THE HARDENING SOIL MODEL - A PRACTICAL GUIDEBOOK

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<table>
<thead>
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<th>Description</th>
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<tr>
<td>$\varepsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$\varepsilon_v$</td>
<td>volumetric strain</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>shear strain</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress</td>
</tr>
<tr>
<td>$p$</td>
<td>total mean stress</td>
</tr>
<tr>
<td>$p'$</td>
<td>mean effective stress</td>
</tr>
<tr>
<td>$q$</td>
<td>deviatoric stress</td>
</tr>
</tbody>
</table>

$\varepsilon_v = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\gamma_s = \frac{1}{3}(\sigma_1 - \sigma_2)^2 + (\sigma_2 + \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)^{1/2}$

Roman Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_u$</td>
<td>undrained shear strength</td>
</tr>
<tr>
<td>$E_0$</td>
<td>maximal soil stiffness</td>
</tr>
<tr>
<td>$e_0$</td>
<td>initial void ratio</td>
</tr>
<tr>
<td>$E_{oed}$</td>
<td>tangent oedometric modulus</td>
</tr>
<tr>
<td>$E_{50}$</td>
<td>secant modulus corresponding to 50% of $q_f$</td>
</tr>
<tr>
<td>$G_0$</td>
<td>(or $G_{max}$) maximal small-strain shear modulus</td>
</tr>
<tr>
<td>$K_0$</td>
<td>coefficient of in situ earth pressure at rest ($K_0 &gt; K_0^{NC}$ for OCR &gt; 1)</td>
</tr>
<tr>
<td>$K_0^{NC}$</td>
<td>coefficient of earth pressure at rest of normally-consolidated soil</td>
</tr>
<tr>
<td>$K_0^{SR}$</td>
<td>stress reversal $K_0$ coefficient defining stress point position at intersection between hardening mechanisms</td>
</tr>
<tr>
<td>$q^{POP}$</td>
<td>($= \sigma_{v0} + \sigma'_e$) preoverburden pressure</td>
</tr>
<tr>
<td>$B_q$</td>
<td>pore pressure parameter for CPTU</td>
</tr>
<tr>
<td>$c$</td>
<td>cohesion intercept</td>
</tr>
<tr>
<td>$c^*$</td>
<td>intercept for $M^*$ slope in $q - p'$ plane ($= 6c \cos \phi/(3 - \sin \phi)$)</td>
</tr>
<tr>
<td>$C_c$</td>
<td>slope of the normal compression line in $\log_{10}$ scale ($= 2.3\lambda$)</td>
</tr>
<tr>
<td>$C_k$</td>
<td>coefficient of curvature ($= d_{50}^3/(d_{10} \cdot d_{60})$)</td>
</tr>
<tr>
<td>$C_N$</td>
<td>overburden correction factor for SPT $N_{60}$-value</td>
</tr>
<tr>
<td>$C_r$</td>
<td>slope of unload-reload consolidation line in $\log_{10}$ scale</td>
</tr>
<tr>
<td>$C_u$</td>
<td>coefficient of uniformity ($= d_{60}/d_{10}$)</td>
</tr>
<tr>
<td>$D$</td>
<td>scaling parameter (by default $= 1.0$ for HS-Std, $= 0.25$ for HS-SmallStrain)</td>
</tr>
<tr>
<td>$D_r$</td>
<td>relative density</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$e$</td>
<td>void ratio</td>
</tr>
<tr>
<td>$E_D$</td>
<td>dilatometer modulus ($= 34.7(p_1 - p_0)$)</td>
</tr>
<tr>
<td>$e_{max}$</td>
<td>maximal void ratio</td>
</tr>
<tr>
<td>$f_t$</td>
<td>limit tensile strength</td>
</tr>
<tr>
<td>$G$</td>
<td>tangent shear modulus</td>
</tr>
<tr>
<td>$G_{ur}$</td>
<td>unload-reload shear modulus</td>
</tr>
<tr>
<td>$G_s$</td>
<td>secant shear modulus</td>
</tr>
<tr>
<td>$H$</td>
<td>parameter which defines the rate of the volumetric plastic strain</td>
</tr>
<tr>
<td>$ID$</td>
<td>dilatometer material index ($= (p_1 - p_0)/(p_0 - u_0)$)</td>
</tr>
<tr>
<td>$IP$</td>
<td>plasticity index ($= w_L - w_P$)</td>
</tr>
<tr>
<td>$K_D$</td>
<td>dilatometer horizontal stress index ($= (p_0 - u_0)/\sigma'_{v0}$)</td>
</tr>
<tr>
<td>$M$</td>
<td>parameter of HS model which defines the shape of the cap surface</td>
</tr>
<tr>
<td>$m$</td>
<td>stiffness exponent for minor stress formulation</td>
</tr>
<tr>
<td>$M^*$</td>
<td>(or $M^*_{c}$) slope of critical state line ($= 6 \sin \phi_c/(3 - \sin \phi_c)$)</td>
</tr>
<tr>
<td>$M_{c}$</td>
<td>slope of critical state line ($= 6 \sin \phi_c/(3 + \sin \phi_c)$)</td>
</tr>
<tr>
<td>$MDMT$</td>
<td>constrained modulus derived from the Marchetti’s dilatometer</td>
</tr>
<tr>
<td>$MD$</td>
<td>one-dimensional drained constrained modulus</td>
</tr>
</tbody>
</table>
Greek Symbols

- $\gamma^{PS}$: plastic strain hardening parameter for deviatoric mechanism
- $\gamma_{SAT}$: saturated unit weight
- $\gamma_D$: dry unit weight
- $\gamma_s$: shear strain
- $\gamma_w$: water unit weight
- $\gamma_{0.7}$: value of small strain for which $G_s/G_0$ reduces to 0.722
- $\kappa$: slope of unload-reload consolidation line in $\ln$ scale
- $\Lambda$: plastic volumetric strain ratio ($= 1 - \kappa/\lambda$)
- $\lambda$: slope of primary consolidation line in $\ln$ scale
- $\nu$: Poisson’s coefficient
- $\nu_{ur}$: unloading/reloading Poisson’s coefficient
- $\phi$: friction angle
- $\phi_c^\prime$: effective friction angle from compression test
- $\phi_{ec}^\prime$: effective friction angle from extension test

Abbreviations

- CK$_{o}$UC: $K_o$ consolidated undrained compression
- CK$_{o}$UE: $K_o$ consolidated undrained extension
- CAP: Cap model with Drucker-Prager failure criterion
- CIDC: consolidated isotropic drained compression
- CIUC: consolidated isotropic undrained compression
- CIUE: consolidated isotropic undrained extension
- CPTU: cone penetration test with pore pressure measurements (electric piezocone)
- CSL: critical state line
- DMT: Marchetti dilatometer test
- DSS: direct simple shear
- FVT: field vane test
- MC: Mohr-Coulomb model
- MCC: Modified Cam clay model
- NCL: normal consolidation line
- OED: oedometric test
- PMT: pressuremeter test
- SBPT: self-boring pressuremeter test
- SCPT: static penetration test with seismic sensor
- SLS: serviceability limit state analysis
- SPT: standard penetration test
- TC: triaxial compression
- UCS: unified classification system
- ULS: ultimate limit state analysis
Sign convention: Throughout this report, the sign convention is the standard convention of soil mechanics, i.e. compression is assigned as positive.
**FAQs**

1. When should the HS model be applied and what are its advantages?
2. Which formulation to describe the stress dependent stiffness should be chosen?
3. How to migrate stiffness moduli between two different formulations for stress dependent stiffness?
4. How to set model parameters for an "undrained" simulation?
5. How to troubleshoot convergence problem at the initial state analysis?
6. What is a typical ratio $E_{ur}^{\text{ref}} / E_{50}^{\text{ref}}$?
7. What is a typical ratio $E_{oed}^{\text{ref}} / E_{50}^{\text{ref}}$?
8. What are typical parameter ranges?
9. Why do three different ratios $K_0$, $K_0^{SR}$ and $K_0^{NC}$ have to be defined in order to run a simulation with the HS model?
10. What should be specified in $\sigma_{oed}^{\text{ref}}$ cell?
11. What should be specified in $\sigma_{\text{ref}}$ cell?
12. What is the difference between preconsolidation defined with OCR and $q^{POP}$?
13. When should I activate the small strain extension?
14. How to identify model parameters?
15. What is the suggested parameter identification sequence?
16. How to use the Virtual Lab v2018?
Chapter 1

Introduction

The use of the finite element analysis has become widespread and popular in geotechnical practice as a mean of controlling and optimizing engineering tasks. However, the quality of any prediction depends on the adequate model adopted in the study. In general, a more realistic prediction of ground movements requires using the models which account for pre-failure behavior of soil. Such behavior, mathematically modeled with non-linear elasticity, is characterized by a strong variation of soil stiffness which depends on the magnitude of strain levels occurring during construction stages. Pre-failure stiffness plays a crucial role in modeling typical geotechnical problems such as deep excavations supported by retaining walls or tunnel excavations in densely built-up urban areas.

The present study completes the ZSoil® report elaborated by Truty (2008) on the Hardening Soil models. The objectives of the present report can be summarized as follows:

- to highlight the need of using advanced constitutive models in daily engineering practice;
- to recall the main features of the Hardening Soil model and to facilitate understanding its mathematical background;
- to provide to practicing engineers who foresee using the Hardening Soil model with a helpful guideline on specifying an appropriate testing program or making use of already acquired experimental results in order to identify or estimate model parameters;
- to show importance of using the Hardening Soil model in typical geotechnical analyses such as shallow footing, retaining wall excavation and tunnel excavation in an urban area.
1.1 Why do we need the HS-SmallStrain model?

It is commonly known that soil behavior is not as simple as its prediction with a simply-formulated linear constitutive models which are commonly used in numerical analyses. Complex soil behavior which stems from the nature of the multi-phase material, exhibits both elastic and plastic non-linearities and, deformations include irreversible plastic strains. Depending on the history of loading, soil may compact or dilate, its stiffness may depend on the magnitude of stress levels, soil deformations are time-dependent, etc. In fact, soil behavior is considered to be truly elastic in the range of small strains as schematically presented in Figure 1.1. In this strain range, soil may exhibit a nonlinear stress-strain relationship. However, its stiffness is almost fully recoverable in unloading conditions. In the aftermath of pre-failure non-linearities of soil behavior, one may observe a strong variation of stiffness starting from very small shear strains, which cannot be reproduced by models such as linear-elastic Mohr-Coulomb model (see Figure 1.2).

![Figure 1.1: Typical representation of stiffness variation in function of the shear strain amplitudes; comparison with the ranges for typical geotechnical problems and different tests (based on Atkinson and Sallfors, 1991, and updated by the author); SCPT - seismic cone penetration test; CPTU - piezocone penetration test; DMT - Marchetti’s dilatometer test; PMT - Pressuremeter test.](image_url)
1.1. WHY DO WE NEED THE HS-SMALLSTRAIN MODEL?

(a) Stress-strain curves

Figure 1.2: Comparison of different model responses for drained triaxial compression condition using equivalent parameters and OCR = 1.2.

(b) Normalized secant stiffness curves
Engineers who are looking for reliable predictions of the engineering system response should be aware that by applying linear-elastic, perfectly plastic models in the finite element analysis, soil ground movements may be underestimated, which may influence the magnitude of efforts which are computed in supporting structural elements. The models which account for high stiffness at very small strains concentrate the development of high amplitudes of strain around the close neighborhood of the source of deformations similarly to what is observed in reality. This can be the case of braced excavations (e.g. Figure 1.4) or tunnel excavations (e.g. Figure 1.3 or 5.13) where the varying stiffness increases soil deformations at the unloading boundaries, appropriately reducing them away from the unloaded zone (Addenbrooke et al., 1997). Furthermore, it is often observed in numerical analyses that not differentiating between loading and unloading stiffness moduli in the Mohr-Coulomb model may result in an unrealistic lifting of the retaining wall associated with unloading of the bottom of the excavation (see e.g. Figure 1.4(c)).

The Hardening Soil (HS) model in its two variants HS-Standard and HS-SmallStrain can be a remedy for modeling of the problems which have been listed above, as they account for most of soil behavior features (see Section 2). Despite the mathematical complexity of the HS model, its parameters have explicit physical meaning and can be determined with conventional soil tests.

![Figure 1.3](image-url)

*Figure 1.3:* Typical model response to the excavation problem using the standard Mohr-Coulomb on the left (unrealistic dominant heaving of the tunnel’s bottom) and the Hardening-Soil model on the right (a realistic trough above the tunnel).
1.1. WHY DO WE NEED THE HS-SMALLSTRAIN MODEL?

Figure 1.4: An example of deep excavation in Berlin Sand (after Truty, 2008). Comparison of model predictions: (a) Hardening-Soil vs Standard Mohr-Coulomb model, (b) wall deflections, (c) surface settlements.
1.2 Application fields of constitutive models

The finite element code ZSoil® includes soil models from simple linear elastic, perfectly plastic (e.g. Mohr-Coulomb), elasto-plastic cap models (e.g. Cap, Modified Cam Clay) to advanced nonlinear-elasto-plastic cap model HS-SmallStrain (ZACE, 2010). Table 1.1 summarizes each class of models in terms of basic model attributes. The table includes the main model features, failure criteria, hardening laws, and a comparison of required and corresponding soil parameters. It can be noticed that different models require a specification of different material properties. However, most of them are common to all presented models.

The choice of a constitutive model depends on many factors but, in general, it is related to the type of analysis that the user intends to perform (e.g. ultimate limit state analysis (ULS) or serviceability limit state analysis (SLS)), expected precision of predictions and available knowledge of the soil. In general, SLS analysis requires an application of advanced constitutive models which predict the stress-strain relation more accurately than simple linear-elastic, perfectly plastic models. A perceived general applicability of constitutive models is schematically proposed in Figure 1.5.

First approximation

Typically, the Mohr-Coulomb model (MC) is used for testing of the FE mesh discretization and should be considered as a first quick approximation in the preliminary analyses. In general, MC model can be applied for the estimation of the ultimate limit state (e.g. stability analyses) or modeling of less influential, massive soil bed layers. The model is often used in the cases where the number of soil tests and the parameter database are limited. The use of MC is not recommended for clays and soft soils because the model overestimates soil stiffness of normally- and lightly consolidated soils\(^1\) (there is no preconsolidation pressure

\(^1\)It is generally assumed that a normally consolidated soil has OCR = 1, lightly overconsolidated OCR...
threshold beyond which important plastic straining occurs) and loading and unloading stiffness are not distinguished.

Soft soils

In many cases, modeling of soft and near normally-consolidated clay type soils can be performed with the family of volumetric cap models, i.e. Cap model and the Modified Cam Clay model, under the assumption that the deformation of the considered soil layer are dominated by the volumetric plastic strains. The Modified Cam clay is however not recommended if the soil exhibits a distinct non-associated (dilatant) behavior. This shortcoming comes from the fact that the direction of strain increment is associated with that of stress increments and the dilatancy cannot be modeled. In addition, natural soils, especially soft clays, may exhibit viscous behavior which can be distinctly observed during secondary consolidation. In the ZSoil®, creep behavior (including swelling) can be modeled by means of constitutive models which exhibit pure linear elastic behavior for stress paths that penetrate the interior of the yield surface (e.g. the Cap model).

All type of soils

Most soil types can be modeled using the family of HS models as their formulation incorporates two hardening mechanisms. The shear mechanism deals with the plastic straining which is dominated by shearing what can be observed in granular soils and in overconsolidated cohesive soils. Having formulated the volumetric hardening mechanism which is governed by the compressing plastic strains, HS models are also suitable for modeling soft soils. It was demonstrated on many examples that the HS models, especially the HS-SmallStrain with high stiffness amplitudes in small strains, give realistic deformations for retaining walls and ground movements behind the wall in modeling excavation problems, e.g. Finno and Calvello (2005); Kempfert (2006); Benz (2007); Truty (2008) and Section 5.1. Since HS models are developed in the isotropic framework for both elastic behavior and hardening mechanisms (uniform expansion of the yielding surfaces in all directions), modeling of heavily overconsolidated soils which exhibit strong anisotropy should be treated carefully.

As regards the HS-Standard model, it does not include the formulation which deal with high amplitudes of stiffness in the small strains, and therefore the stiffness parameters should be chosen according to dominant strain levels in the modeled task. The HS-Std model is not able to reproduce hysteretic elastic behavior nor cyclic mobility (gradual softening due to cyclic loading). Since the HS-SmallStrain model reproduces the hysteretic elastic behavior, it can be applied to a certain extent for cycling loading as long as the cyclic mobility is not crucial for a given application and as long as dynamically-induced liquefaction effects are not considered.

General limitations

Note that none of the models mentioned above is able to reproduce debonding (destructuration between 1 and 3, whereas heavily overconsolidated OCR = 6 – 8 (Bowles, 1997).
CHAPTER 1. INTRODUCTION

tion) effects which can be observed as softening in the sensitive soils. It should also be noted that the cap hardening parameter (preconsolidation pressure) is not coupled with the degree of saturation, and therefore modeling of collapsible behavior of partially saturated soils is not possible with the implemented models.
Table 1.1: Comparison of selected soil models implemented in ZSoil®.

