</td>
</tr>
<tr>
<td>$n$</td>
<td>[-]</td>
<td>0.5</td>
<td>Exponent in expression for stiffness modulus (applied to $\beta_{cc}$) (EC2)</td>
</tr>
</tbody>
</table>

Window 6-2
Chapter 7

Benchmarks

7.1 Gopalaratnam and Shah monotonic and cyclic uniaxial tensile tests (1985)

Files: CPDM-UNIAX-TENS-GOP.inp, CPDM-UNIAX-TENS-CYCLIC-GOP.inp

Comparison of the experimental and numerical prediction of the $\sigma - \varepsilon$ curves in the uniaxial (monotonic and cyclic) tensile tests (after Gopalaratnam and Shah) is the aim of this benchmark. This test is run with a single B8 element of size 82.6 mm x 82.6 mm x 82.6 mm[4] using displacement driven loading program. The test setup is shown in the following figure. Imposed displacements in X-direction are set at nodes 2, 3, 6 and 7.

![Figure 7.1: Uniaxial tensile test: mesh and boundary conditions](image)

Material properties (after Lee and Fenves, 1998) are summarized in the table below. It should be mentioned that all parameters with the * are not meaningful in the considered test.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
</table>


### Elasticity

<table>
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<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
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<tr>
<td>$E$</td>
<td>[MPa]</td>
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</tr>
<tr>
<td>$\nu$</td>
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<td>0.18</td>
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</table>

### Nonlinear Compression

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<th>Value</th>
</tr>
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<tr>
<td>$f$</td>
<td>[MPa]</td>
<td>27.6*</td>
</tr>
<tr>
<td>$f_{cp}/f_c$</td>
<td></td>
<td>0.4*</td>
</tr>
<tr>
<td>$f_{cbo}/f_{co}$</td>
<td></td>
<td>1.16*</td>
</tr>
<tr>
<td>$\sigma_{c,D}/f_c$</td>
<td></td>
<td>0.4*</td>
</tr>
<tr>
<td>$\sigma_c/f_c$</td>
<td>[MPa]</td>
<td>1.0*</td>
</tr>
<tr>
<td>$D_c$</td>
<td></td>
<td>0.4*</td>
</tr>
<tr>
<td>$G_c$</td>
<td>[MN/m]</td>
<td>5.69*10^{-3}</td>
</tr>
</tbody>
</table>

### Tension

<table>
<thead>
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<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$f_t$</td>
<td>[MPa]</td>
<td>3.48</td>
</tr>
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<td>$\sigma_t/f_t$</td>
<td></td>
<td>0.5</td>
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<tr>
<td>$D_t$</td>
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<td>0.5</td>
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### Dilatancy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$G_t$</td>
<td>[MN/m]</td>
<td>4*10^{-5}</td>
</tr>
<tr>
<td>$s$</td>
<td></td>
<td>0.2*</td>
</tr>
</tbody>
</table>

### Calibration

- Preserve $G_c$ & $D_c$.
- $\varepsilon_{c1}$ unused.

### Dilatancy Type

- Constant

<table>
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<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>$\sigma_{c,dil}/f_c$</td>
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</tr>
<tr>
<td>$\alpha_p$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha_{po}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The resulting $\sigma_1 - \varepsilon_1$ curves for monotonic and cyclic tests are shown in the next two figures.
7.1. GOPALARATNAM AND SHAH MONOTONIC AND CYCLIC UNIAXIAL TENSILE TESTS (1985)

Figure 7.2: Uniaxial monotonic tensile test: comparison of $\sigma_1 - \varepsilon_1$ curves

Figure 7.3: Uniaxial cyclic tensile test: comparison of $\sigma_1 - \varepsilon_1$ curves
7.2 Karsan and Jirsa monotonic and cyclic uniaxial compression tests (1969)

In this benchmark both the monotonic and cyclic uniaxial compression tests (after Karsan and Jirsa) are reproduced with aid of the plastic damage model. These tests are run with a single B8 element of size 82.6 mm x 82.6 mm x 82.6 mm [4] using displacement driven loading program. The test setup is shown in the following figure.