<table>
<thead>
<tr>
<th>Model</th>
<th>Hardening Soil</th>
<th>Cap</th>
<th>Mohr-Coulomb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-linear elasto-plastic, shear and compression strain hardening</td>
<td>elasto-plastic, compression strain hardening</td>
<td>elastic-perfectly plastic</td>
<td></td>
</tr>
<tr>
<td>Basic features</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• hyperbolic stress-strain relation</td>
<td>• non-linear stress-strain relation for normally and lightly overconsolidated material</td>
<td>• linear stress-strain relation</td>
<td></td>
</tr>
<tr>
<td>• stress dependent stiffness according to power law</td>
<td>• constant stiffness (possibility of introducing linear stress dependent stiffness through superelements)</td>
<td>• constant stiffness (possibility of introducing linear stress dependent stiffness through superelements)</td>
<td></td>
</tr>
<tr>
<td>• non-linear dilatancy according to Rowe’s law (+ cut-off for excessive plastic dilatancy)</td>
<td>• linear soil dilatancy</td>
<td>• linear soil dilatancy (+ cut-off for excessive plastic dilatancy)</td>
<td></td>
</tr>
<tr>
<td>• distinction between primary loading and unloading</td>
<td>• evolution of preconsolidation pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• evolution of preconsolidation pressure</td>
<td>• plastic straining in primary compression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• plastic straining in primary deviatoric loading</td>
<td>• plastic straining in primary deviatoric loading</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• small strain stiffness (HS-SmallStrain only)</td>
<td>• distinction between primary loading and unloading</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• hysteretic, nonlinear elastic stress-strain relation (small strains only)</td>
<td>• evolution of preconsolidation pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• plastic straining in primary compression</td>
<td></td>
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<tr>
<td></td>
<td>• plastic straining in primary deviatoric loading</td>
<td></td>
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</tr>
<tr>
<td>Failure criterion</td>
<td>Mohr-Coulomb (hexagon in π-plane)</td>
<td>Drucker-Prager (circular in π-plane)</td>
<td>Mohr-Coulomb (hexagon in π-plane)</td>
</tr>
<tr>
<td>Cap yield surface</td>
<td>ellipsoidal, defined by van Eekelen criterion in π-plane</td>
<td>ellipsoidal, circular in π-plane</td>
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<tr>
<td>Hardening</td>
<td>deviatoric-shear and isotropic-compaction</td>
<td>isotropic-compaction</td>
<td>none</td>
</tr>
<tr>
<td>Flow rule</td>
<td>non-associated for shear hardening associated for isotropic hardening</td>
<td>non-associated for shear hardening associated for isotropic hardening</td>
<td>non-associated</td>
</tr>
<tr>
<td>Corresponding soil parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small strain stiffness</td>
<td>$E_0$ and $E_{0.1}$ (HS-SmallStrain only)</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>Elastic characteristics</td>
<td>$v_{ur}$ and $E_{ur}$</td>
<td>$v$ and $E$</td>
<td>$v$ and $E$</td>
</tr>
<tr>
<td>$E_{0.1}$ and $m$</td>
<td></td>
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<tr>
<td>Failure criterion</td>
<td>$\phi$ and $c$</td>
<td>$\phi$ and $c$</td>
<td>$\phi$ and $c$</td>
</tr>
<tr>
<td>Dilatancy</td>
<td>$\psi$ and $\psi_{max}$</td>
<td>$\psi$</td>
<td>$\psi$ and $\psi_{max}$</td>
</tr>
<tr>
<td>$\psi_{max}$ (can be determined from $\lambda$)</td>
<td></td>
<td>$\lambda$</td>
<td></td>
</tr>
<tr>
<td>Cap surface parameters</td>
<td>OCR and $K_{OCR}$</td>
<td>OCR</td>
<td>none</td>
</tr>
</tbody>
</table>
Chapter 2

Short introduction to the HS models

The Hardening Soil model (HS-Standard) was designed by Schanz (1998); Schanz et al. (1999) in order to reproduce basic macroscopic phenomena exhibited by soils such as:

- **densification**, i.e. a decrease of voids volume in soil due to plastic deformations, e.g. Figure 2.11;
- **stress dependent stiffness**, i.e. observed phenomena of increasing stiffness moduli with increasing stress level (mean stress), e.g. Figure 2.4;
- **soil stress history**, i.e. accounting for preconsolidation effects;
- **plastic yielding**, i.e. development of irreversible strains with reaching a yield criterion, e.g. Figure 2.2;
- **dilatation**, i.e. an occurrence of negative volumetric strains during shearing, e.g. Figure 2.11.

Contrary to other models such us the Cap model or the Modified Cam Clay (let alone the Mohr-Coulmb model), the magnitude of soil deformations can be modeled more accurately by incorporating three input stiffness parameters corresponding to the triaxial loading stiffness ($E_{50}$), the triaxial unloading-reloading stiffness ($E_{ur}$), and the oedometer loading modulus ($E_{oed}$).

An enhanced version of the HS-Standard, the Hardening Soil Small model (HS-SmallStrain) was formulated by Benz (2007) in order to handle a commonly observed phenomena of:

- **strong stiffness variation** with increasing shear strain amplitudes in the domain of small strains (Figure 1.1);
- **hysteretic, nonlinear elastic stress-strain relationship** which is applicable in the range of small strains (Figure 2.20).

These features mean that the HS-SmallStrain is able to produce more accurate and reliable approximation of displacements which can be useful for dynamic applications or in modeling unloading-conditioned problems, e.g. deep excavations with retaining walls.
Although both models can be considered as advanced soil models which are able to faithfully approximate complex soil behavior, they include some limitations related to specific behavior observed for certain soils. The models are not able to reproduce softening effects associated with soil dilatancy and soil destructuration (debonding of cemented particles) which can be observed, for instance, in sensitive soils. As opposed to the HS-SmallStrain model, the HS-Standard does not account for large amplitudes of soil stiffness related to transition from very small strain to engineering strain levels ($\varepsilon \approx 10^{-3} - 10^{-2}$). Therefore, the user should adapt the stiffness characteristics to the strain levels which are expected to take place in conditions of the analyzed problem. Moreover, the HS-Standard model is not capable to reproduce hysteretic soil behavior observed during cycling loading. As an enhanced version of the HS-Standard model, HS-SmallStrain accounts for small strain stiffness and therefore, it can be used to some extent to model hysteretic soil behavior under cyclic loading conditions with the exception of gradual softening which is experimentally observed with an increasing number of loading cycles.
2.1 Hardening Soil-Standard model

2.1.1 Shear mechanism

The shear mechanism is introduced in order to handle the soil hardening which is induced by the plastic shear strains. Domination of plastic shear strains can be typically observed for granular materials such as sands, and heavily consolidated cohesive soils.

2.1.1.1 Shear yield mechanism

The hardening yield function for shear mechanism \( f_1 \), is described using the concept of hyperbolic approximation of the relation between the vertical strain \( \varepsilon_1 \) and deviatoric stress \( q \) for a standard triaxial drained compression test (Figure 2.2). The yield condition is thus expressed as follows:

\[
\begin{align*}
    f_1 &= \frac{q_a}{E_{50}} \frac{q}{q_a - q} - 2 \frac{q}{E_{ur}} - \gamma^{PS} \\
    & \quad \text{for } q < q_f
\end{align*}
\]

where \( \gamma^{PS} \) is the plastic strain hardening parameter, \( q_a \) is the asymptotic deviatoric stress which is defined by the ultimate deviatoric stress \( q_f \) and the failure ratio \(^1 R_f \) is defines as:

\[
q_a = \frac{q_f}{R_f}
\]

\(^1\)A suitable value of the failure ratio is set by default \( R_f = 0.9 \). For most soils, the value of \( R_f \) falls between 0.75 and 1. See also Section 3.3.3.

Figure 2.2: Hyperbolic stress-strain relationship and the definition of different moduli in the triaxial drained test condition.
It means that for larger values of the hardening parameter $\gamma_{PS}$, the hyperbolic relation is restrained by the ultimate deviatoric stress $q_f$ described by the Mohr-Coulomb criterion (Figure 2.2 and Figure 2.3):

$$q_f = \frac{2 \sin(\phi)}{1 - \sin(\phi)} (\sigma_3 + c \cot(\phi))$$

(2.3)

which is defined by the friction angle $\phi$ and the cohesion $c$. 

\[\text{Table 2.1: Shear mechanism properties}\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle $\phi$</td>
<td>30°</td>
</tr>
<tr>
<td>Dilatancy angle $\psi$</td>
<td>0°</td>
</tr>
<tr>
<td>Cohesion $c$</td>
<td>20 [kN/m$^2$]</td>
</tr>
<tr>
<td>Failure ratio $R_f$</td>
<td>0.9</td>
</tr>
<tr>
<td>Tensile cut-off $f_t$</td>
<td>0 [kN/m$^2$]</td>
</tr>
<tr>
<td>Dilatancy cut-off $e_{max}$</td>
<td>1</td>
</tr>
<tr>
<td>Multiplier for Fowle’s dilatancy law in contractant domain $D$</td>
<td>0</td>
</tr>
</tbody>
</table>

\[\text{Figure 2.3: Cap surface of the volumetric hardening mechanism, yield loci for the different values of the hardening parameter } \gamma_{PS} \text{ and the Mohr-Coulomb criterion limiting the larger values of } \gamma_{PS}.\]
2.1.2 Stress dependent stiffness

The secant modulus $E_{50}$ which corresponds to 50% of the value of $q_f$ is defined to be minor stress dependent using the frequently adopted power law:

$$E_{50} = \frac{E_{\text{ref}}}{E_{50}^{\text{ref}}} \left( \frac{\sigma_{3}^* + c \cot \phi}{\sigma_{\text{ref}} + c \cot \phi} \right)^m$$

(2.4)

where $\sigma_{3}^* = \max(\sigma_3, \sigma_L)$, i.e. stiffness degrades with decreasing $\sigma_3$ up to the limit minor stress $\sigma_L$ which can by assumed be default $\sigma_L = 10$ kPa; and $\sigma_{\text{ref}}$ is the minor stress at which $E_{\text{ref}}^{50}$ has been identified. In the triaxial compression test, $\sigma_{\text{ref}}$ corresponds to the confining stress $\sigma_3$ (cf. Figure 3.4).

Note that $E_{50}$ largely controls the magnitude of the plastic strains which are related to the shear yield mechanism. In natural soil, the exponent $m$ varies between 0.3 and 1.0. Janbu (1963) reported values of 0.5 for Norwegian sands and silts, whereas Kempfert (2006) provided values between 0.38 and 0.84 for soft lacustrine clays (see also Section 3.3.7). The user may set the material stiffness to be independent on the stress level by setting $m = 0$ (i.e. constant stiffness like in the standard Mohr-Coulomb model).

By analogy with $E_{50}$, the modulus $E_{ur}$ which defines the slope of the unloading-reloading curve is also defined as minor stress dependent:

$$E_{ur} = \frac{E_{\text{ref}}^{ur}}{E_{ur}^{\text{ref}}} \left( \frac{\sigma_{3}^* + c \cot \phi}{\sigma_{\text{ref}} + c \cot \phi} \right)^m$$

(2.5)

Note that the same $\sigma_{\text{ref}}$ applies to the stiffness moduli $E_{50}^{\text{ref}}$, $E_{ur}^{\text{ref}}$ and $E_0^{\text{ref}}$.

<table>
<thead>
<tr>
<th>Stiffness setup</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Demended secant reference E modulus at 50% of qf</td>
<td>25000</td>
<td>[kN/m^2]</td>
</tr>
<tr>
<td>Reference stress for Young modulus</td>
<td>100</td>
<td>[kN/m^2]</td>
</tr>
</tbody>
</table>

An example of stress dependency is graphically presented in Figure 2.4.
Figure 2.4: Example of stress dependency at initial state defined by Eq. (2.5) for different values of parameter (a) $m$, (b) $\phi$, and (c) $c$. 
2.1.2.1 Stress dependent stiffness based on the mean effective stress.

It can be observed that in the case of dynamic analyzes or modeling of excavation problems with overconsolidation soils which exhibit $K_0 > 1$, a rotation of principal stress may occur resulting in spurious oscillations of stiffness moduli. For example, in the case of excavation of a circular tunnel, one may observe the rotation of principal stresses at the tunnel sides, and the vertical stress which initially defines the soil stiffness $\sigma_3 = \sigma''_v$ becomes $\sigma_3 = \sigma''_h$, which may lead to an underestimation of soil stiffness. This is because $\sigma''_h$ decreases to zero dropping the unloading-reloading stiffness to its minimal value which is limited by $\sigma_L$. Moreover, at the bottom of an excavated tunnel $\sigma_3$ which is equal to the vertical stress may also drop to zero resulting in slightly overestimated swelling of the tunnel bottom.

In order to remedy this problem, the user can use the stress dependency formulation which depend on the mean effective stress $p'$ which can be written in a general form as:

$$E = E_{\text{ref}} \left( \frac{p'}{\sigma_{\text{ref}}} \right)^{m_p} \quad \text{with} \quad p' = \max(p', \sigma_L) \quad (2.6)$$

where:

- $\sigma_{\text{ref}}$ - reference stress
- $E_{\text{ref}}$ - reference modulus corresponding to the reference stress $\sigma_{\text{ref}}$
- $m_p$ - stiffness exponent for $p'$-formulation which is equal to $m_p = m$ if $c = 0$
- otherwise $m_p \neq m$
- $p'$ - mean effective stress $(\sigma'_1 + \sigma'_2 + \sigma'_3)/3$
- $\sigma_L$ - the limiting stress (in order to avoid zero stiffness when $p'$ is close to 0)

A unique $\sigma_{\text{ref}}$ applies to the stiffness moduli $E_{50}^{\text{ref}}, E_{ur}^{\text{ref}}$ and $E_0^{\text{ref}}$.

In the current version (ZSoil v2018), the evolution of the preconsolidation parameter (refer to Eq. 2.23) remains the same for both formulations of stiffness dependency and it contains the component $c \cot \phi$. Note that parameter transformation does not apply to $E_0^{\text{ref}}$ because this input parameter is not a model parameter but it serves as the reference value for calibrating parameters $M$ and $H$.

<table>
<thead>
<tr>
<th>Minor effective stress $\sigma''_3$</th>
<th>Mean effective stress $p'$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Drawback:</strong> Spurious oscillations of stiffness moduli in the case of dynamic analyzes and excavation problems in materials with $K_0 &gt; 1$</td>
<td>Stiffness oscillations independent of the principal stresses rotation</td>
</tr>
<tr>
<td><strong>Drawback:</strong> Underestimation of unloading-reloading stiffness in excavation problems</td>
<td>Unloading-reloading stiffness depends on $p'$ which is larger than $\sigma_3$ in excavation problems</td>
</tr>
<tr>
<td>Direct identification of the stiffness moduli $E_{50}$ and $E_{ur}$ for the constant $\sigma_3$ during triaxial compression test</td>
<td><strong>Drawback:</strong> $E_{50}$ and $E_{ur}$ are not directly identifiable from triaxial test as $p'$ varies during triaxial compression test. A simple transformation method is provided in Figure 2.5.</td>
</tr>
</tbody>
</table>
In ZSoil v2018, \( p \)-stress dependency can be activated in Elastic dialog window (✓ Advanced option has to be checked), as illustrated below.

### 2.1.2.2 Parameter migration between minor and mean stress formulations

Note that the initial material stiffness described by \( \sigma_3 \)-formulation and \( p' \)-formulation may be different across the soil profile for the same \( \sigma_{\text{ref}} \) if the same reference moduli \( E_0^{\sigma,\text{ref}}(\sigma_{\text{ref}}) = E_0^{p,\text{ref}}(\sigma_{\text{ref}}) \) and the same stiffness exponent \( m = m_p \) are taken. In the other words, assuming the same departing stiffness profile for both formulations one should be aware that:

- the reference moduli are different \( E_0^{\sigma,\text{ref}}(\sigma_{\text{ref}}) \neq E_0^{p,\text{ref}}(\sigma_{\text{ref}}) \) if \( K_0 \neq 1 \)
- the stiffness exponents \( m \) and \( m_p \) are different if \( c > 0 \)

In order to preserve the same or similar variation of \( E_0 \) across the soil profile obtained for \( \sigma_3 \)-formulation, one may adjust \( E_0^{p,\text{ref}} \) to the same value of \( \sigma_{\text{ref}} \) by proceeding the following procedure:

1. Determine \( E^{\text{ref}}, m \) for a given \( \sigma_{\text{ref}}, \phi' \) and, \( c' \) assuming \( \sigma_3 \)-formulation

   e.g. for \( \sigma_{\text{ref}} = 100 \text{kPa} \), the following parameters were obtained from triaxial compression tests: \( E_{\text{ur}} = 260000 \text{kPa} \), \( m = 0.5 \), \( \phi' = 30^\circ \) and \( c = 5 \text{kPa} \)

2. Specify \( n \)-number of different effective vertical stresses \( \sigma'_{v0} \) and in situ stress ratio \( K_0 \) (in order to determine \( \sigma_3 \))
2.1. HARDENING SOIL-STANDARD MODEL

e.g. \( \sigma'_{v0} = \{50, 100, 200\} \text{kPa} \) and \( K_0 = 0.6 \) stress dependen stresses \( \sigma_3 = \min(\sigma'_{h0}, \sigma'_{v0}) \)

3. Compute effective mean stresses, effective horizontal stresses \( \sigma'_{h0} = K_0 \sigma'_{v0} \) and the minor e.g. \( p' = \{36.6, 73.3, 146.6\} \text{kPa} \), \( \sigma_3 = \{30, 60, 90\} \text{kPa} \)

4. Make an assumption that for each effective vertical stress, the stiffness moduli computed using \( p' \)-formulation should be equal to those obtained wit \( \sigma_3 \)-formulation. It yields in a system of \( n \)-equations with \( E_{p, \text{ref}} \) and \( m_p \) as unknowns:

\[
E_{\text{ref}} \left( \frac{\sigma_3 + c' \cot \phi'}{\sigma_{\text{ref}} + c' \cot \phi'} \right)^m - E_{p, \text{ref}} \left( \frac{p'}{\sigma_{\text{ref}}} \right)^{m_p} = 0
\]

5. Find \( E_{p, \text{ref}} \) and \( m_p \) which minimize the error for a system of nonlinear equations.

\( \text{e.g. } E_{p, \text{ref}} = 229938 \text{kPa}, \ m_p = 0.458 \)

Parameter migration between the \( \sigma_3 \)-formulation and \( p' \)-formulation can also be carried out by performing a simple regression analysis using a spreadsheet (refer to Figure 2.5). This method requires plotting the evolution of the stiffness modulus computed using \( \sigma_3 \)-formulation against the normalized mean effective stress. Two coefficients which define the power-type trendline \( y = ax^b \) give the values of \( E_{p, \text{ref}} \) and \( m_p \), respectively.

⚠️ Note that the above procedure applies only to the initial stiffness described by the maximal modulus \( E_0 \). Parameter migration for \( E_{50} \) and \( E_{ur} \) is presented in the next paragraph.
CHAPTER 2. SHORT INTRODUCTION TO THE HS MODELS

(a) Input variables and computed data in a spreadsheet

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
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<td></td>
<td>c1p - dependent stiffness</td>
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<tr>
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</tr>
<tr>
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<td>a0^eff</td>
<td>100 kPa</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>c</td>
<td>5 kPa</td>
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<td>7</td>
<td>φ</td>
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<tr>
<td>8</td>
<td>cD(c4)</td>
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<tr>
<td>9</td>
<td>a = c' cot φ</td>
<td>8.685 kPa</td>
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<td>Input</td>
<td>Computed variables</td>
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<td>c1p = Kp φ'</td>
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<td>a1 = min(c1p, c1p')</td>
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<td>E0 (sig-3)</td>
<td>p' = (c1p' + 2a1)c0</td>
<td>p'/c0</td>
<td>E0/E0^eff (sig-3)</td>
</tr>
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<td>27</td>
<td>28</td>
<td>29</td>
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</tbody>
</table>
```

(b) Determination of p'-dependency parameters using regression analysis

```
\[ E = E_0 \left( \frac{c_3 + a}{c_{3p} + a} \right) \]
```

Figure 2.5: Migration between the σ₃-formulation and p'-formulation for the initial stiffness parameter E₀^eff using a spreadsheet.
2.1. HARDENING SOIL-STANDARD MODEL

Stiffness migration between $\sigma_3$- and $p'$-formulation for $E_{50}$ and $E_{ur}$. In the case of the remaining moduli which define $\varepsilon_1 - q$ curve at larger strains, i.e. $E_{50}^{\text{ref}}$ and $E_{ur}^{\text{ref}}$, a different procedure is required since during triaxial compression $p'$ varies with the increasing $q$ contrary to $\sigma_3$ which remains constant. It means, for example, that assuming the same value of $E_{50}^{\text{ref}}$ for both formulations the secant stiffness obtained at $0.5q_f$ will be larger for $p'$-formulation even though $K_0 = 1$ and $c = 0$ have been assumed in the simulation, refer to Figure 2.6(a).

In order to transform the remaining moduli $E_{50}^{\text{ref}}$ and $E_{ur}^{\text{ref}}$ obtained for $\sigma_3$-formulation to $p'$-formulation preserving the same value for $\sigma_\text{ref}$ one may proceed the following procedure:

1. Evaluate the failure deviatoric stress for $\sigma_3 = \sigma_\text{ref}$, and given $\phi$ and $c$ from:
   \[
   q_f = \frac{2\sin\phi}{1 - \sin\phi} (\sigma_3 + c \cot\phi)
   \]

2. Take $p'$-formulation for dependency law:
   \[
   E_{50}^{p'} = E_{50}^{p,\text{ref}} \left( \frac{p}{\sigma_\text{ref}} \right)^{m_p}
   \]

3. Make an assumption that for the same $\sigma_\text{ref}$ the secant moduli are equal for both formulations:
   \[
   E_{50}^{p,\text{ref}} = E_{50}^{\sigma,\text{ref}} \text{ at } 0.5q_f
   \]

4. Approximate $E_{50}^{p,\text{ref}}$ using the $p'$-stress dependency formulation:
   \[
   \begin{array}{c|c}
   \text{if } 0.5q_f > \sigma_\text{ref} & E_{50}^{p,\text{ref}} \approx E_{50}^{\sigma,\text{ref}} \left( \frac{0.5q_f}{\sigma_\text{ref}} \right)^{m_p} \\
   \text{else} & E_{50}^{p,\text{ref}} \approx E_{50}^{\sigma,\text{ref}} \left( \frac{\sigma_\text{ref}}{0.5q_f} \right)^{m_p}
   \end{array}
   \]

An example of the above transformation procedure is illustrated in Figure 2.6(b). The same approach can be applied to transform the unloading-reloading modulus, i.e. $E_{ur}^{\sigma,\text{ref}}$ to $E_{ur}^{p,\text{ref}}$.

- **Figure 2.6:** Comparison of simulations carried out for $\sigma_3$-formulation and $p'$-formulation with the HS-Standard assuming the confining pressure $\sigma_3 = 345$ kPa ($K_0 = 1$) and $c = 0$ kPa.
2.1.2.2.1 Integrated tool for moduli migration

Virtual Lab v2018 offers an integrated toolbox for moduli migration (Figure 2.7). The tool includes the migration approaches which are presented above.

Figure 2.7: Parameter migration between the two formulations for stress dependent stiffness in Virtual Lab v2018.
2.1.3 Shear hardening law

The shear hardening yield function \( f_1 \) can be decomposed into part which is a function of stress - two first components, whereas the last component is a function of plastic strains \( \gamma^{PS} = \varepsilon_1^p - \varepsilon_2^p - \varepsilon_3^p \). Assuming that in the contractancy domain, the volumetric plastic strain \( \varepsilon_v^p = \varepsilon_1^p + \varepsilon_2^p + \varepsilon_3^p \) is observed to be very small \( \varepsilon_v^p \approx 0 \), it can thus be written:

\[
\gamma^{PS} \approx 2 \varepsilon_1^p
\]  

(2.7)

Hence, for the primary loading in drained triaxial conditions, \( \varepsilon_1 \) is evaluated using the yield condition (Eq.(2.1)) and decomposition of the elastic and the plastic strains:

\[
\varepsilon_1 = \varepsilon_1^e + \varepsilon_1^p = \frac{q}{E_{ur}} + \frac{1}{2} \left( \frac{q_a}{E_50} q_a - q - 2q \frac{q_a}{E_{ur}} \right) = \frac{q_a}{2E_50} q_a - q
\]  

(2.8)

For the drained triaxial conditions and the confining stress remaining constant (i.e. \( \sigma_2 = \sigma_3 = \text{const.} \)), the modulus \( E_{ur} \) remains constant and the elastic strains can be computed from:

\[
\varepsilon_1^e = \frac{q}{E_{ur}} \quad \text{and} \quad \varepsilon_2^e = \varepsilon_3^e = \nu_{ur} \frac{q}{E_{ur}}
\]  

(2.9)

where \( \nu_{ur} \) denotes unloading/reloading Poisson’s ratio.

The hyperbolic relation between the axial strain and the deviatoric stress presented in Equation 2.8 can be rearranged into:

\[
q = \frac{\varepsilon_1}{2E_50} + \frac{\varepsilon_1 R_f}{q_f}
\]  

(2.10)

which can also be rewritten in the following form:

\[
q = \frac{\varepsilon_1}{a + b \varepsilon_1}
\]  

(2.11)

These equations are graphically presented in Figure 2.8.

Note that for an anisotropically consolidated clay, the initial state deviatoric stress (after consolidation but before compression) which corresponds to the state of zero strains is:

\[
q_0 = \sigma_3 \frac{1 - K_0}{K_0}
\]  

(2.12)

so the the deviatoric stress after consolidation becomes:

\[
q_m = q - q_0
\]  

(2.13)

and Eq.(2.10) can be rewritten as:

\[
q_m = \frac{\varepsilon_1}{2E_50 + \varepsilon_1 R_f} q_{m,f}
\]  

(2.14)

with \( q_{m,f} \) denoting \( q_m \) at failure.

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2.1.4 Plastic flow rule and dilatancy

The plastic flow rule is derived from the plastic potential:

\[ g_1 = \frac{\sigma_1 - \sigma_3}{2} + \frac{\sigma_1 + \sigma_3}{2} \sin \psi_m \]  

(2.15)

and it takes the linear form:

\[ \dot{\varepsilon}_p^v = \dot{\gamma}^p \sin \psi_m \]  

(2.16)

where the mobilized dilatancy angle \( \psi_m \) is calculated in the HS-Standard model according to:

\[ \sin \psi_m = 0 \quad \text{if} \quad \phi_m < \phi_{cs} \quad \text{(cut-off in contractancy domain)} \]  

(2.17a)

\[ \sin \psi_m = \frac{\sin \phi_m - \sin \phi_{cs}}{1 - \sin \phi_m \sin \phi_{cs}} \quad \text{if} \quad \phi_m \geq \phi_{cs} \quad \text{(Rowe’s dilatancy)} \]  

(2.17b)

where the mobilized friction angle \( \phi_m \) is computed from:

\[ \sin \phi_m = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2c \cot \phi} \]  

(2.18)

and the critical state friction angle which is a material property and is independent of the stress conditions, is defined by the friction angle \( \phi \) and the ultimate dilatancy angle \( \psi \) as:

\[ \sin \phi_{cs} = \frac{\sin \phi - \sin \psi}{1 - \sin \phi \sin \psi} \]  

(2.19)

It means that dilatancy may occur for the larger values of the mobilized friction angle \( \phi_m > \phi_{cs} \), whereas for smaller stress ratios (\( \phi_m < \phi_{cs} \)), the material contracts and the mobilized dilatancy angle \( \psi_m \) is calculated according to Rowe’s dilatancy.

---

2 The mobilized friction angle \( \phi_m \) describes the stress ratio \( \tau/\sigma \) (at the critical state \( \phi_m = \phi_{cs} \)).
2.1. HARDENING SOIL-STANDARD MODEL

dilatancy angle is controlled by the cut-off criterion as presented in Figure 2.9. A graphical explanation of the mobilized dilatancy for deviatoric mechanism and the associated flow rule for the volumetric mechanism are presented in Figure 2.9.

Figure 2.9: Rowe’s dilatancy law and the cut-off criterion in the contractant domain for the HS-Standard model.

The ultimate dilatancy angle \( \psi \) defines the dilatancy parameter \( d \) which defines the maximal slope of the \( \varepsilon_1 - \varepsilon_v \) curve (Figure 2.11):

\[
d = -\frac{d\varepsilon_v}{d\varepsilon_1} \approx -\frac{d\varepsilon^p_v}{d\varepsilon^p_1} = \frac{2\sin \psi}{1 - \sin \psi}
\]  

(2.20)

In order to avoid an extensive dilatancy which is produced by Rowe’s law for the larger shear strains at the critical state, an additional cut-off criterion is introduced to respect the maximal defined void ratio \( e_{max} \) (Figure 2.11):

\[
\text{if } e \geq e_{max} \quad \sin \psi_m = 0 \quad \text{(cut-off)}
\]

(2.21)

otherwise Eq.(2.17b) is used to calculate \( \sin \psi_m \).
CHAPTER 2. SHORT INTRODUCTION TO THE HS MODELS

(a) Non-associated flow for deviatoric mechanism

(b) Associated flow for volumetric mechanism

Figure 2.10: Plastic flow rules in the HS model (a) graphical explanation of mobilized dilatancy $\psi_m$ which increases from 0 up to the input dilatancy angle $\psi$ once M-C line is reached, (b) contractancy increases with compressive $p'$ stress from zero to maximum value at M-C failure only when cap is mobilized.

Figure 2.11: Strain curve for a standard triaxial drained compression test with the dilatancy cut-off.
2.1.5 Volumetric mechanism

The volumetric plastic mechanism is introduced to account for a threshold point beyond (preconsolidation pressure) which important plastic straining occur characterizing a normally-consolidated state of soil. Since the shear mechanism generates no volumetric plastic strain in the contractant domain, the model without volumetric mechanism could significantly over-estimate soil stiffness in virgin compression conditions particularly for normally- and lightly overconsolidated cohesive soils. Such a problem can be observed when using, for instance, the Mohr-Coulomb model.

The second yield mechanism is proposed in the form of the cap surface similarly to other hardening models available in ZSoil®, e.g. Modified Cam Clay or Cap. The yield function which is graphically presented in Figure 2.12 and 2.3, is thus defined as:

\[
f_2 = \frac{q^2}{M^2 r^2(\theta)} + p^2 + p_c^2
\]

where \( r(\theta) \) obeys van Eekelen’s formula in order to assure a smooth and convex yield surface (cf. also the formulation of the Modified Cam Clay model); \( M \) is the model parameter which defines the shape of the cap surface and is related to \( K_{NC}^0 \), and \( p_c \) denotes the preconsolidation pressure which defines an intersection of the cap surface with the hydrostatic axis \( p' \).

![Figure 2.12: 3D representation of strength anisotropy in the HS model with the Mohr-Coulomb failure surface and the cap surface which obeys van Eekelen’s formula.](image)

Evolution of the hardening parameter \( p_c \) is described by the hardening law:

\[
dp_c = -H \left( \frac{p_c + c \cot \phi}{\sigma_{ref} + c \cot \phi} \right)^m \, \text{d}e_v^p
\]

where \( H \) is the parameter which controls the rate of volumetric plastic strains and is related to the tangent oedometric modulus \( E_{oed}^t \) at given reference oedometric (vertical) stress level (see Figure 3.7(a)).
The rate of the volumetric plastic strain is then computed:

\[ d\varepsilon_v^p = d\lambda_22H \left( \frac{p_c + c \cot \phi}{\sigma_{ref} + c \cot \phi} \right)^m p' \]  

(2.24)

Note that the parameters \( M \) and \( H \) can be easily calculated with the aid of the internal ZSoil® calculator by providing the values of \( K_{NC}^{\text{ref}} \) and the tangent oedometric modulus \( E_{\text{oed}}^{\text{ref}} \) corresponding to the reference oedometric vertical stress \( \sigma_{\text{oed}}^{\text{ref}} \); both must be captured from the primary loading curve (Normal Consolidation Line NCL); note that \( \sigma_{\text{oed}}^{\text{ref}} \) is used to compute the initial stress point defined by \( p^* \) and \( q^* \) (see Figure 3.7).

The automatic optimization of \( M \) and \( H \) must fulfill two conditions:

- \( K_{NC}^0 \) produced by the model in the oedometric conditions is the same as \( K_{NC}^{\text{ref}} \) specified by the user.
- \( E_{\text{oed}} \) generated by the model in the oedometric conditions is the same \( E_{\text{oed}}^{\text{ref}} \) specified by the user.

The internal optimization procedure runs a strain driven oedometer test with the vertical strain amplitude \( \Delta \varepsilon_v = 10^{-5} \) and the tangent oedometric modulus is computed as \( E_{\text{oed}} = \delta \sigma_v / \delta \varepsilon_v \approx \Delta \sigma / \Delta \varepsilon_v \).

The plastic potential in the volumetric mechanism is derived from the yield criterion neglecting \( r(\theta) \) term (Truty, 2008).
2.1.6 Additional strength criterion

Sometimes, it is necessary to control excessive tensile stresses which are built up during the analysis, particularly when using materials with high values of cohesion. The tensile strength condition is thus described with the Rankine’s criterion:

\[ f_3 = \sigma_3 + f_t = 0 \]  \hspace{1cm} (2.25)

where \( f_t \) is the user-defined tensile strength (default value \( f_t = 0 \)) and \( \sigma_3 \) denotes the minimal principal stress.

The plastic potential is associated with the cut-off condition.

<table>
<thead>
<tr>
<th>Shear mechanism</th>
<th>( \phi )</th>
<th>( \psi )</th>
<th>( c )</th>
<th>( R_f )</th>
<th>( f_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle</td>
<td>30 [deg]</td>
<td>2 [deg]</td>
<td>20 [kN/m^2]</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Dilatancy angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohesion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile cut-off</td>
<td>☑️</td>
<td></td>
<td></td>
<td></td>
<td>5 [kN/m^2]</td>
</tr>
<tr>
<td>Dilatancy cut-off</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplier for Rowe’s dilatancy law in contractant domain</td>
<td>D</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.1.7 Initial state variables

Setting the initial stress state is necessary for calculating the initial values of hardening parameters $\gamma_{PS}$ and $p_{c0}$. This calculation can be performed by a numerical procedure at the beginning of FE analysis based on the initial effective stress conditions $\sigma'_0 = \sigma_0(\sigma'_{x0}, \sigma'_{y0}, \sigma'_{z0})$ and its distance from the maximal stress point $\sigma^{SR}$ which is supposed to be experienced by the soil (see Figure 2.15). In order to calculate this distance, the user has to define the following variables:

1. **Stress history** variable which can be set in two ways:
   - through the overconsolidation ratio $OCR = \sigma_{vc}/\sigma'_{v0}$ where $\sigma_{vc}$ is the vertical pre-consolidation stress (see Figure 2.13). By applying this option, a constant OCR profile is obtained.
   - through the maximal preoverburden pressure offset $q^{POP} = \sigma_{vc} - \sigma'_{v0}$ (see Figure 2.13). This option is useful to describe the deposits with varying OCR over the depth (typically superficial layers of natural soil subject to mechanical overconsolidation or dessication).

![Setting initial state variables with respect to the initial stress state](image)

2. **Historical coefficient of earth pressure at rest** $K_{SR}^{0}$ which corresponds to the maximal stress point $\sigma^{SR}$ (Figure 2.15), and its value can be assumed as:
   - $K_{0}^{NC}$ consolidation: $K_{SR}^{0} = K_{0}^{NC}$ (automatically copied from $K_{0}^{NC}$ cell), applicable to most study cases if soil was subject to $K_{0}^{NC}$ natural consolidation (oedometric conditions), or for simulating a triaxial compression or extension triaxial on a $K_{0}^{NC}$-consolidated sample.
   - Isotropic consolidation: $K_{SR}^{0} = 1$ (automatically assigned) when simulating isotropically consolidated compression or extension triaxial tests.
   - Anisotropic consolidation: $K_{SR}^{0}$ (user-defined) in situations when historical consolidation was different than $K_{0}^{NC}$-consolidation specified for $E_{oom}$.

3. **Current in situ stress configuration** $\sigma'_0 = \sigma'_0(\sigma'_{x0}, \sigma'_{y0}, \sigma'_{z0})$
   - $\sigma'_{y0} = \rho \cdot g \cdot y$
   - $\sigma'_{x0} = \sigma'_{y0} \cdot K_{0x}$
   - $\sigma'_{z0} = \sigma'_{y0} \cdot K_{0x}$
2.1. HARDENING SOIL- STANDARD MODEL

**Figure 2.13:** Definition the initial preconsolidation state by means of a constant OCR and the resulting vertical preconsolidation stress $\sigma_{vc}$.

**Figure 2.14:** Definition the initial preconsolidation state by means of preoverburden pressure $q^{\text{POP}}$ and the resulting variable OCR profile (typically observed for superficial soil layers). In such a case, a variable $K_0$ can also be considered by applying, for instance, Eq. (3.89).
At the beginning of a FE analysis, ZSoil® sets the stress reversal point (SR) with:

\[
\sigma_{SR}^y = \sigma_y \cdot OCR \quad \text{or} \quad \sigma_{SR}^y = \sigma_{y0} + q^{POP}
\]  

and

\[