Figure 7.4: Uniaxial compression test: mesh and boundary conditions

Material properties (after Lee and Fenves, 1998) are summarized in the table below. It should be mentioned that all parameters with the * are not meaningful in the considered test. Value of the parameter \( \frac{f_{co}}{f_c} \) (in the original formulation by Lee and Fenves) is not given in the article therefore it is assumed as \( \frac{f_{co}}{f_c} = 0.62 \).

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
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<tbody>
<tr>
<td>Elastic</td>
<td></td>
<td>E</td>
<td>[MPa]</td>
<td>31000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu )</td>
<td>[-]</td>
<td>0.18</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>Compression</td>
<td>( f_c )</td>
<td>[MPa]</td>
<td>27.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{f_{co}}{f_c} )</td>
<td>[-]</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{f_{cbo}}{f_{co}} )</td>
<td>[-]</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_{c,D}/f_c )</td>
<td>[-]</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tilde{\sigma}_c/f_c )</td>
<td>[-]</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D_c</td>
<td>[-]</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G_c</td>
<td>[MN/m]</td>
<td>5.69*10^{-3}</td>
</tr>
<tr>
<td>Tension</td>
<td></td>
<td>f_t</td>
<td>[MPa]</td>
<td>3.48*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_t/f_t )</td>
<td>[-]</td>
<td>0.5*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D_t</td>
<td>[-]</td>
<td>0.5*</td>
</tr>
</tbody>
</table>
### Dilatancy

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_t$</td>
<td>[MN/m]</td>
<td>$4 \times 10^{-5}$*</td>
</tr>
<tr>
<td>$s$</td>
<td>[-]</td>
<td>0.2</td>
</tr>
<tr>
<td>Type</td>
<td></td>
<td>Constant</td>
</tr>
<tr>
<td>$\sigma_{c,dil}/f_c$</td>
<td>[-]</td>
<td>0.62</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>[-]</td>
<td>0.20</td>
</tr>
<tr>
<td>$\alpha_{po}$</td>
<td>[-]</td>
<td>0.20</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>[-]</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The resulting $\sigma_1 - \varepsilon_1$ curves are shown in the next two figures.
CHAPTER 7. BENCHMARKS

Figure 7.5: Uniaxial compression test: comparison of $\sigma_1 - \varepsilon_1$ curves for monotonic compression test

Figure 7.6: Uniaxial compression test: comparison of $\sigma_1 - \varepsilon_1$ curves for cyclic uniaxial compression test
7.3 Kupfer’s tests

Files: CPDM-Kupfer-1-0.inp, CPDM-Kupfer-1-0_52.inp, CPDM-Kupfer-1-0_226.inp, CPDM-Kupfer-1-1.inp, CPDM-Kupfer-1-t-0_052.inp

In this benchmark the monotonic biaxial compression-compression and compression-tension tests (after Kupfer) are reproduced with aid of the plastic damage model. These tests are run with a 2x2 B8 elements of size 100.0 mm x 100.0 mm x 50.0 mm each (as in [4]) using spherical arc-length displacement control driver (node A is used to control displacements). To avoid nonuniform deformations all nodes in face A have same Z-displacement as node A (using periodic BC option). The test setup is shown in the following figure. Five tests are run for different $q_1/q_2$ ratios i.e. $q_1/q_2 = -1/0$ (uniaxial compression), $q_1/q_2 = -1/-1$ (biaxial compression), $q_1/q_2 = -1/-0.52$, $q_1/q_2 = -1/-0.226$, $q_1/q_2 = -1/+0.052$ (compression-tension).