\sigma_{x}^{SR} = \sigma_{y}^{SR} K_0^{SR} \quad \text{and} \quad \sigma_{z}^{SR} = \sigma_{y}^{SR} K_0^{SR}
\]

Then ZSoil® uses the calculated \(\sigma^{SR}\) stress state to compute initial values of the hardening parameter \(\gamma_{PS0}\) from the condition \(f_1 = 0\), and \(p_{c0}\) from \(f_2 = 0\).

\[\text{Figure 2.15: Initial stress state setup. Note that for normally-consolidated soil } \sigma_{SR}^{0} \text{ coincides with } \sigma_{0}^{0}, \text{ and therefore the Initial } K_0^{0} \text{ state which is required to be set by the user is equal to } K_{0}^{NC} \text{ specified in the Non-linear material menu. For overconsolidated soil, the initial state coefficient } K_0^{0} \text{ is typically larger than } K_{0}^{NC} \text{ (cf. Section 3.3.9).}
\]

Note that \(\sigma_{0}^{0}\) in ZSoil® is computed with the Initial State driver based on gravity-induced vertical stress and on the user-specified \(K_0^{0}\) (Initial Ko State menu):

Using the automatic \(K_0^{0}\) evaluation option, the initial \(K_0^{0}\) profile is computed in as the function of the preconsolidation state and the effective friction angle (Mayne and Kulhawy, 1982):

\[
K_0 = K_{0}^{NC} OCR \sin \phi'
\]
with:

\[ K_{0}^{NC} = 1 - \sin \phi' \]  

(2.28)

and the upper bound value for \( K_0 \) is limited to the passive lateral earth pressure coefficient:

\[ K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \]  

(2.29)

The initial \( K_0 \) state can also be set via the PrePro by applying effective stresses \( \sigma_x' \), \( \sigma_y' \), \( \sigma_z' \) using *Initial stresses* option, Fig. 2.16. This option can be useful in the case when imposing a variable \( K_0 \) is needed (see an example in Section 5.3). However, in order to obtain the imposed initial stress field, the Initial State driver has be omitted in the analysis.

**Figure 2.16:** Imposing *Initial stresses* option in PrePro.

NB. In ZSoil®, compressive stress is negative. In the case of Deformation+Flow analysis, the effective stresses have to be introduced when using *Initial stresses* option.
2.1.8 Troubleshooting for Initial State analysis

It may happen that the FE analysis does not converge using the Initial State driver. In such a case the following double-check sequence is suggested to be performed:

1. Control whether Initial $K_0$ State is specified for each material which is defined by the material formulation Hardening-Soil.

2. Control whether $K_0$ defined in Initial $K_0$ State is larger than $K_0^{SR}$ defined in Non linear menu of the HS-small strain stiffness model.

3. Try to start Initial State analysis from a very small Initial load factor (e.g. 0.2) applying a very small Increment (e.g. 0.1 to 0.4, see below).

Another efficient way to converge and accelerate the initial state analysis is to define the initial stress state via the PrePro by means of Initial stresses option (see Fig. 2.17). In the Initial stresses dialog window, you can put default values for the simplified stress definition which is applied on the whole model domain (i.e. all the soil layers). This option will help to find the first guess but the final solution for each soil layer will be computed based on the user-imposed $K_0$ defined locally in the material definition, Fig. 2.1.7.

![Figure 2.17: Defining initial stresses using the simplified definition.](image)
2.2 Hardening Soil-SmallStrain model

The basic Hardening Soil-Standard model which is implemented in ZSoil® can be extended with the Hardening Soil-SmallStrain model which allows accounting for S-shaped stiffness reduction which is presented in Figure 1.1. In such a case the stress paths which return to the elastic domain during unloading can be modeled as a non-linear stress-strain relationship.

Since HS-SmallStrain is implemented as an extension of HS-Std model, the list of model parameters remains the same as for HS-Std (cf. Table 2.2) and it is only extended with two parameters which define small strain behavior, i.e. the maximal shear modulus \( G_0 \) and a characteristic shear strain level \( \gamma_{0.7} \) at which the secant shear modulus \( G_s \) reduces to 70% of the initial shear modulus \( G_0 \). The effect of small strain stiffness is taken into account in the HS model once the small strain extension is activated (see below).

The study cases presented in Section 5.1, 5.2 and 5.3 demonstrate sensitivity of numerical simulations to small strain extension. For example, Figures 5.30(a) and 5.31(a) demonstrate that it is better to run a simulation with underestimated small-strain parameters than not to account for the small-strain stiffness at all.

2.2.1 Non-linear elasticity for small strains

In order to describe the nonlinear S-shaped stiffness reduction, the commonly known in soil dynamics, hyperbolic Hardin-Drnevich relation is adopted. This relation relates the current secant shear modulus \( G_s \) with an equivalent monotonic shear strain level \( \gamma_{\text{hist}} \), and it takes the following forms:

for primary loading:

\[
\frac{G_s}{G_0} = \frac{1}{1 + a \frac{\gamma_{\text{hist}}}{\gamma_{0.7}}} \tag{2.30a}
\]

for unloading/reloading:

\[
\frac{G_s}{G_0} = \frac{1}{1 + a \frac{\gamma_{\text{hist}}}{2\gamma_{0.7}}} \tag{2.30b}
\]

with \( a = 0.385 \) modifying the original Hardin-Drnevich formula. Note that for \( \gamma_{\text{hist}} = \gamma_{0.7} \), the ratio \( G_s/G_0 \) is equal to 0.722 which means 72.2% reduction in the case of more accurate considerations (see Figure 2.18).

The equivalent monotonic shear strain is computed from:

\[
\gamma_{\text{hist}} = \frac{3}{2} \varepsilon_q \tag{2.31}
\]

3Note that during unloading/reloading, Hardening Soil Standard model reproduces linear elasticity only.
Figure 2.18: Reduction of the secant shear modulus \( G_s \) using Eq. (2.30a) and interpretation of the parameter \( \gamma_{0.7} \).

with \( \varepsilon_q \) denoting the second deviatoric strain invariant, and in triaxial test conditions \( \gamma_{\text{hist}} \) can be expressed as:

\[
\gamma_{\text{hist}} = \varepsilon_1 - \varepsilon_3
\]  

(2.32)

The corresponding tangent shear modulus \( G \) can be expressed as:

\[
\frac{G}{G_0} = \left( \frac{\gamma_{0.7}}{\gamma_{0.7} + a\gamma_{\text{hist}}} \right)^2
\]  

(2.33)

The modified Hardin-Drneovich formula is only valid if \( \gamma_{\text{hist}} \leq \gamma_c \), with \( \gamma_c \) being the cut-off shear strain at which:

\[
G = G_{\text{ur}} \quad \text{where} \quad G_{\text{ur}} = \frac{E_{\text{ur}}}{1 + \nu_{\text{ur}}}
\]  

(2.34)

The stiffness cut-off allows applying the Hardin-Drneovich formula in the elastic domain (see Figure 2.19), whereas further stiffness reduction is governed by the hardening mechanism. The cut-off shear strain can be computed from:

\[
\gamma_c = \frac{\gamma_{0.7}}{a} \left( \sqrt{\frac{G_0}{G_{\text{ur}}}} - 1 \right)
\]

(2.35)

In Eq. (2.30b), the term \( 2\gamma_{0.7} \) replaces \( \gamma_{0.7} \) appearing in Eq. (2.30a) for virgin loading in order to fulfill Masing’s rule which describes the hysteretic behavior in loading/unloading conditions (see Figure). The rule assumes that (i) initial tangent shear modulus in unloading is equal to the initial tangent shear modulus during initial loading, and (ii) size of the unloading and reloading curves is twice of the initial loading curve.

Further details on implementation of Hardin-Drneovich relationship is provided in Truty (2008).
2.2. HARDENING SOIL-SMALLSTRAIN MODEL

Figure 2.19: Reduction of the tangent shear modulus $G$ in the HS-SmallStrain model based on Hardin-Drnevich formula (Eq.(2.33)).

Figure 2.20: Hysteretic soil behavior using Masing’s rule.

2.2.2 Modifications of the plastic part

HS-SmallStrain model also requires some modifications in the plastic part of the HS-Standard code. These modifications concern the plastic flow rule and dilatancy in the domain of contractancy.

Introducing the cut-off for the contractancy domain (as it is in the HS-Standard model, cf. Eq.(2.17a)) could yield too small volumetric strains. Therefore, allowing a certain amount of contractancy for the mobilized friction angle $\phi_m$ before it reaches the critical state ($\phi_m < \phi_{cs}$). Introducing the scaling parameter $D$ into Eq.(2.17b) match Rowe’s dilatancy in the contractancy domain to the formula proposed by Li and Dafalias (2000), see Figure 2.21.
Rowe’s dilatancy law for HS-SmallStrain model is thus formulated as:

\[
\sin \psi_m = D \frac{\sin \phi_m - \sin \phi_{cs}}{1 - \sin \phi_m \sin \phi_{cs}}
\]  

(2.36a)

where:

\[
D = 0.25 \quad \text{if} \quad \sin \psi_m < \sin \phi_{cs} 
\]  

(2.36b)

\[
D = 1.00 \quad \text{if} \quad \sin \psi_m \geq \sin \phi_{cs} 
\]  

(2.36c)

Parameter \(D\) is automatically updated to the value 0.25, once the small strain extension is activated.

Another modification concerns the hardening laws for parameters \(\gamma_{PS}\) and \(p_c\). The modification is executed by introducing \(h_i\) function which is required for an appropriate approximation of \(\gamma - G\) curve in the case when a stress path starts directly from one or two yield surfaces. Evolution of the hardening parameters is defined as follows:

\[
d\gamma_{PS} = d\lambda_1 h_i \left( \frac{\partial g_1}{\partial \sigma_1} - \frac{\partial g_1}{\partial \sigma_2} - \frac{\partial g_1}{\partial \sigma_3} \right) = d\lambda_1 h_i \quad \text{for shear mechanism} 
\]  

(2.37)

and

\[
dp = d\lambda_2 2H h_i \left( \frac{p_c + c \cot \phi}{\sigma_{\text{ref}} + c \cot \phi} \right)^2 p \quad \text{for volumetric mechanism} 
\]  

(2.38)
with the function $h_i$ being defined as:

$$h_i = G_m \left(1 + \frac{E_{ur}}{2E_{50}}\right)^{1+\frac{E_{ur}}{2E_{50}}}$$  \hspace{1cm} (2.39)

where the stiffness multiplier $G_m$ is calculated as:

$$G_m = \frac{G_{\min}}{G_{ur}}$$  \hspace{1cm} (2.40)

with the minimum stiffness in loading history:

$$G_{\min} = \frac{G_0}{1 + a \gamma_{\text{max}} \gamma_{0.7}}$$  \hspace{1cm} (2.41)

By substituting Eq.(2.41) into Eq.(2.40), the following formula can be obtained:

$$G_m = \frac{G_0/G_{ur}}{1 + a \gamma_{\text{hist}} \gamma_{0.7}/\gamma_{0.7}} = \frac{E_{0\text{ref}}/E_{ur\text{ref}}}{1 + a \gamma_{\text{hist}} \gamma_{0.7}/\gamma_{0.7}}$$  \hspace{1cm} (2.42)
Table 2.2: List of parameters defining the HS-Standard ans HS-SmallStrain models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>HS-Std.</th>
<th>HS-SmallStrain</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{0}^{\text{ref}}$</td>
<td>[kPa]</td>
<td>-</td>
<td>✓</td>
<td>defines the initial tangent slope of $\varepsilon_{1} - q$ curve at the reference minor principal stress $\sigma_{3}^{\text{ref}}$</td>
</tr>
<tr>
<td>$\gamma_{0.7}$</td>
<td>[-]</td>
<td>-</td>
<td>✓</td>
<td>defines a characteristic shear strain level $\gamma_{s}$ at which the ratio $G_{s}/G_{0} = 0.722$</td>
</tr>
<tr>
<td>$E_{\text{ref}}^{ur}$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>defines unloading/reloading stiffness at engineering strains ($\varepsilon \approx 10^{-3}$) at the reference minor principal stress $\sigma_{3}^{\text{ref}}$</td>
</tr>
<tr>
<td>$E_{50}^{\text{ref}}$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>defines the secant stiffness at 50% of the ultimate deviatoric stress $q_{f}$ at the reference minor principal stress $\sigma_{3}^{\text{ref}}$</td>
</tr>
<tr>
<td>$\sigma_{\text{ref}}$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>reference stress used to scale stiffness moduli $E_{0}^{\text{ref}}, E_{\text{ref}}^{ur}, E_{50}^{\text{ref}}$ to current values with respect to a current minor principal stress $\sigma_{3}$ (or a current mean stress $p'$ if this formulation is selected)</td>
</tr>
<tr>
<td>$m$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>defines stress dependent stiffness through Eq. (2.5)</td>
</tr>
<tr>
<td>$\nu_{ur}$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>defines the ratio $\varepsilon_{3}/\varepsilon_{1}$ in an unloading-reloading cycle (elastic deformations)</td>
</tr>
<tr>
<td>$R_{f}$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>used to compute the hardening parameter $\gamma_{PS}$ with the use of the asymptotic deviatoric stress $q_{a}$ defining the hyperbolic function $f_{2}$ (default $R_{f} = 0.9$)</td>
</tr>
<tr>
<td>$c'$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>defines the intercept of the Mohr-Coulomb line at null stress condition</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>[°]</td>
<td>✓</td>
<td>✓</td>
<td>defines the slope of the Mohr-Coulomb yield criterion</td>
</tr>
<tr>
<td>$\psi$</td>
<td>[°]</td>
<td>✓</td>
<td>✓</td>
<td>defines the maximal slope of $\varepsilon_{1} - \varepsilon_{v}$ curve for dilatancy</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>defines the cut-off limit corresponding to the maximal void ratio observed in material at the ultimate state</td>
</tr>
<tr>
<td>$f_{t}$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>defines the maximal tensile strength for material</td>
</tr>
<tr>
<td>$D$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>controls Rowe’s dilatancy law in the contractancy domain (default $D = 0$ for HS-Std, $D = 0.25$ for HS-SmallStrain)</td>
</tr>
<tr>
<td>$M$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>defines the shape of the elliptical cap yield surface</td>
</tr>
<tr>
<td>$H$</td>
<td>[kPa]</td>
<td>✓</td>
<td>✓</td>
<td>defines the rate of the plastic volumetric strain and the preconsolidation pressure</td>
</tr>
<tr>
<td>OCR or $q_{\text{POP}}$</td>
<td>[-] or [kPa]</td>
<td>✓</td>
<td>✓</td>
<td>sets the initial position of stress with respect to the cap surface and it is used to compute the hardening parameter $\gamma_{PS}$ and $p_{c0}$</td>
</tr>
<tr>
<td>$K_{0}^{\text{SR}}$</td>
<td>[-]</td>
<td>✓</td>
<td>✓</td>
<td>sets a historical position of the stress point $\sigma_{\text{SR}}^{\text{SR}} (K_{0}^{\text{SR}} = \sigma_{h}^{\text{SR}}/\sigma_{v}^{\text{SR}})$ with respect to the initial stress configuration for an overconsolidated material and it is used to compute the hardening parameter $\gamma_{PS}$ and $p_{c0}$</td>
</tr>
</tbody>
</table>
**Figure 2.22:** Dialog window for the *Elastic* group of parameters which define the HS model including the small strain extension.
Figure 2.23: Dialog window for the Nonlinear group of parameters which define the HS model including the initial state setup.
2.3 Model parameters

Although the HS model is mathematically complex, its parameters have the physical meaning and they can be derived from the standard laboratory test, i.e. the triaxial compression and oedometer tests. A complete list of parameters that the user needs to specify before running application is provided in Table 2.2. The details related to the identification of specific parameters are provided in the subsequent Chapter 3.

The following abbreviations apply to Table 2.3:

- CICD - triaxial test: consolidated isotropically compression drained
- CICU - triaxial test: consolidated isotropically compression undrained
- OED - oedometer test
- CPT - cone penetration test
- CPTU - piezocone cone penetration test
- DMT - Marchetti’s dilatometer test
- SCPTU - piezocone cone penetration test with seismic sensor
- DMT - Marchetti’s dilatometer test with seismic sensor
- SPT - standard penetration test
Table 2.3: List of parameters which should be provided by the user (advanced parameters in gray).

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Unit</th>
<th>Direct estimation test</th>
<th>Alternative test or solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{r}^{ref}$</td>
<td>[kPa]</td>
<td>SCPT, DMT or bore-hole, cross-hole or other geophysical method</td>
<td>unloading-reloading branch of CICD; geotechnical evidence; sands: CPT geotechnical evidence</td>
</tr>
<tr>
<td>$\gamma_{0.7}$</td>
<td>[-]</td>
<td>CICD with local gauges</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$E_{r}^{ref}$ ($\sigma_{3}^{ref}$)</td>
<td>[kPa]</td>
<td>min. 1 CICD at $\sigma_{3}^{ref}$</td>
<td>sands: CPT</td>
</tr>
<tr>
<td>$E_{ur}^{ref}$ ($\sigma_{3}^{ref}$)</td>
<td>[kPa]</td>
<td>min. 1 CICD at $\sigma_{3}^{ref}$</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$\sigma_{3}^{ref}$</td>
<td>[kPa]</td>
<td>1 CICD</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$\nu_{ur}$</td>
<td>[-]</td>
<td>min. 1 CICD with unloading-reloading curve</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$m$</td>
<td>[-]</td>
<td>3 CICD at different $\sigma_{3}$</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$c$</td>
<td>[kPa]</td>
<td>3 CICD or CICU at different $\sigma_{3}$</td>
<td>geotechnical evidence; sand: CPT, DMT, PMT, SPT geotechnical evidence; default $R_{f} = 0.9$, geotechnical evidence</td>
</tr>
<tr>
<td>$\phi$</td>
<td>[°]</td>
<td>3 CICD or CICU at different $\sigma_{3}$</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>[°]</td>
<td>min. 1 CICD</td>
<td></td>
</tr>
<tr>
<td>$R_{f}$</td>
<td>[-]</td>
<td>min. 1 CICD</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{max}$</td>
<td>[-]</td>
<td>min. 1 CICD on a dense or preconsolidated soil specimen</td>
<td>geotechnical evidence</td>
</tr>
<tr>
<td>$f_{t}$</td>
<td>[kPa]</td>
<td>isotropic extension</td>
<td>default $f_{t} = 0$</td>
</tr>
<tr>
<td>$D$</td>
<td>[-]</td>
<td>min. 1 CICD</td>
<td>default $D=0$ for HS-Standard and $D=0.25$ for HS-SmallStrain</td>
</tr>
</tbody>
</table>

Volumetric (cap) mechanism

| $E_{oed}^{ref}$ ($\sigma_{oed}^{ref}$) | [kPa]| min. 1 OED | clays: CPT, DMT |
| $\sigma_{oed}$ | [kPa]| idem | idem |

Initial state variables (soil history)

| OCR or $q^{POP}$ | [-/kPa]| min. 1 OED | clay: CPT, CPTU, DMT |
| $K_{0}^{SR}$     | [-]   | $K_{0}$-consolidation | "Jaky’s formula" |