Material properties, that were not given in the cited article, are summarized in the table below.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
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<td>[MPa]</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$\nu$</td>
<td>[-]</td>
<td>0.20</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>Compression</td>
<td>$f_c$</td>
<td>[MPa]</td>
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</tr>
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<td></td>
<td></td>
<td>$f_{co}/f_c$</td>
<td>[-]</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{to}/f_{co}$</td>
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CHAPTER 7. BENCHMARKS

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<th>Value</th>
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<tr>
<td></td>
<td>$\sigma_{c}/f_c$</td>
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<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$D_c$</td>
<td>[-]</td>
<td>0.44</td>
</tr>
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<td></td>
<td>$G_c$</td>
<td>[MN/m]</td>
<td>$4.5 \times 10^{-3}$</td>
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<tr>
<td>Calibration</td>
<td>[-]</td>
<td>Preserve $G_c$ &amp; $D_c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>[MPa]</td>
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</tr>
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<td>$\bar{\sigma}_t/f_t$</td>
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<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$D_t$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$G_t$</td>
<td>[MN/m]</td>
<td>$1.5 \times 10^3$</td>
</tr>
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<td>Dilatancy</td>
<td>$s$</td>
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<td></td>
<td>Type</td>
<td>[-]</td>
<td>Variable</td>
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<tr>
<td></td>
<td>$\sigma_{c,dil}/f_c$</td>
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<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\alpha_p$</td>
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</tr>
<tr>
<td></td>
<td>$\alpha_{po}$</td>
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<td>0.20</td>
</tr>
<tr>
<td></td>
<td>$\alpha_d$</td>
<td>[-]</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The resulting stress-strain diagrams are shown in the following figures:

**Figure 7.8:** $q_1/q_2 = -1/0$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_2$ (right) diagrams

**Figure 7.9:** $q_1/q_2 = -1/0$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_2$ (right) diagrams for $\alpha_p = 0.5$
7.3. KUPFER’S TESTS

Figure 7.10: \( q_1/q_2 = -1/ -0.52: \sigma_1 - \varepsilon_1 \) (left) and \( \sigma_1 - \varepsilon_3 \) (right) diagrams

Figure 7.11: \( q_1/q_2 = -1/ -0.52: \sigma_1 - \varepsilon_1 \) (left) and \( \sigma_1 - \varepsilon_2 \) (right) diagrams

Figure 7.12: \( q_1/q_2 = -1/ -1: \sigma_1 - \varepsilon_1 \) (left) and \( \sigma_1 - \varepsilon_3 \) (right) diagrams
CHAPTER 7. BENCHMARKS

Figure 7.13: $q_1/q_2 = -1/ -0.226$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_3$ (right) diagrams

Figure 7.14: $q_1/q_2 = -1/ -0.226$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_2$ (right) diagrams

Figure 7.15: $q_1/q_2 = -1/ +0.052$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_3$ (right) diagrams
7.3. **KUPFER’S TESTS**

Figure 7.16: $q_1/q_2 = -1/ + 0.052$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_3$ (right) diagrams without modification of the yield condition (reference model, $\rho = 0.0$)

Figure 7.17: $q_1/q_2 = -1/ + 0.052$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_2$ (right) diagrams

Figure 7.18: $q_1/q_2 = -1/ + 0.052$: $\sigma_1 - \varepsilon_1$ (left) and $\sigma_1 - \varepsilon_2$ (right) diagrams without modification of the yield condition (reference model, $\rho = 0.0$)
CHAPTER 7. BENCHMARKS

7.4 Three point bending test

Files: Malvar-Warren-ps.inp

The three point bending test for plain concrete, carried out experimentally by Malvar and Warren [6], is analyzed in this section. Geometry of the notched specimen, mesh and boundary conditions used in the test, run as a plane-strain problem, are shown in the following figure. The element adjacent to the fixed node in the right bottom point is modeled as elastic to cancel local plastic effects. A displacement driven loading program, with the maximum assumed deflection of 5mm is used with 50 equal steps.