The table below presents typical ranges of HS-model parameters in soils. It also indicates relevant sections where the interested user may find more information about parameter estimation in case of lack laboratory data.

Table 2.4: Typical values and ranges for parameters of the HS model.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Unit</th>
<th>Coarse soils</th>
<th>Fine soils</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Small stiffness (HS-SmallStrain only)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_0^{\text{ref}}$</td>
<td>[kPa]</td>
<td>$7 \cdot 10^{-5} &lt; \gamma_{0.7} &lt; 4 \cdot 10^{-5}$</td>
<td>$\gamma_{0.7} &gt; 9 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\gamma_{0.7}$</td>
<td>[-]</td>
<td>cf. Sec.3.2.1</td>
<td>cf. Sec.3.3.1</td>
</tr>
<tr>
<td><strong>Elastic constants</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{50}^{\text{ref}}(\sigma_{\text{ref}})$</td>
<td>[kPa]</td>
<td>$E_{ur}^{\text{ref}}/3$, cf. 3.2.2</td>
<td>$E_{ur}^{\text{ref}}/3$, cf. Sec.3.3.5</td>
</tr>
<tr>
<td>$E_{ur}^{\text{ref}}(\sigma_{\text{ref}})$</td>
<td>[kPa]</td>
<td>Sec.3.2.2</td>
<td>Sec.3.3.5</td>
</tr>
<tr>
<td>$\sigma_{\text{ref}}$</td>
<td>[kPa]</td>
<td>typically taken as 100 kPa, cf. also in Virtual Lab report</td>
<td></td>
</tr>
<tr>
<td>$\nu_{ur}$</td>
<td>[-]</td>
<td>$0.15 &lt; \nu_{ur} &lt; 0.25$, cf. Sec.3.2.4</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>[-]</td>
<td>$0.5$, cf. Sec.3.2.5</td>
<td>$0.5 &lt; m &lt; 1.0$, cf. Sec.3.3.7</td>
</tr>
<tr>
<td><strong>Shear mechanism</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c'$</td>
<td>[kPa]</td>
<td>$0 \div 5$</td>
<td>$\geq 0$</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>[°]</td>
<td>$25° &lt; \phi &lt; 50°$</td>
<td>$18° &lt; \phi &lt; 42°$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>cf. Sec.3.2.6</td>
<td>cf. Sec.3.3.2</td>
</tr>
<tr>
<td>$\psi$</td>
<td>[°]</td>
<td>Sec.3.2.7</td>
<td>cf. Sec.3.3.4</td>
</tr>
<tr>
<td>$R_f$</td>
<td>[-]</td>
<td>$0.75 &lt; R_f &lt; 1$ with average $R_f = 0.9$, cf. Sec.3.3.3</td>
<td></td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>[-]</td>
<td>Sec.3.2.9</td>
<td>Sec.3.3.10</td>
</tr>
<tr>
<td>$f_t$</td>
<td>[kPa]</td>
<td>default $f_t = 0$</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>[-]</td>
<td>default $D=0$ for HS-Standard and $D=0.25$ HS-SmallStrain</td>
<td></td>
</tr>
<tr>
<td><strong>Volumetric (cap) mechanism</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{\text{oed}}^{\text{ref}}(\sigma_{\text{oed}})$</td>
<td>[kPa]</td>
<td>$\approx E_{50}^{\text{ref}}(\sigma_{\text{ref}})$, cf. Sec.3.2.3 and 3.3.6</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{oed}}^{\text{ref}}$</td>
<td>[kPa]</td>
<td>$\approx \sigma_{\text{ref}}/K_{0}^{\text{NC}}$, cf. Sec.3.2.3</td>
<td></td>
</tr>
<tr>
<td>$K_{0}^{\text{NC}}$</td>
<td>[-]</td>
<td>good-working equation: $K_{0}^{\text{NC}} = 1 - \sin \phi'$</td>
<td></td>
</tr>
<tr>
<td><strong>Initial state variables (soil history)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCR or $q_{\text{POP}}^{\text{POP}}$</td>
<td>[-/kPa]</td>
<td>OCR $\geq 1$ or $q_{\text{POP}}^{\text{POP}} \geq 0$</td>
<td></td>
</tr>
<tr>
<td>$K_{0}^S$</td>
<td>[-]</td>
<td>$= K_{0}^{\text{NC}}$ for natural soils, cf. Sec. 2.1.7</td>
<td></td>
</tr>
<tr>
<td>$K_{0}$</td>
<td>[-]</td>
<td>good-working equation: $K_{0} = K_{0}^{\text{NC}} \text{OCR}^{\sin \phi'}$, cf. Sec.3.3.9</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 3

Parameter determination

As most of the constitutive models for soils, the **Hardening-Soil Standard** model has been designed based on behavior of soil specimen which is observed during laboratory tests with the use of standard devices such as triaxial cell and oedometer. Therefore, still responding to certain test requirements such as drained compression, model parameters can be derived directly from the experimental curves. Direct parameter identification is presented in Section 3.1. Sometimes, the test requirements cannot be fulfilled (e.g. performing drained compression test on low permeable clay specimen may prove to be too time consuming). Then, the model can still be calibrated using, for instance, the measurements derived from the undrained triaxial compression test or the model parameters can be estimated based on results obtained through *in situ* tests or approximated using parameter correlations observed in geotechnical practice. Such an indirect parameter determination is presented in Section 3.2 for sand type materials, and in Section 3.3 for cohesive soils.

Additional parameter which describes the small stain stiffness in the **Hardening-Soil Small** model can be easily determined using the measurements derived from one of the *in situ* probes equipped with a seismic sensor which allows measuring the velocity of shear waves. Owing to time and economical constraints of laboratory testing, and the effect of specimen disturbances during soil sampling, the use of laboratory devices to determine $G_0$ seems less reasonable. Nevertheless, an approximate value of $G_0$ can be derived from unloading-reloading branch derived from the triaxial compression test.

The following sections provide a comprehensive guideline on parameter identification which may help the user to effectively apply the advanced constitutive models. In this context, the guideline may be helpful in specifying an appropriate testing program or making use of already acquired experimental results which need a specific treatment in order to estimate model parameters.
3.1 Experimental testing requirements for direct parameter identification

3.1.1 Direct parameter identification for the Hardening-Soil Standard

Direct parameter identification for the Hardening-Soil Standard model requires the use of two commonly used laboratory devices:

- **triaxial cell** with consolidated isotropically drained compression test (CICD); three programmed compression tests at different confining pressures $\sigma_3$ should provide:
  - stress paths in $p' - q$ plane which are used to determine strength parameters $\phi' (\equiv \phi)$ and $c' (\equiv c)$, according to Figure 3.1 or Mohr’s circles; Note that $\phi$ and $c$ can also be derived from the undrained compression test\(^1\) (CU) considering that the failure stress envelopes derived from drained and undrained tests are essentially similar, see an example in Figure 3.2;
  - relationships $\varepsilon_1 - q$ which is used to determine unloading-reloding modulus $E_{ur}\text{,}^2$, as shown in Figure 3.4; secant modulus $E_{50}$ and failure ratio $R_f$ according to Figure 3.3; and stiffness stress dependency parameter $m$ according to Figure 3.5;
  - relationships $\varepsilon_1 - \varepsilon_v$, which is used to determine the dilatancy angle $\psi$ and the maximal void ratio $e_{\text{max}}$, as shown in Figure 3.6.

- **oedometer**; the test should provide pre- and post-yield evolution of the void ratio (or specimen height) with respect to changes of vertical effective stress, $\sigma_v'$, which is used to estimate:
  - the preconsolidation pressure $\sigma_c'$ for cohesive deposits, which is then used to determine the overconsolidation ratio $\text{OCR}$ defined as:
    \[ \text{OCR} = \frac{\sigma_c'}{\sigma_{v0}} \] (3.1)
  - the tangent oedometric stiffness $E_{\text{oed}}$ for corresponding $\sigma_{\text{oed}}$ which have to be captured from the primary loading curve\(^4\) (postyield branch), see Figure 3.7.

The preconsolidation pressure $\sigma_c'$ understood as a threshold point beyond which the important plastic straining occur, is difficult to establish unambiguously. Among a number of methods proposed in literature for determining $\sigma_c'$, the following ones are commonly used owing to their simplicity:

\(^1\)In the case of the undrained test, the maximum principal stress ratio $(\sigma_1' / \sigma_3')_{\text{max}}$ and the maximum deviatoric stress $(\sigma_1 - \sigma_3)_{\text{max}}$ can be considered as the failure criteria.
\(^2\)Note that $E_{ur}$ corresponds to Young’s modulus $E$ which is specified by the user in Mohr-Coulomb or Cap model.
\(^3\)Note that $E_0 > E_{ur} > E_{50}$ or $G_0 > G_{ur} > G_{50}$.
\(^4\)This condition implies that at given oedometric pressure, both shear and volumetric mechanisms are active following $K_0^{\text{NC}}$ consolidation line, as shown in Figure 3.7(b).
3.1. EXPERIMENTAL TESTING REQUIREMENTS FOR DIRECT PARAMETER IDENTIFICATION

- empirical, graphical Casagrande’s method (Casagrande, 1936), see Figure 3.9(a),
- simple graphical method proposed by Pacheco-Silva (1970), see Figure 3.9(b),
- and as a last resort, $\sigma'_c$ can be taken as the vertical stress which corresponds to the intersection point of the reloading and the virgin compression lines, cf. Figure 3.9(a).

It should be noticed that the in situ preconsolidation pressure may vary from that derived from laboratory tests considering specimen disturbances due to sampling, transporting or specimen trimming, etc. Leroueil et al. (1983a) demonstrated that the in situ preconsolidation pressure is observed as:

$$\sigma'_{c,\text{in situ}} = \alpha \cdot \sigma'_{c,\text{lab}} \quad (3.2)$$

where $\alpha = 1.1$ for normally consolidated clays ($\text{OCR} < 1.2$), $\alpha = 1.0$ for lightly consolidated clays ($1.2 < \text{OCR} < 2.5$), and $\alpha = 0.9$ for overconsolidated clays ($2.5 < \text{OCR} < 4.5$).

$E_{\text{oed}}$ and $\sigma_{\text{oed}}^{\text{ref}}$ are the input variables which are used to calculate parameters $M$ and $H$ with the aid of the internal ZSoil® calculator, see material interface for nonlinear characteristics of the HS model.

Clearly, $E_{\text{oed}}$ can also be determined from:

$$E_{\text{oed}} = \left( 1 + \frac{\varepsilon_{\text{ref}}}{C_c} \right) \sigma^* \quad (3.3)$$

where $C_c$ is the compression index (see Figure 3.8), $\varepsilon_{\text{ref}}$ denotes the void ratio corresponding to $\sigma_{\text{oed}}^{\text{ref}}$, and:

$$\sigma^* = \frac{\Delta \sigma'}{\log_{10} \left( \frac{\sigma_{\text{oed}}^{\text{ref}} + \Delta \sigma'}{\sigma_{\text{oed}}^{\text{ref}}} \right)} \quad (3.4)$$

Since we look for the tangent modulus $E_{\text{oed}}$, $\Delta \sigma'$ tends to 0, and $\sigma^*$ is equal to $2.303 \sigma_{\text{oed}}$.

In this case, $E_{\text{oed}}$ can be derived from:

$$E_{\text{oed}} = \frac{2.3(1 + \varepsilon_{\text{ref}})}{C_c} \sigma_{\text{oed}}^{\text{ref}} \quad (3.5)$$

Note that $\sigma_{\text{oed}}^{\text{ref}}$ and $\varepsilon_{\text{ref}}$ are relevant to material which undergoes plastic straining, i.e. the stress point lies on the primary loading curve.

In the case of incompleteness of experimental results, the input model parameters can be estimated using approximative parameter correlations which are provided in Section 3.2 and 3.3.

Sometimes, the compression index $C_c$ can also be expressed through the isotropic compression index $\lambda$ which is the slope of the virgin compression line plotted in $\ln p' - e$ axes. Since $\log_{10} x = 0.43 \ln x$, one can derive:

$$C_c = 2.3 \lambda \quad (3.6)$$

A number of correlations for estimating $C_c$ are provided in Appendix B.
then \( \phi = \arcsin \left( \frac{3M^*}{6 + M^*} \right) \) 
\( c = c^* \frac{3 - \sin \phi}{6 \cos \phi} \)  \hspace{1cm} (3.7)

Figure 3.1: Determination of the residual Mohr-Coulomb envelope and strength parameters \( \phi \) and \( c \) from typical stress paths derived from the triaxial drained compression tests driven at three different confining pressures \( \sigma_3 \).

Figure 3.2: Compatibility of strength envelopes derived from drained and undrained triaxial tests (from Kempfert, 2006).
3.1. EXPERIMENTAL TESTING REQUIREMENTS FOR DIRECT PARAMETER IDENTIFICATION

Figure 3.3: Determination of the slope $a$ and $b$ for identification of the secant modulus $E_{50}$ and failure ratio $R_f$ from typical triaxial drained compression results $\varepsilon_1 - q$. Best precision of the interpreted parameters is obtained by plotting the trendline for two closest data points adjacent to $0.5q_f$. 

Figure 3.4: Determination of $E$ moduli (input model parameters) from a typical curve derived from the triaxial drained compression tests.
CHAPTER 3. PARAMETER DETERMINATION

Identification algorithm:
1. Find three values of $E^{(i)}_{50}$ corresponding to $\sigma_3^{(i)}$ respectively.
2. Find a trend line $y = ax + b$ by assigning variables $y$ as $\ln E^{(i)}_{50}$ and $x$ as $\ln \left( \frac{\sigma^{(i)} + c \cot \phi}{\sigma_{ref} + c \cot \phi} \right)$ and assuming $\sigma_{ref}$ (typically equal to 100kPa)
3. Then the determined slope of the trend line $a$ is the parameter $m$.

Figure 3.5: Determination of the stiffness stress dependency parameter $m$ from three curves derived from the triaxial drained compression tests.
3.1. EXPERIMENTAL TESTING REQUIREMENTS FOR DIRECT PARAMETER IDENTIFICATION

\[
\psi = -\arcsin \left( \frac{d}{2 - d} \right)
\]

assuming negative sign (-) for an increase of the specimen height and compressive axial strain with positive sign (+).

**Figure 3.6:** Determination of the dilatancy angle \( \psi \) from \( \varepsilon_v - \varepsilon_1 \) curve obtained in the triaxial drained compression test.

\[
p^* = \frac{1 + 2K_0^{NC}}{3}\sigma_{oed}^{\text{ref}} \quad \text{and} \quad q^* = (1 - K_0^{NC})\sigma_{oed}^{\text{ref}}
\]

**Figure 3.7:** Assumptions to the automatic determination of parameters \( M \) and \( H \): at given \( \sigma_{oed}^{\text{ref}} \) which is located at post-yield plastic curve, both shear and volumetric mechanism are active.
CHAPTER 3. PARAMETER DETERMINATION

Figure 3.8: Determination of the compression index $C_c$ from typical results derived from oedometer test for estimating the tangent modulus $E_{oed}$.

Figure 3.9: Estimation of preconsolidation pressure $\sigma'_c$ (a) Casagrande’s method, (b) Pacheco Silva’s method.
3.1.2 Direct parameter identification for the Hardening-Soil Small

Direct parameter identification for the Hardening-Soil Small model requires the measurements derived from geophysical tests or one of the advanced *in situ* probes equipped with a seismic sensor which allows measuring the shear wave velocity $V_s$ in the subsoil. Two commonly known devices, i.e. the seismic piezocone (SCPT or SCPTU) or seismic Marchetti’s dilatometer (SDMT), can be used to determine small strain stiffness $G_0$ (or $G_{max}$) from the following expression:

$$G_0 = \rho V_s^2 \quad (3.8)$$

where $\rho$ is a density of soil. Typical ranges for $V_s$ in different types of soils, as well as a variety of methods that can be used to estimate $V_s$ from *in situ* tests (SPT, DMT, CPT, PMT) are given in Appendix C.

Note that in natural conditions $G_0$ is stress dependent and, in the HS-SmallStrain model, this parameter is defined by analogy to other stiffness moduli with:

$$G_0 = G_0^{ref} \left( \frac{\sigma_s^{ref} + c \cot \phi}{\sigma_s^{ref} + c \cot \phi} \right)^m \quad (3.9)$$

Having determined $G_0$, the parameter $E_0$ which is defined by the user in the material dialog, can be calculated from:

$$E_0 = 2(1 + \nu_{ur})G_0 \quad (3.10)$$

assuming that Poisson’s coefficient $\nu_{ur}$ is a constant in the model.

Soil stiffness at very small strains can also be approximated based on the initial part of the $\varepsilon_1 - q$ curve or the unloading-reloading branch derived from the triaxial compression test, as demonstrated on Figure 3.4. However, an exact determination of the initial soil stiffness $E_i$ may prove to be difficult, especially in soft soils. Therefore, one should realize that the initial slope $E_i$ derived from triaxial test can be more than once lower than soil stiffness $E_0$ observed in natural conditions.

Identification of the parameter $\gamma_{0.7}$ at which the secant shear modulus $G_s^{ref}$ is reduced to $0.722G_0^{ref}$, requires the use of advanced laboratory devices in order to determine the S-shape curve at very small strain levels. In practice, it may prove to be time-consuming and expensive and therefore, it is suggested to estimate $\gamma_{0.7}$ by means of typically observed experimental curves. In the case of granular materials $\gamma_{0.7}$ mostly depends on the mean effective stress $p'$ (see Figure 3.19) but also on overconsolidation. In cohesive materials, $\gamma_{0.7}$ may mostly depend on the plasticity index $I_P$ (PI) (see Figures 3.38 or 3.40), however the stress level ($p'$) and the overconsolidation (OCR) may also increase the value of $\gamma_{0.7}$. Having assumed all other model parameters, it is also recommended to run a one-element simulation of the triaxial compression test in order to examine the shape of $\log(\varepsilon_1) - G$ (or $E$) curve derived from the computed $\varepsilon_1 - q$ results.
CHAPTER 3. PARAMETER DETERMINATION

3.1.3 Parameter identification sequence

The following sequence should be followed during parameter identification for the HS model:

1. Identify $\phi'$ and $c'$ (e.g. Figure 3.1)
2. Identify $\psi$ (e.g. Figure 3.6)
3. Identify $R_f$ and $E_{50}^{(i)}$ for different confining pressures $\sigma_3^{(i)}$ (e.g. Figure 3.3)
4. Identify $E_{ur}^{(i)}$ for different confining pressures $\sigma_3^{(i)}$ (the value of $\nu_{ur}$ can be assigned between 0.1 and 0.2)
5. Identify $m$ based on the identified values of $E_{50}^{(i)}$ or and $E_{ur}^{(i)}$ (e.g. Figure 3.5)
6. Assign the reference stress $\sigma_{ref}$ (it can be the confining pressure in the triaxial test that best corresponds to in situ stress conditions)
7. Evaluate $E_0$ and $\gamma_{0.7}$
8. Calculate $E_{ref}^0$, $E_{ref}^{ur}$, $E_{ref}^{50}$ in terms of $\sigma_{ref}$ (note that by applying the stress stiffness dependency law such as Eq.(2.4) the stiffness moduli also depend on $\phi$, $c$ and $m$)
9. Double-check the following relationships:
   - $E_{ref}^0 > E_{ref}^{ur} > E_{ref}^{50}$
   - $E_{ref}^{ur}/E_{ref}^{50} > 2$
   - $3.6 < E_0^{ref}/E_{50}^{ref} < 30$ (typically $6 < E_0^{ref}/E_{50}^{ref} < 14$)
10. Evaluate $K_{0NC}^{NC}$ and $E_{oed}^{ref}$ for the corresponding vertical reference stress $\sigma_{oed}^{ref}$ in order to compute $M$ and $H$ (note that $M$ and $H$ should be recomputed if one of the following parameters has been changed: $E_{ur}$, $\nu_{ur}$, $m$, $E_{50}$, $\phi$, $\psi$, $c$, $R_f$)
11. Evaluate the profile of soil preconsolidation in order to specify a constant OCR or a variable OCR profile by means of $q^{POP}$
12. Set $K_0^{SR} = K_0^{NC}$
13. Evaluate the in situ stress state in order to specify $K_0$ (for example accounting for preconsolidation state Eq.(3.89))

An example of parameter identification using a spreadsheet is given in Table 5.8.
3.1. EXPERIMENTAL TESTING REQUIREMENTS FOR DIRECT PARAMETER IDENTIFICATION

3.1.4 Model parameters for "undrained" simulations

In ZSoil®, the undrained behavior can be obtained in the two-phase analysis (Deformation+flow) by running one of the following drivers:

- Consolidation driver (undrained or partially-drained conditions can be obtained depending on the action time and soil permeability)
- Driven Load (Undrained) (perfectly undrained conditions)

Since the constitutive models are formulated in effective stresses, it is recommended to use the effective stress parameters:

- Effective stiffness parameters \(E'_0, E'_u, E'_{g0}, \nu'_u, \) permeability
- Effective strength parameters \(\phi', c'\)

The main advantages of working with the effective parameters are as follows:

- Deviatoric stress at failure (that corresponding to the undrained shear strength \(q_u^f = 2S_u\)) depends on preconsolidation history (cf. Figure ??)
- Undrained shear strength is stress dependent

Another approach which can be considered for simulating the undrained behavior is that relying on "undrained" strength parameters, i.e. \(\phi = \phi^u = 0, c = S_u, \nu'_u = 0.4999\).

Note, however, that for this approach the undrained shear strength is constant and stress independent.

In such approach, the following parameters should be considered:

- Effective strength parameters \(\phi = \phi^u = 0, c = S_u\) (\(\psi = 0\) for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point)
- Effective stiffness parameters \(E'_0, E'_u, E'_{g0}, \nu'_u = 0.4999\)
- High OCR, e.g. 1000 to skip the cap mechanism in computations (no plastic volumetric deformations)

The following sequence should be respected when setting the "undrained" parameters:

1. Insert the effective parameters \(E'_0, E'_u, \) in Elastic dialog window and the "undrained" Poisson’s coefficient \(\nu'_u = 0.4999\).

---

5 In order to obtain perfectly undrained conditions for consolidation driver, use very small values of the permeability coefficient, say \(10^{-12} \div 10^{-15}\) m/s, and short action times in order to disable evolution of the cap mechanism that imposes no volume changes.
CHAPTER 3. PARAMETER DETERMINATION

In the Nonlinear dialog window, disable Automatic evaluation of H and M parameters in order to avoid unfeasible calculation of M and H parameters for the null friction angle.

2. Set "undrained" parameters $\phi^u = 0^\circ$, $c = S_u$, and $\psi = 0$ for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point (cf. Truty and Obrzud (2015)).

3. Set high OCR, e.g. 1000, in order to skip the cap mechanism during analysis

4. Set input $E_{50}$ equal to undrained one $E_{50}^u$. The "undrained" secant modulus can be computed from:

$$E_{50}^u = \frac{3E'_{50}}{2(1 + \nu)} \tag{3.11}$$

where $\nu$ should correspond to the ratio $\varepsilon_3/\varepsilon_1$ obtained for plastic straining and the effective secant modulus $E'_{50}$, i.e. $\nu \approx 0.3$. 

5. Set "undrained" parameters $\phi^u = 0^\circ$, $c = S_u$, and $\psi = 0$ for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point (cf. Truty and Obrzud (2015)).

3. Set high OCR, e.g.1000, in order to skip the cap mechanism during analysis

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where $\nu$ should correspond to the ratio $\varepsilon_3/\varepsilon_1$ obtained for plastic straining and the effective secant modulus $E'_{50}$, i.e. $\nu \approx 0.3$. 

5. Set "undrained" parameters $\phi^u = 0^\circ$, $c = S_u$, and $\psi = 0$ for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point (cf. Truty and Obrzud (2015)).

3. Set high OCR, e.g.1000, in order to skip the cap mechanism during analysis

4. Set input $E_{50}$ equal to undrained one $E_{50}^u$. The "undrained" secant modulus can be computed from:

$$E_{50}^u = \frac{3E'_{50}}{2(1 + \nu)} \tag{3.11}$$

where $\nu$ should correspond to the ratio $\varepsilon_3/\varepsilon_1$ obtained for plastic straining and the effective secant modulus $E'_{50}$, i.e. $\nu \approx 0.3$. 

5. Set "undrained" parameters $\phi^u = 0^\circ$, $c = S_u$, and $\psi = 0$ for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point (cf. Truty and Obrzud (2015)).

3. Set high OCR, e.g.1000, in order to skip the cap mechanism during analysis

4. Set input $E_{50}$ equal to undrained one $E_{50}^u$. The "undrained" secant modulus can be computed from:

$$E_{50}^u = \frac{3E'_{50}}{2(1 + \nu)} \tag{3.11}$$

where $\nu$ should correspond to the ratio $\varepsilon_3/\varepsilon_1$ obtained for plastic straining and the effective secant modulus $E'_{50}$, i.e. $\nu \approx 0.3$. 

5. Set "undrained" parameters $\phi^u = 0^\circ$, $c = S_u$, and $\psi = 0$ for normally- and lightly cohesive soils in order to avoid excessive gain in material resistance after reaching the failure stress point (cf. Truty and Obrzud (2015)).
3.2 Alternative parameter estimation for granular materials

3.2.1 Initial stiffness modulus and small strain threshold

The present section provides a number of approaches for estimating the initial soil stiffness and the small strain threshold $\gamma_{0.7}$. Some correlations allow to directly approximate the input parameter $E_0$, the others provide solutions for estimating the initial shear modulus $G_0$. Then the input parameter $E_0$ can be obtained through:

$$E_0 = 2(1 + \nu_{ur})G_0$$

(3.12)

assuming a constant value of the unloading-reloading Poisson's coefficient $\nu_{ur}$.

Geotechnical evidence. Experimental data shows that the initial stiffness of soils may depend on the stress level, soil porosity and overconsolidation. These factors can be taken into account using a modified equation proposed by Hardin and Black (1969):

$$G_0 = A \cdot f(e) \cdot \text{OCR}^k \left(\frac{p'}{p_{ref}}\right)^m, \text{ in [MPa]}$$

(3.13)

where $G_0$ is the maximum small-strain shear modulus in MPa, $p'$ is the mean effective stress in kPa, $p_{ref}$ is the reference stress equal to the atmospheric pressure $p_{ref} = 100$ kPa, OCR is the overconsolidation ratio and $A$, $f(e)$, $k$, $m$ are the correlated functions and parameters which are given in Table 3.1 and 3.2 for different types of soils. It is observed that the empirical exponent $k$ varies from 0 for sands and 0.5 for high plasticity clays.

Biarez and Hicher (1994) proposed a simple relationship for all soils with $w_L < 50\%$:

$$E_0 = \frac{140}{e} \left(\frac{p'}{p_{ref}}\right)^{0.5}, \text{ in [MPa]}$$

(3.14)
### Table 3.1: Parameters for estimation of $G_0$ in different types of granular soils using Eq. (3.13).

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>$D_{50}$ [mm]</th>
<th>$C_u$ [-]</th>
<th>$A$ [-]</th>
<th>$f(e)$ [-]</th>
<th>$k$ [-]</th>
<th>$m$ [-]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenya carbonate sand</td>
<td>0.13</td>
<td>1.86</td>
<td>101-129</td>
<td>$e^{-0.8}$</td>
<td>0</td>
<td>0.45-0.52</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td>Quiou carbonate sand</td>
<td>0.75</td>
<td>4.40</td>
<td>71</td>
<td>$e^{-1.3}$</td>
<td>0</td>
<td>0.62</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>Ottawa sand No.20-30</td>
<td>0.72</td>
<td>1.20</td>
<td>69</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.50</td>
<td>Hardin and Richart Jr (1963)</td>
</tr>
<tr>
<td>SLB sand (subangular)</td>
<td>0.62</td>
<td>1.11</td>
<td>82-130</td>
<td>$1 + e$</td>
<td>0</td>
<td>0.44-0.53</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.16</td>
<td>1.46</td>
<td>71-87</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.41-0.51</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.19</td>
<td>1.56</td>
<td>84-104</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.50-0.57</td>
<td>Chaudary et al. (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.22</td>
<td>1.35</td>
<td>72</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>$e^{-1.3}$</td>
<td>0.45</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.50</td>
<td>1.33</td>
<td>61-64</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.44-0.53</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.54</td>
<td>1.50</td>
<td>71</td>
<td>$(2.27 - e)^2 / (1 + e)$</td>
<td>$e^{-0.8}$</td>
<td>0.43-0.48</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.55</td>
<td>1.66</td>
<td>79-90</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.43-0.48</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td>Ham River sand (subangular)</td>
<td>0.27</td>
<td>1.67</td>
<td>72-81</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.5-0.52</td>
<td>Kuwano and Jardine (2002)</td>
</tr>
<tr>
<td>Silica sand (subangular)</td>
<td>0.20</td>
<td>1.10</td>
<td>80</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.5</td>
<td>Kallioglou et al. (2003)</td>
</tr>
<tr>
<td>Hostun sand (angular)</td>
<td>0.31</td>
<td>1.94</td>
<td>80</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.47</td>
<td>Hoque and Tatsuoka (2000)</td>
</tr>
<tr>
<td>Silica sand (angular)</td>
<td>0.32</td>
<td>2.80</td>
<td>48</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.50</td>
<td>Kallioglou et al. (2003)</td>
</tr>
<tr>
<td>Silica sand</td>
<td>0.55</td>
<td>1.80</td>
<td>275</td>
<td>$(1.46 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.42</td>
<td>Wichtmann and Triantafyllidis (2004)</td>
</tr>
<tr>
<td>Hime gravel (subround)</td>
<td>1.73</td>
<td>1.33</td>
<td>53-94</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.45-0.51</td>
<td>Chaudary et al. (2004)</td>
</tr>
<tr>
<td>Chiba gravel (subround)</td>
<td>7.90</td>
<td>10</td>
<td>76</td>
<td>$(2.17 - e)^2 / (1 + e)$</td>
<td>0</td>
<td>0.50</td>
<td>Modoni et al. (1999)</td>
</tr>
</tbody>
</table>

### Table 3.2: Parameters for estimation of $G_0$ in granular soils using Eq. (3.13).

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>$e_{\min}$ [-]</th>
<th>$e_{\max}$ [-]</th>
<th>$A$ [-]</th>
<th>$f(e)$ [-]</th>
<th>$m$ [-]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean uniform sands with $C_u &lt; 1.8$</td>
<td>0.5</td>
<td>1.1</td>
<td>57</td>
<td>$(2.17 - e)^2 / (1 + e)^2$</td>
<td>0.4</td>
<td>Iwasaki and Tatsuoka (1977)</td>
</tr>
<tr>
<td>All soils with $w_L &lt; 50%$</td>
<td>0.4</td>
<td>1.8</td>
<td>58*</td>
<td>$1 / e$</td>
<td>0.5</td>
<td>Biarez and Hicher (1994)</td>
</tr>
<tr>
<td>Undisturbed crushed sands</td>
<td>0.6</td>
<td>1.5</td>
<td>33</td>
<td>$(2.97 - e)^2 / (1 + e)^2$</td>
<td>0.5</td>
<td>Hardin and Black (1969)</td>
</tr>
</tbody>
</table>

*obtained from Eq. 3.14 assuming $\nu = 0.2$
Figure 3.10: Graphical representation of empirical relations presented in Table 3.6 and 3.18 (after Benz, 2007).
Geotechnical evidence. In case of lack of test data at very small strain levels, $E_0$ can be evaluated from an empirical relation proposed by Alpan (1970). This relation which is presented in Figure 3.11, relates so-called "static" modulus $E_s$ to the "dynamic" modulus $E_d$. For the sake of HS-SmallStrain model, $E_s$ can be considered as $E_{ur}$ obtained at engineering strain levels ($\varepsilon \approx 10^{-3}$), whereas $E_0$ can be considered as $\approx E_d$.

![Figure 3.11](image)

*Figure 3.11: Approximative relation between "static" soil stiffness (here $E_s \approx E_{ur}$) and "dynamic" modulus $E_d$ corresponding to $E_0$ proposed by Alpan (1970).*

Geotechnical evidence. It can be observed in laboratory test that secant stiffness reduces with mobilization of the shear strength. Mayne (2007) provides a selection of secant modulus curves, represented by the ratio $G_s/G_0$ or $E_s/E_0$. The collected results were derived from monotonic laboratory shear tests performed on an sorted mix of clayey ans sandy materials, and they are presented in Figure 3.13. Such experimental results can be approximated with a hyperbolic model by Fahey and Carter (1993) (see Figure 3.12):

$$\frac{G_s}{G_0} = 1 - F \left( \frac{q}{q_{max}} \right)^g$$  

(3.15)

where $f$ and $g$ are soil-specific model parameters (typically $0.8 < f < 1.0$).

Experimental results reported in Lo Presti et al. (1998) show that $g$ increases ($E_0/E_{50}$ decreases) with overconsolidation, especially for quartz and calcereous sands. It has also been recognized that $E_0/E_{50}$ increases with soil cementation and structurization. Therefore, higher values of $E_0/E_{50}$ can be expected for sensitive clays.

Experimental results and Eq. (3.15) have been used in this report to develop a method which provides reasonable first-guess values of $E_0$ based on a known value of $E_{50}$. Considering that $q/q_{max}$ corresponding to $E_{50}$ is equal to 0.5, and assuming that $F = 1$, the hyperbolic equation can be used to find lower and upper limits of $E_{50}/E_0$ by adjusting $g$ parameter. The adjustment of $g$ with respect to overconsolidation or the consolidation stress ratio $K_c$ (which for natural soils increases with overconsolidation) was carried out using the experimental results for Toyura sand presented in Figure 3.14. A summary of adjusted $g$ values and corresponding $E_0/E_{50}$ ratios with respect to overconsolidation is provided in Table 3.3.
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Figure 3.12: Reduction of $G_0/G_{50}$ by the hyperbolic model - Equation (3.15) (assuming $F = 1$).

Figure 3.13: Observed secant stiffness modulus reduction curves from static torsional and triaxial shear data on clays and sands (from Mayne, 2007) and the superposed hyperbolic curves obtained for different $g$ values. The adjusted $g$ values provides reasonable first-guess lower and upper limits for estimating $E_0$ from a determined $E_{50}$ value (see Table 3.3).
Figure 3.14: Normalized secant modulus of Toyura vs mobilized shear strength (Lo Presti et al., 1998, from) and adjusting of $g$ parameter with respect to stress consolidation ratio.

Table 3.3: Typical values of $E_0/E_{50}$ ratio for granular soils with respect to the preconsolidation state (this table has been developed based on literature review and Fahey’s stiffness reduction model).

<table>
<thead>
<tr>
<th>Degree of preconsolidation</th>
<th>OCR [-]</th>
<th>$E_0/E_{50}$ [-]</th>
<th>Corresponding $g$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&quot;Min&quot;</td>
<td>&quot;Max&quot;</td>
</tr>
<tr>
<td>Unknown</td>
<td>4.1</td>
<td>18.0</td>
<td>0.40</td>
</tr>
<tr>
<td>Normally consolidated</td>
<td>1 $\div$ 1.2</td>
<td>6.3</td>
<td>18.0</td>
</tr>
<tr>
<td>Lightly overconsolidated</td>
<td>1.2 $\div$ 2.5</td>
<td>4.6</td>
<td>12.5</td>
</tr>
<tr>
<td>Overconsolidated</td>
<td>2.5 $\div$ 4.5</td>
<td>3.9</td>
<td>8.1</td>
</tr>
<tr>
<td>Heavily overconsolidated</td>
<td>4.5 $\div$ 10</td>
<td>3.6</td>
<td>5.5</td>
</tr>
</tbody>
</table>

The higher values of $E_0/E_{50}$ ratio are suggested for aged, cemented and structured sands.
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Figure 3.15: Cone resistance vs. maximal shear modulus $G_0$ for sands (after Robertson and Campanella, 1983).

**CPT.** Initial small strain stiffness for sands can be approximated from cone resistance measurements $q_c$ derived from CPT. Robertson and Campanella (1983) related the maximal shear modulus $G_0$ with $q_c$ for different effective vertical stresses $\sigma_{v0}'$, as presented in Figure 3.15.

Based on calibration chamber and field measurements Rix and Stokoe (1992) proposed the correlation for uncemented quartz sands (cf. Figure 3.16). The wide range of $G_0/q_c$ at low values of normalized cone resistance is explained by variations in soil compressibility; more compressible sands may give lower values of $Q_t$ and hence higher values of $G_0/q_c$ (Lunne et al., 1997).

\[
\left( \frac{G_0}{q_c} \right)_{avg} = 1634 \left( \frac{q_c}{\sqrt{\sigma_{v0}'}} \right)^{-0.75}
\]

\[(3.16)\]

with $G_0$, $q_c$, and $\sigma_{v0}'$ in kPa.

**DMT.** In general, $G_0$ in sands can be estimated based on dilatometer modulus $E_D$ using the correlations obtained based on calibration chamber tests by Baldi et al. (1986) and on field tests by Belotti et al. (1986):

\[
\frac{G_0}{E_D} = 2.72 \pm 0.59 \quad (3.17a)
\]

\[
\frac{G_0}{E_D} = 2.20 \pm 0.7 \quad (3.17b)
\]

The relations are graphically presented in Figure 3.17.
Figure 3.16: Comparison of empirical correlations for estimating $G_0\ (G_{\text{max}})$ in sands (Eq.(3.16)) and clays (Eq.(3.47)) from CPT data (correlations interpreted for typical $q_t$ ranges).

Figure 3.17: Comparison of empirical correlations for estimating $G_0\ (G_{\text{max}})$ in sands from DMT data (Eq.(3.17a) and Eq.(3.17b)).
SPT. $G_0$ can be estimated for sands from the correlation proposed by Ohta and Goto (1976, as referred in Kramer (1996)), cf. Figure 3.18:

$$G_0 = 438 N_{1.60}^{0.3333} p_a \left( \frac{p'}{p_a} \right)^{0.5}$$ \hspace{1cm} (3.18)