![Three point bending test: geometry, mesh and boundary conditions](image)

Material properties are summarized in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>[MPa]</td>
<td>21700</td>
</tr>
<tr>
<td>$\nu$</td>
<td>[-]</td>
<td>0.20</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>[MPa]</td>
<td>29.0</td>
</tr>
<tr>
<td>$\sigma_{c,t}/f_c$</td>
<td>[-]</td>
<td>0.6</td>
</tr>
<tr>
<td>$\sigma_{c,D}/f_c$</td>
<td>[-]</td>
<td>1.16</td>
</tr>
<tr>
<td>$\sigma_{c,t}/f_c$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{c,D}/f_c$</td>
<td>[-]</td>
<td>1.0</td>
</tr>
<tr>
<td>$D_i$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$G_{c}$</td>
<td>[MN/m]</td>
<td>4.5*10^{-3}</td>
</tr>
<tr>
<td>$f_t$</td>
<td>[MPa]</td>
<td>3.1</td>
</tr>
<tr>
<td>$\sigma_{t}/f_t$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$D_t$</td>
<td>[-]</td>
<td>0.55</td>
</tr>
<tr>
<td>$G_{t}$</td>
<td>[MN/m]</td>
<td>0.65*10^{-4}</td>
</tr>
<tr>
<td>$s$</td>
<td>[-]</td>
<td>0.2</td>
</tr>
<tr>
<td>Dilatancy</td>
<td>Constant</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{c,dil}/f_c$</td>
<td>[-]</td>
<td>0.6</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>[-]</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{po}$</td>
<td>[-]</td>
<td>0.20</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>[-]</td>
<td>1.0</td>
</tr>
</tbody>
</table>
7.5 RC SLAB UNDER POINT LOADING

Comparison of the experimental and numerical force-deflection diagrams is shown in Fig. 7.20. The ZSoil prediction quite well matches the experimental curve reported by Malvar and Warren. The peak is predicted at same deflection as in the experiment and a small overshoot is visible just after the peak. This effect can be explained by a specific form of the softening described by an exponential function. It has to be mentioned that Lee and Fenves obtained their result by diminishing the tensile strength from 3.1 MPa to 2.4 MPa and tensile fracture energy from 70 [N/m] to 30 [N/m]. This effect can only be explained by the fact that their elements did not suffer from severe locking phenomenon.

![Figure 7.20: Three point bending test: comparison of experimental and numerical force-deflection diagrams](image)

7.5 RC slab under point loading

Files: Jofriet-3x3.inp, Jofriet-6x6.inp, Jofriet-12x12.inp

A square reinforced concrete slab with dimensions [2] 91.44 cm x 91.44 cm, 4.45cm thick, simply supported at four corners, and loaded by a concentrated force at the center, is analyzed here (see Fig. 7.21). Due to dual symmetry only one quarter is discretized. The slab is reinforced by an orthogonal reinforcement with density $\rho = 0.85\%$ same in both directions. The averaged effective depth of the cross section is equal to 3.33 cm (to simplify the analysis both reinforcement layers are placed at 1.12 cm from the bottom fibers). It is very important to cancel membrane forces by proper setting of boundary conditions. Material properties for
concrete, taken from publication published by Krätzig et al. [3], are as follows:

Material properties are summarized in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>[MPa]</td>
<td>28613</td>
</tr>
<tr>
<td>$\nu$</td>
<td>[-]</td>
<td>0.15</td>
</tr>
<tr>
<td>$f_{c}$</td>
<td>[MPa]</td>
<td>37.92</td>
</tr>
<tr>
<td>$f_{co}/f_{c}$</td>
<td>[-]</td>
<td>1.16</td>
</tr>
<tr>
<td>$\sigma_{c,D}/f_{c}$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{c}/f_{c}$</td>
<td>[-]</td>
<td>1.0</td>
</tr>
<tr>
<td>$D_{c}$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$G_{c}$</td>
<td>[MN/m]</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$f_{t}$</td>
<td>[MPa]</td>
<td>2.91</td>
</tr>
<tr>
<td>$\sigma_{t}/f_{t}$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
<tr>
<td>$D_{t}$</td>
<td>[-]</td>
<td>0.7</td>
</tr>
<tr>
<td>$G_{t}$</td>
<td>[MN/m]</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$s$</td>
<td>[-]</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Steel is modeled with an elastic-plastic model characterized by Young modulus $E_s = 201300$ MPa and strength $f_y = 345.4$ MPa.