with $G_0$ and $p'$-mean effective stress in kPa, $p_a = 100\text{kPa}, N_{1.60}$ - "overburden-corrected" $N_{60}$-value (cf. Table 3.10).

Another correlation for sand based on SPT date was proposed by Imai and Tonouchi (1982, as refered in Kramer (1996)), cf. Figure 3.18:

$$G_0 = 15600 N_{60}^{0.68}$$ \hspace{1cm} (3.19)

($G_0$ in kPa.)

Figure 3.18: Comparison of empirical correlations for estimating $G_0$ ($G_{\text{max}}$) in sands from SPT data, Eq.(3.18) plotted for $K_0 = 0.4$ and Eq.(3.19).
Geotechnical evidence. It has been proved experimentally that the strain threshold \( \gamma_{0.7} \) does not depend on soil density in the case of non-cohesive granular soils (cf. 3.19). On the other hand, \( \gamma_{0.7} \) can be affected by the magnitude of the confining pressure \( \sigma_0' \) which corresponds to the mean effective stress \( p' \) for in situ conditions (cf. Darendeli and Stokoe (2001); Wichmann and Triantafyllidis (2004)). Hence, the parameter \( \gamma_{0.7} \) can be approximated from a diagram presented in Figure 3.19. Note that the model formulation does not account for stress dependency of \( \gamma_{0.7} \). If needed this parameter can be incorporated into boundary value problems through definition average mean effective stress for defined sub-layers.

![Figure 3.19: Influence of relative density \( I_D \) and the confining pressure \( p' \) on strain threshold \( \gamma_{0.7} \) for sands (from Wichmann and Triantafyllidis, 2004).](image)

Experimental evidence for sands reported in Darendeli and Stokoe (2001), Figure 3.20, allows to write the following approximating relationship:

\[
\gamma_{0.7} = \gamma_{0.7}^{\text{ref}} \left( \frac{p'}{p_0} \right)^{0.35}
\]

with

\[
\gamma_{0.7}^{\text{ref}} (p_0) = 1.26 \cdot 10^{-4}
\]

- reference strain threshold at \( p_0 \)

\( p_0 = 4 \text{ atm} \approx 400 \text{kPa} \)

- atmospheric pressure

An estimation of \( \gamma_{0.7} \) for granular soils can also be carried out using a linear interpolation which is obtained through interpretation of the results presented in Figure 3.19:

\[
\gamma_{0.7} = 8.75 \cdot 10^{-5} \frac{p'}{p_{\text{ref}}} + \gamma_{0.7}^{\text{ref}} \quad \text{for } p' \leq 400 \text{kPa}
\]

with

\[
\gamma_{0.7}^{\text{ref}} (p_{\text{ref}}) = 1.0 \cdot 10^{-4}
\]

- reference strain threshold at \( p_{\text{ref}} \)

\( p_{\text{ref}} = 100 \text{kPa} \)

- reference pressure

where \( p_{\text{ref}} = 100 \text{kPa} \).

A fit of the above correlation to the experimental curves is presented in Figure 3.21.
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Figure 3.20: Comparison of predictions for $\gamma_{0.7}$ at different mean stresses $p'$ using Eq. (3.20) for the data reported in Darendeli and Stokoe (2001).

Figure 3.21: Comparison of predictions for $\gamma_{0.7}$ at different mean stresses $p'$ using Eq. (3.21) with experimental data for sands reported in Wichmann and Triantafyllidis (2004).
In the case of granular soils, it is also observed that the strain threshold may be affected by OCR and for sands with high content of fines additionally on $I_P$. In order to account these effects, the correlation presented in Eq. (3.54) can be used (this generalized formula by Darendeli (2001) was developed based on a database containing four groups of soils, i.e. "clean" sands, sands with high content of fines, silts and clays.)
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

3.2.2 Secant and unloading-reloading moduli

In general, the relation between the stiffness moduli is as follows:

\[ E_{50} < E_s < E_{ur} \]  \hspace{1cm} (3.22)

where \( E_s \) denotes "static" modulus or secant modulus taken from the initial part of the \( \varepsilon_1 - q \) experimental curve at \( \varepsilon_1 = 0.1\% \).

In the case when \( E_{50} \) or \( E_{ur} \) cannot be directly determined from experimental curves, it may be relevant for many practical cases to set:

\[ \frac{E_{ur}^{ref}}{E_{50}^{ref}} = 2 \text{ to } 6 \text{ (an average can be assumed equal to 3)} \]  \hspace{1cm} (3.23)

Higher ratios can be assumed for loose sands (3 to 6), whereas lower ones for dense sands (2 to 4). However, note that the following condition should be satisfied: \( \frac{E_{ur}^{ref}}{E_{50}^{ref}} > 2 \).

However, note that the following condition should be satisfied: \( \frac{E_{ur}^{ref}}{E_{50}^{ref}} > 2 \).

In absence of laboratory results, the stiffness moduli can be approximated based on typically observed order of magnitudes of "static" modulus \( E_s \) which are given in Table 3.4. Assuming that soil behavior follows the stress-strain relation described by Equation 2.8 and assuming \( \varepsilon_1 = 0.1\% \), the "static" modulus can be represented with:

\[ E_s = \frac{q}{0.001} = \frac{1}{2E_{50}^{ref}} + \frac{0.001 \cdot R_f}{q_f(\phi, c)} \]  \hspace{1cm} (3.24)

The above equation can be represented graphically in Figure 3.22, and can be used to estimate \( E_{50} \) from the known value of \( E_s \).

Table 3.4: Typical values for the "static" modulus \( E_s \) [MPa] (compiled from Kezdi, 1974; Prat et al., 1995, and extended by the authors).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Very loose</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
<th>Very dense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>Gravels/Sand well-graded</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Sand, uniform</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Sand/Gravel silty</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Sand/Gravel clayey</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
CHAPTER 3. PARAMETER DETERMINATION

Figure 3.22: Estimating the ratio between the static modulus $E_s$ and secant modulus $E_{50}$.

For sands, the secant modulus $E_{50}$ can be estimated based on the known porosity, Figure 3.23.

Figure 3.23: Normalized secant modulus $E_{50}$ vs. porosity $n_0$ for different sands (after Schanz and Vermeer, 1998).
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

CPT. Secant modulus $E_{50}$ for sands can be approximated from cone resistance measurements $q_c$ derived from CPT. Robertson and Campanella (1983) related $E_{50}$ with $q_c$ for different effective vertical stresses $\sigma'_v$, as presented in Figure 3.24.

![Figure 3.24: Cone resistance $q_c$ vs. secant $E_{50}$ modulus for sands (after Robertson and Campanella, 1983).](image)

DMT. The "static" modulus corresponding to $E_{ur}$ can be evaluated for silty sands and sand from the vertical drained constrained modulus $M_{DMT}$ which is derived from three intermediate dilatometer parameters, i.e. the material index $I_D$, the horizontal stress index $K_D$, and the dilatometer modulus $E_D$. Note that $M_{DMT}$ may correspond to $E_{oed}$ only for normally-consolidated soil. The unloading-reloading modulus $E_{ur}$ can be evaluated assuming that:

$$E_{ur} = E_s = aE_{oed} = aM_{DMT}$$

(3.25)

where $a \approx 0.9$ as $a = (1 + \nu_{ur})(1 - 2\nu_{ur})/(1 - \nu_{ur})$ and the dilatometer vertical drained constrained modulus $M_{DMT}$:

$$M_{DMT} = R_ME_D$$

(3.26)

with $E_D = 34.7(p_1 - p_0)$ and $R_M$ depending on soil type behavior, i.e.

- silty sand ($1.8 < I_D < 3$): $R_M = R_{M,0} + (2.5 - R_{M,0})\log K_D$ and $R_{M,0} = 0.14 + 0.15(I_D - 0.6)$
- sand ($I_D \geq 3$): $R_M = 0.5 + 2\log K_D$
- if $K_D > 10$: $R_M = 0.32 + 2.18\log K_D$
- if $R_M < 0.85$: set $R_M = 0.85$
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**SPT.** Table 3.5 gives some empirical equations which can be used to evaluate the "static" modulus based on SPT $N$-value. A graphical interpretation of correlation performance and a comparison with typical values for the "static" modulus $E_s$ is presented in Figure 3.25.

*Table 3.5: Empirical correlations relating the "static" modulus $E_s$ with SPT $N$-value.*

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Empirical formula $E_s$ in [kPa]</th>
<th>Remarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravels/Sands</td>
<td>$6000N_{60}$</td>
<td>can be considered as a good approximation for $E_{ur}$ in well-graded sands and gravels</td>
<td>Bowles (1997)</td>
</tr>
<tr>
<td>well-graded</td>
<td>$1200(N_{60} + 6) + 4000$</td>
<td>seems to give too low values for $E_{ur}$ in gravelly sand</td>
<td>Begemann (1974)</td>
</tr>
<tr>
<td>Gravelly sand</td>
<td>$600(N_{55} + 6)$</td>
<td>seems to give too low values for $E_{ur}$ in gravelly sand</td>
<td>Bowles (1997)</td>
</tr>
<tr>
<td></td>
<td>$+2000$ if $N &gt; 15$</td>
<td>low values for $E_{ur}$ in gravelly sands</td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>$(3.5$ to $5.0) \times 10^4\log(N_{60})$</td>
<td>perceived as a good approximation of lower and upper bound for $E_{ur}$ in uniform sands</td>
<td>Tromienkov (1974)</td>
</tr>
<tr>
<td>Sand NC</td>
<td>$2750N_{55}$</td>
<td>can be considered as a good approximation for $E_{ur}$ in loose to medium NC sands</td>
<td>Bowles (1997)</td>
</tr>
<tr>
<td></td>
<td>$7000\sqrt{N_{55}}$</td>
<td>can be considered as a lower-bound approximation for $E_{ur}$ in NC uniform sands</td>
<td>Bowles (1997)</td>
</tr>
<tr>
<td>Sand NC or sand</td>
<td>$780N_{60} + 200000$</td>
<td>can be considered as a good approximation for $E_{ur}$ in uniform NC sands</td>
<td>authors’ equation from plot of D’Appolonia et al. (1970)</td>
</tr>
<tr>
<td>&amp; gravel</td>
<td>$\frac{1}{1 - \nu^2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand OC</td>
<td>$1100N_{60} + 400000$</td>
<td>can be considered as a good approximation for $E_{ur}$ in uniform NC sands</td>
<td>authors’ equation from plot of D’Appolonia et al. (1970)</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{1 - \nu^2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand saturated</td>
<td>$500(N_{60} + 15)$</td>
<td>gives too low values for $E_{ur}$</td>
<td>Webb (1969)</td>
</tr>
<tr>
<td>Fine sands and</td>
<td>$100(44N_{60})^{0.75} \pm 5000$</td>
<td>can be considered as the lower-bound approximation for $E_{ur}$ in silty sands</td>
<td>Schultze and Menzenbach (1961)</td>
</tr>
<tr>
<td>silty sands</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clayey sand</td>
<td>$333(N_{60} + 15)$</td>
<td>can be considered as the lower-bound approximation for $E_{ur}$ in clayey sands</td>
<td>Webb (1969)</td>
</tr>
</tbody>
</table>

$N_{60}$ corresponds to the energy ratio $E_r = 60$. Since the energy $\times$ blow count should be a constant for any soil, the following equation can be applied $E_{r1} \times N_1 = E_{r2} \times N_2$ (Bowles, 1997). For example, $N_{55} = N_{60} \times 60/55$.  

3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Figure 3.25: Performance of empirical correlations from Table 3.5 compared with typical values for the "static" modulus $E_s$ from Table 3.4.
3.2.3 Oedometric modulus

In case of lack of oedometric test data for granular material the oedometric modulus can approximately be taken as:

$$E_{\text{oed}}^{\text{ref}} \approx E_{50}^{\text{ref}}$$  \hspace{1cm} (3.27)

In such a case, the oedometric vertical reference stress $\sigma_{\text{oed}}^{\text{ref}}$ should be matched to the reference minor stress $\sigma_{\text{ref}}$ since the latter typically corresponds to the confining (horizontal) pressure $\sigma_{\text{ref}} = \sigma_3 = \sigma_{h'}$:

$$\sigma_{\text{oed}}^{\text{ref}} = \sigma_{\text{ref}} / K_{\text{NC}}$$  \hspace{1cm} (3.28)

On the other hand, when defining $\sigma_{\text{oed}}^{\text{ref}} = \sigma_{\text{ref}}$ in the model, the following relationship should be taken:

$$E_{\text{oed}}^{\text{ref}} \approx E_{50}^{\text{ref}} \left( K_{0\text{NC}} \right)^m$$  \hspace{1cm} (3.29)

![Figure 3.26: Comparison of reference stiffness moduli for sands from oedometer and triaxial tests (after Schanz and Vermeer, 1998).](image1)

![Figure 3.27: Normalized stiffness modulus of various sands derived from oedometer tests (after Schanz and Vermeer, 1998).](image2)
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

**DMT.** The constrained tangent modulus $M_D$ (corresponding to $E_{oed}$ in oedometer test) can be interpreted from three intermediate DMT parameters, i.e. the material index $I_D$, the horizontal stress index $K_D$, and the dilatometer modulus $E_D$, by applying the correlation presented in Eq. (3.26).

**Important note.** In the case of the HS model, $E_{oed}^{ref}$ can be taken as equal to $M_{DMT}$ if the latter has been derived from DMT but **only for normally-consolidated soil.** In such a case $\sigma_{oed}$ can be taken as $\sigma'_{v0}$ which corresponds to the testing depth for which $M_{DMT}$ has been evaluated.
3.2.4 Unloading-reloading Poisson’s ratio

Experimental measurements from local strain gauges show that the initial values of Poisson’s ratio in terms of small mobilized stress levels ($q/q_{\text{max}}$) varies between 0.1 and 0.2 for clays, sands and rocks (Figure 3.28). Therefore, the characteristic value for the elastic unloading/reloading Poisson’s ratio of $\nu_{ur} = 0.2$ can be adopted for most soils.

![Figure 3.28: Poisson’s ratio $\nu$ vs. mobilized stress level derived from local strain measurements on sand, clay and soft rock (after Mayne et al., 2009).](image-url)
3.2.5 Stiffness exponent

Geotechnical evidence. In natural soil, the exponent $m$ varies between 0.3 and 1.0. Janbu (1963) reported values of 0.5 for Norwegian sands ans silts. Typical values for $m$ obtained in clean sands and gravels are provided in Table 3.6.

![Figure 3.29](image1)

**Figure 3.29:** Typical values for $m$ obtained for sands from triaxial test vs. initial porosity $n_0$ (from Schanz and Vermeer, 1998).

![Figure 3.30](image2)

**Figure 3.30:** Typical values for $m$ obtained for sands from oedometric test vs. initial porosity $n_0$ (after Schanz and Vermeer, 1998).
CHAPTER 3. PARAMETER DETERMINATION

Table 3.6: Typical values for $m$ observed in clean sands and gravels for the shear modulus $G_0$ (from Benz, 2007).

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>m [-]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenya carbonate sand</td>
<td>0.45-0.52</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td>Quiou carbonate sand</td>
<td>0.62</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>Ottawa sand No.20-30</td>
<td>0.50</td>
<td>Hardin and Richart Jr (1963)</td>
</tr>
<tr>
<td>SLB sand (subround)</td>
<td>0.44-0.53</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.41-0.51</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.50-0.57</td>
<td>Chaudary et al. (2004)</td>
</tr>
<tr>
<td>Toyoura sand (subangular)</td>
<td>0.45</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.44-0.53</td>
<td>Hoque and Tatsuoka (2004)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.43</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Ticino sand (subangular)</td>
<td>0.43-0.48</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td>H.River sand (subangular)</td>
<td>0.5-0.52</td>
<td>Kuwano and Jardine (2002)</td>
</tr>
<tr>
<td>Silica sand (subangular)</td>
<td>0.5</td>
<td>Kallioglou et al. (2003)</td>
</tr>
<tr>
<td>Hostun sand (angular)</td>
<td>0.47</td>
<td>Hoque and Tatsuoka (2000)</td>
</tr>
<tr>
<td>Silica sand (angular)</td>
<td>0.50</td>
<td>Kallioglou et al. (2003)</td>
</tr>
<tr>
<td>Silica sand</td>
<td>0.42</td>
<td>Wichtmann and Triantafyllidis (2004)</td>
</tr>
<tr>
<td>Hime gravel (subround)</td>
<td>0.45-0.51</td>
<td>Chaudary et al. (2004)</td>
</tr>
<tr>
<td>Chiba gravel (subround)</td>
<td>0.50</td>
<td>Modoni et al. (1999)</td>
</tr>
</tbody>
</table>

Table 3.7: Suggested ranges of stiffness exponent $m$ observed for oedometric modulus $E_{oed}$ (von Soos, 1991).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>m [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel: poorly-graded (uniform)</td>
<td>$0.4 \div 0.6$</td>
</tr>
<tr>
<td>Gravel: sandy, well-graded</td>
<td>$0.5 \div 0.7$</td>
</tr>
<tr>
<td>Gravel: silty or clayey, well-graded, not crushed</td>
<td>$0.5 \div 0.7$</td>
</tr>
<tr>
<td>Gravel-sand-clay mixture, crushed</td>
<td>$0.7 \div 0.9$</td>
</tr>
<tr>
<td>Sand: fine, uniform</td>
<td>$0.6 \div 0.75$</td>
</tr>
<tr>
<td>Sand: coarse, uniform</td>
<td>$0.55 \div 0.7$</td>
</tr>
<tr>
<td>Sand: well-graded and gravelly sand</td>
<td>$0.55 \div 0.7$</td>
</tr>
<tr>
<td>Sand: with fines, not crushed</td>
<td>$0.65 \div 0.8$</td>
</tr>
<tr>
<td>Sand: with fines, crushed</td>
<td>$0.75 \div 0.9$</td>
</tr>
</tbody>
</table>
3.2.6 Friction angle

In coarse soils, the value of friction angle is mostly influenced by soil density, shape of particles and soil gradation. Friction angle can be approximated from Table 3.8 or estimated based on in situ test results from SPT, CPT or DMT.

Brinch Hansen and Lundgren (1958) proposed to estimate the friction angle with the following empirical correlation:

$$\phi = 36^\circ + \phi_1 + \phi_2 + \phi_3 + \phi_4$$  \hspace{1cm} (3.30)

using the following corrections to account for different soil features:

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$\phi_1$</th>
<th>Relative density</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravel</td>
<td>$+2^\circ$</td>
<td>very loose</td>
<td>$-6^\circ$</td>
</tr>
<tr>
<td>gravel + sand</td>
<td>$+1^\circ$</td>
<td>loose</td>
<td>$-3^\circ$</td>
</tr>
<tr>
<td>sand</td>
<td>$0^\circ$</td>
<td>medium</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dense</td>
<td>$+3^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>very dense</td>
<td>$+6^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soil gradation</th>
<th>$\phi_3$</th>
<th>Particles shape</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>well-graded</td>
<td>$+3^\circ$</td>
<td>angular</td>