Comparison of the experimental and numerical force-displacement diagrams is shown in Fig. 7.23. A typical mismatch between the model and experiment is observed at deflections in range $1 \div 2$mm. It can be reduced by diminishing the tensile strength up to $f_t = 2.3$ MPa (see Krätzig et. al). Better prediction can be obtained for fixed value of the characteristic length $l_{RC} = 0.13$ m.
7.5. RC SLAB UNDER POINT LOADING

Figure 7.22: RC slab: comparison of experimental and numerical force-deflection diagrams ($l_c = h^c$)

Figure 7.23: RC slab: comparison of experimental and numerical force-deflection diagrams for fixed value of characteristic length $l_{RC}^c = 0.13$ m
7.6 Creep in monotonic tensile test

Files:
CPDM-aging-creep-cont-2D-to-2days.inp,
CPDM-aging-creep-cont-2D-to-3days.inp,
CPDM-aging-creep-cont-2D-to-7days.inp,
CPDM-aging-creep-cont-2D-to-14days.inp,
CPDM-aging-creep-cont-2D-to-28days.inp,
CPDM-aging-creep-cont-2D-to-90days.inp
CPDM-aging-creep-shell-to-2-days

The test setup is shown in the following figure.

![Test setup for uniaxial creep tensile test](image)

Elastic and nonlinear material properties are the same as in the Kupfer test (7.3) except Poisson value that is equal to zero here. Creep properties are summarized in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$[1/\text{MPa}]$</td>
<td>$9 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B$</td>
<td>[day]</td>
<td>500</td>
</tr>
<tr>
<td>Initial age ($t_0$)</td>
<td>[day]</td>
<td>$2/3/7/14/28/90$</td>
</tr>
<tr>
<td>Equivalent time flag</td>
<td></td>
<td>OFF</td>
</tr>
<tr>
<td>$Q$</td>
<td>[K]</td>
<td>unused</td>
</tr>
<tr>
<td>$R$</td>
<td>[C]</td>
<td>unused</td>
</tr>
<tr>
<td>$T_{\text{ref}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>[-]</td>
<td>0.38</td>
</tr>
<tr>
<td>$t_{28}$</td>
<td>[day]</td>
<td>28</td>
</tr>
<tr>
<td>$n$</td>
<td>[-]</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Load time function associated with the uniform loading is defined as

<table>
<thead>
<tr>
<th>time [days]</th>
<th>LTF (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.01</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The resulting displacement time histories are shown in the figure below. Perfect agreement between theory and EC2 creep model is observed.

![Figure 7.25: Evolution of $u_x$ displacement in time for different loading times (linear range)](image)

Same result is obtained when layered Q4-MITC shell element is used (here only $t_o = 2$ days is considered)
Figure 7.26: Evolution of \( u_x \) displacement in time for \( t_o = 2 \) [days] (shell element is used here)

### 7.7 Creep under variable loading conditions

**Files:**

CPDM-aging-creep-cont-2D-superp.inp

Showing the effect of creep superposition for a single element tensile test is the aim of this benchmark. The test setup and material data is exactly the same as in the benchmark shown in section 7.6 except the age of concrete that is equal to \( t_o = 90 \) [days] here. The element is loaded at time \( t = 0 \) (time at beginning of the analysis while age of concrete is equal to 90 days) with \( q(t = 0) = 0.1 \) MN/m\(^2\). At time \( t = 5 \) days loading \( q(t = 5) = 0.2 \) MN/m\(^2\). Comparison of the analytical and numerical solutions is shown in figure below.
7.7. CREEP UNDER VARIABLE LOADING CONDITIONS

Figure 7.27: Evolution of $u_x$ displacement in time
Bibliography