<td>$+1^\circ$</td>
</tr>
<tr>
<td>medium</td>
<td>$0^\circ$</td>
<td>subangular</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>poorly-graded</td>
<td>$-3^\circ$</td>
<td>subrounded</td>
<td>$-3^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rounded</td>
<td>$-5^\circ$</td>
</tr>
</tbody>
</table>

Table 3.8: Empirical values for $\phi$ and $D_r$ of granular soils based on SPT at about 6 m depth and normally consolidated (after Bowles, 1997, and modified by the authors).

<table>
<thead>
<tr>
<th>Description</th>
<th>Very loose</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
<th>Very Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative density $D_r$</td>
<td>0</td>
<td>0.15</td>
<td>0.35</td>
<td>0.65</td>
<td>0.85</td>
</tr>
<tr>
<td>SPT $N_{70}$ :</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fine</td>
<td>1-2</td>
<td>3-6</td>
<td>7-15</td>
<td>16-30</td>
<td></td>
</tr>
<tr>
<td>medium</td>
<td>2-3</td>
<td>4-7</td>
<td>8-20</td>
<td>21-40</td>
<td>&gt; 40</td>
</tr>
<tr>
<td>coarse</td>
<td>3-6</td>
<td>5-9</td>
<td>10-25</td>
<td>26-45</td>
<td>&gt; 45</td>
</tr>
<tr>
<td>SPT $\phi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fine</td>
<td>26-28</td>
<td>28-30</td>
<td>30-34</td>
<td>33-38</td>
<td></td>
</tr>
<tr>
<td>medium</td>
<td>27-28</td>
<td>30-32</td>
<td>32-36</td>
<td>36-42</td>
<td>&lt; 50</td>
</tr>
<tr>
<td>coarse</td>
<td>28-30</td>
<td>30-34</td>
<td>33-40</td>
<td>40-50</td>
<td></td>
</tr>
</tbody>
</table>

$N_{70}$ corresponds to the energy ratio $E_r = 70$. Since the energy x blow count should be a constant for any soil, the following equation can be applied:

$$E_{r1} \times N_1 = E_{r2} \times N_2$$  \hspace{1cm} (Bowles, 1997).

For example, $N_{60} = N_{70} \times 70/60$.

SPT. The friction angle for granular soils with a small content of fine grains can be determined using the chart suggested by Peck et al. (1974). This chart also correlates the SPT number with the bearing factors $N_\gamma$ and $N_q$ which are standardly used for dimensioning of foundations.
Soil density
Dense Very dense Compact Loose Very loose

Bearing factors $N, N_q$

Figure 3.31: Determination of the friction angle $\phi'$ and bearing factors for granular soils based on the SPT number (from Peck et al., 1974).

Table 3.9: Estimation of the friction angle $\phi'$ from the SPT number.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Standard Penetration Resistance $N_{60}$, blows/0.3m</th>
<th>Friction angle, $\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose sand</td>
<td>&lt; 4</td>
<td>&lt; 29</td>
</tr>
<tr>
<td>Loose sand</td>
<td>4-10</td>
<td>29-30</td>
</tr>
<tr>
<td>Medium sand</td>
<td>10-30</td>
<td>30-36</td>
</tr>
<tr>
<td>Dense sand</td>
<td>30-50</td>
<td>36-41</td>
</tr>
<tr>
<td>Very dense sand</td>
<td>&gt; 50</td>
<td>&gt; 41</td>
</tr>
</tbody>
</table>
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

<table>
<thead>
<tr>
<th>Empirical formula</th>
<th>Reference</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi'<em>{\text{peak}} = \sqrt{15.4N</em>{60} + 20^\circ} )</td>
<td>Hatanaka and Uchida (1996)</td>
<td>tends to overestimate ( \phi' ) in very dense sands ( \sigma'_{e0} &lt; 100\text{kPa} )</td>
</tr>
<tr>
<td>( \phi'<em>{\text{peak}} = \sqrt{15.4N</em>{60} + 20^\circ(\pm3^\circ)} )</td>
<td>Hatanaka and Uchida (1996) without overburden correction</td>
<td>simplification: ( \sigma'_{e0} = p_a )</td>
</tr>
<tr>
<td>( \phi' = \sqrt{20N_{1,60} + 15^\circ} )</td>
<td>Teixeira (1996)</td>
<td>tends to overestimate ( \phi' ) in very dense sands ( \sigma'_{e0} &lt; 100\text{kPa} )</td>
</tr>
<tr>
<td>( \phi' = \tan^{-1} \left( \frac{N_{60}}{12.2 + 20.3\frac{\sigma'_{e0}}{p_a}} \right)^{0.34} )</td>
<td>Schmertmann (1975)</td>
<td>tends to overestimate ( \phi' ) in very dense sands ( \sigma'_{e0} &lt; 100\text{kPa} )</td>
</tr>
<tr>
<td>( \phi' = \tan^{-1} \left( \frac{N_{60}}{32.5} \right)^{0.34} )</td>
<td>Schmertmann (1975) without overburden correction</td>
<td>simplification: ( \sigma'_{e0} = p_a )</td>
</tr>
<tr>
<td>( \phi' = 25^\circ + 28\sqrt{\frac{N_{55}}{\sigma'_{e0}}} )</td>
<td>after Townsend et al. (2003)</td>
<td>tends to overestimate ( \phi' ) in very dense sands ( \sigma'_{e0} &lt; 100\text{kPa} )</td>
</tr>
<tr>
<td>( \phi' = 15^\circ + \sqrt{18N_{70}} )</td>
<td>Shioi and Fukui (1982)</td>
<td>Japan Road Association standards</td>
</tr>
<tr>
<td>( \phi' = 27^\circ + 0.36N_{70} )</td>
<td>Shioi and Fukui (1982)</td>
<td>Japanese National Railway standards</td>
</tr>
</tbody>
</table>

\( N_{60} \) corresponds to the energy ratio \( E_r = 60 \). Since the energy \times\ blow count should be a constant for any soil, the following equation can be applied \( E_{r1} \times N_1 = E_{r2} \times N_2 \) (Bowles, 1997). For example, \( N_{55} = N_{60} \times 60/55 \).

\( N_{1,60} = N_{60}C_N \) ("overburden-corrected" \( N_{60} \)-value) with the overburden correction factor \( C_N = (p_a/\sigma'_{e0})^{0.5} \) (Liao and Whitman, 1986) use \( C_N = 1.7 \) if \( C_N > 1.7 \).

\( p_a \) - atmospheric pressure (average sea-level pressure is 101.325 kPa)
Figure 3.32: Cone resistance vs. peak friction angle $\phi'$ for sands (after Robertson and Campanella, 1983).

**CPT.** The most widely accepted relationship which relates the cone resistance $q_t$ with $\phi'$ for granular materials is the expression proposed by Robertson and Campanella (1983) (Figure 3.32):

$$\phi' = \arctan \left[ 0.10 + 0.38 \log \left( \frac{q_t}{\sigma'_{v0}} \right) \right]$$  \hspace{1cm} (3.31)

**DMT.** Two direct empirical correlations suggested in Totani et al. (1999) can be used to estimate lower and upper bounds of the range of the friction angle:

$$\phi'_{\text{max}} = 31 + K_D / (0.236 + 0.066 K_D)$$  \hspace{1cm} (3.32)

$$\phi'_{\text{min}} = 28 + 14.6 \log K_D - 2.1 (\log K_D)^2$$  \hspace{1cm} (3.33)

with $K_D$ denoting horizontal stress index which is calculated based on the first dilatometer reading $p_0$, i.e. $K_D = (p_0 - u_0) / \sigma'_{v0}$.

**Geotechnical evidence.** Typical values of the friction angle for granular soils are provided in Tables 3.11, 3.12, 3.13 and 3.14.
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Table 3.11: Representative values of $\phi$ observed in sands (after Schmertmann, 1978).

<table>
<thead>
<tr>
<th>Relative density $D_r$ [%]</th>
<th>Fine Grained Uniform</th>
<th>Well-graded</th>
<th>Friction angle $\phi$ [°] Medium Grained Uniform</th>
<th>Well-graded</th>
<th>Coarse Grained Uniform</th>
<th>Well-graded</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>34</td>
<td>36</td>
<td>36</td>
<td>38</td>
<td>38</td>
<td>41</td>
</tr>
<tr>
<td>60</td>
<td>36</td>
<td>38</td>
<td>38</td>
<td>41</td>
<td>41</td>
<td>43</td>
</tr>
<tr>
<td>80</td>
<td>39</td>
<td>41</td>
<td>41</td>
<td>43</td>
<td>43</td>
<td>44</td>
</tr>
<tr>
<td>100</td>
<td>42</td>
<td>43</td>
<td>43</td>
<td>44</td>
<td>44</td>
<td>46</td>
</tr>
</tbody>
</table>

Table 3.12: Representative values of $\phi$ observed in cohesionless soils (after Carter and Bentley, 1991).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform sand, round grains</td>
<td>Loose</td>
</tr>
<tr>
<td>Well-graded sand, angular grains</td>
<td>27</td>
</tr>
<tr>
<td>Sandy gravels</td>
<td>33</td>
</tr>
<tr>
<td>Silty sand</td>
<td>35</td>
</tr>
<tr>
<td>Inorganic silt</td>
<td>27-33</td>
</tr>
</tbody>
</table>

Table 3.13: Representative values of $\phi$ observed in compacted sands and gravels (after Carter and Bentley, 1991).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>UCS class</th>
<th>$\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-graded sand-gravel mixtures</td>
<td>GW</td>
<td>&gt; 38</td>
</tr>
<tr>
<td>Poorly-graded sand gravel mixtures</td>
<td>GP</td>
<td>&gt; 37</td>
</tr>
<tr>
<td>Silty gravels, poorly graded sand-gravel-clay</td>
<td>GM</td>
<td>&gt; 34</td>
</tr>
<tr>
<td>Clayey gravels, poorly graded sand-gravel-clay</td>
<td>GC</td>
<td>&gt; 31</td>
</tr>
<tr>
<td>Well-graded clean sand, gravelly sands</td>
<td>SW</td>
<td>38</td>
</tr>
<tr>
<td>Poorly-graded clean sands</td>
<td>SP</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 3.14: Representative relationships between relative density $D_r$ and friction angle $\phi$ for granular soils.

<table>
<thead>
<tr>
<th>State of compaction</th>
<th>Relative density $D_r$ [%]</th>
<th>$\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose</td>
<td>0-15</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>Loose</td>
<td>15-35</td>
<td>25-30</td>
</tr>
<tr>
<td>Medium</td>
<td>35-65</td>
<td>30-37</td>
</tr>
<tr>
<td>Dense</td>
<td>65-85</td>
<td>37-43</td>
</tr>
<tr>
<td>Very dense</td>
<td>85-100</td>
<td>&gt; 42</td>
</tr>
</tbody>
</table>
3.2.7 Dilatancy angle

It is typically observed in laboratory tests that for a very dense sand the value of the dilatancy angle $\psi$ is about $1/3$ of the peak friction angle $\phi'$. In the case of loose sands, the dilatancy angle reduces to a few degrees, where as normally consolidated clays may exhibit no dilatancy at all. For example, Bolton (1986) reports $\psi_{\text{max}} = 14.7^\circ$ for a dense sand which corresponded to the peak friction angle $\phi'_{\text{max}} = 44.8^\circ$ derived from a drained, plane strain compression test.

Bolton (1986) proposed that for well-compacted granular soils, the maximal dilatancy angle can be estimated from:

$$\psi = 3.75I_R \quad \text{under triaxial conditions} \quad (3.34a)$$
$$\psi = 6.25I_R \quad \text{under plain strain conditions} \quad (3.34b)$$

with the relative dilatancy index $I_R$ can be estimated for well-compacted granular soils from:

$$I_R = 5D_r - 1 \quad (0 < I_R < 4) \quad (3.35)$$

with $D_r$ denoting the relative density index \(\frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}\).

Another reasonable approximation of $\psi$ can be obtained using the following equation:

$$\psi = \phi - 30^\circ \quad (3.36)$$

for the values of friction angle larger than $30^\circ$.

Notice that the above approximation gives $\psi \approx \phi' / 3$ for a dense gravel, whereas small values of $\psi$ will be obtained for the the friction angles that correspond to loose sands (typically $\phi \approx 30^\circ$ to $32^\circ$).
3.2.8 Coefficient of earth pressure "at rest"

Various relationships can be found in literature for estimation of the coefficient of earth pressure "at rest" for normally consolidated soils. They commonly relate the value of \( K_{0}^{NC} \) to the effective friction angle \( \phi' \), and the most popular are:\(^6\)

\[
K_{0}^{NC} = 1 - \sin \phi' \tag{3.37a}
\]

\[
K_{0}^{NC} = \left( \sqrt{2} - \sin \phi' \right) / \left( \sqrt{2} + \sin \phi' \right) \tag{3.37b}
\] (Simpson, 1992)

These equations are illustrated in Fig.3.33.

\[
Figure 3.33: \text{Typical relationships between } K_{0}^{NC} \text{ and } \phi' \text{ observed for soils.}
\]

In the case of sands, the notion of preconsolidation pressure is not as meaningful as for cohesive soils, and therefore OCR = 1 (i.e. \( K_{0}^{SR} = K_{0}^{NC} \)) can be assumed when calculating parameters \( H \) and \( M \).

In the case of running a simulation of isotropic consolidation (the case of isotropically consolidated triaxial compression tests, i.e. CIU or CID), the coefficient should be assumed as \( K_{0}^{SR} = 1 \).

\(^6\)Note that Eq.(3.37a) is often erroneously called "Jaky's equation" as it is a simplified form of his original expression \( K_{0}^{NC} = (1 - \sin \phi')/(1 + \sin \phi')(1 + 2/3 \sin \phi') \) (Jaky, 1947) which gives essentially the same results as Eq.(3.87).
3.2.9 Void ratio

Void ratio for a saturated soil can be calculated from:

\[ e = w_n G_s \]  

(3.38)

where \( w_n \) is the water content, \( G_s \) is the specific gravity of soil solids and \( S \) is the saturation ratio.

In the case of partially unsaturated soil, the void ratio can be obtained from:

\[ \gamma = \frac{G_s \gamma_w (1 + w_n)}{1 + e} \quad \text{or} \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 \quad \text{or} \quad e = w_n G_s / S \]  

(3.39)

where \( \gamma_d \) is the dry unit weight.

Hence, the maximum void ratio \( e_{\text{max}} \) can be calculated from:

\[ e_{\text{max}} = \frac{G_s \gamma_w}{\gamma_{d,\text{min}}} - 1 \]  

(3.40)

The maximum void ratio \( e_{\text{max}} \) can also be estimated according to approximate relationship presented in Figure 3.34 between the void ratio the coefficient of uniformity for different granular soils.

Typical values of void ratios and dry unit weights observed in granular soils are provided in Table 3.15.

*Table 3.15: Typical values of void ratio and dry unit weights observed in granular soils (after Das, 2008).*

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Voids ratio ( e ) [-]</th>
<th>Dry unit weight ( \gamma ) [kN/m(^3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>0.75</td>
<td>0.35</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.85</td>
<td>0.4</td>
</tr>
<tr>
<td>Standard Ottawa sand</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Gravelly sand</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>Silty sand</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>Silty sand and gravel</td>
<td>0.85</td>
<td>0.15</td>
</tr>
</tbody>
</table>
3.2. ALTERNATIVE PARAMETER ESTIMATION FOR GRANULAR MATERIALS

Figure 3.34: Generalized charts for estimating $e_{max}$ and $e_{min}$ from gradational and particle shape characteristics (from UFC, 2004; Das, 2008).
3.2.10 Overconsolidation ratio

CPT. Estimation of OCR from CPT data in sands can be carried out using a relationship developed based on statistical multiple regression analysis of 26 separate sands worldwide with flexible-walled calibration chambers (Kulhawy and Mayne, 1990; Lunne et al., 1997; Mayne et al., 2001; Mayne, 2007). Primarily clean siliceous (quartz and feldspar) sand samples where subject to normally-consolidated and overconsolidated states $1 \leq \text{OCR} \leq 15$. Sands in the test were usually dry or saturated without back pressures. The obtained empirical equation is the following:

$$\text{OCR} = \left[ \frac{0.192 \left( \frac{q_t}{p_a} \right)^{0.22}}{(1 - \sin \phi') \left( \frac{\sigma_{v0}'}{p_a} \right)^{0.31}} \right] \left( \frac{1}{\sin \phi' - 0.27} \right)$$

(3.41)

with $q_t$ denoting cone tip resistance, $\sigma_{v0}'$ effective overburden stress, and atmospheric pressure is $p_a = 100$ kPa.
3.2.11 Coefficient of earth pressure "at rest"

**CPT.** Estimation of $K_0$ from CPT data in sands can be carried out using a relationship developed based on statistical multiple regression analysis of 26 separate sands worldwide with flexible-walled calibration chambers (Kulhawy and Mayne, 1990; Lunne et al., 1997; Mayne et al., 2001; Mayne, 2007). Primarily clean siliceous (quartz and feldspar) sand samples where subject to normally-consolidated and overconsolidated states $1 \leq OCR \leq 15$. Sands in the test were usually dry or saturated without back pressures. The obtained empirical equation is the following:

$$K_0 = 0.192 \left( \frac{q_t}{p_a} \right)^{0.22} \left( \frac{p_a}{\sigma'_{v0}} \right)^{0.31} OCR^{0.27} \quad (3.42)$$

with $q_t$ denoting cone tip resistance, OCR overconsolidation ratio, and atmospheric pressure is $p_a = 100$ kPa.

**DMT-CPT.** In 1980’s, researchers reported that a unique correlation for $K_0$ from $K_D$ cannot be established for sands as data points showed that such correlation in sand also depends on $\phi$ or $D_r$. Initially developed Marchetti’s chart $K_0 - q_c - K_D$ (Marchetti, 1985) was updated by Baldi et al. (1986) by incorporating all subsequent calibration chamber research and converted into a simple algebraic equations:

$$K_0 = 0.376 + 0.095 K_D - 0.0017 q_c / \sigma'_{v0} \quad \text{for "freshly" deposited sand} \quad (3.43a)$$

$$K_0 = 0.376 + 0.095 K_D - 0.0046 q_c / \sigma_{v0} \quad \text{for "seasoned" sand} \quad (3.43b)$$

Marchetti et al. (2001) recommend to use Eq.(3.43a) in “freshly deposited” sand, whereas for “seasoned” sands Eq.(3.43b).
3.3 Alternative parameter estimation for cohesive materials

3.3.1 Initial stiffness modulus and small strain threshold

The present section provides a number of approaches for estimating the initial soil stiffness and the small strain threshold $\gamma_{0,7}$. Some correlations allow to directly approximate the input parameter $E_0$, the others provide solutions for estimating the initial shear modulus $G_0$. Then the input parameter $E_0$ can be obtained through:

$$E_0 = 2(1 + \nu_{ur})G_0$$  \hspace{1cm} (3.44)

assuming a constant value of the unloading-reloading Poisson’s coefficient $\nu_{ur}$.

Geotechnical evidence. Experimental data shows that the initial stiffness of soils may depend on the stress level, soil porosity and overconsolidation. These factors can be taken into account using a modified equation proposed by Hardin and Black (1969):

$$G_0 = A \cdot f(e) \cdot OCR^k \left( \frac{p'}{p_{ref}} \right)^m, \text{ in [MPa]}$$  \hspace{1cm} (3.45)

where $G_0$ is the maximum small-strain shear modulus in MPa, $p'$ is the mean effective stress in kPa, $p_{ref}$ is the reference stress equal to the atmospheric pressure $p_{ref} = 100$ kPa, OCR is the overconsolidation ratio and $A$, $f(e)$, $k$, $m$ are the correlated functions and parameters which are given in Table 3.16 and 3.18 for different types of soils. It is observed that the empirical exponent $k$ varies from 0 for sands and 0.5 for high plasticity clays. It means that $k$ may increase with soil plasticity and its value can be taken from Table 3.17.

Biarez and Hicher (1994) proposed a simple relationship for all soils with $w_L < 50\%$:

$$E_0 = \frac{140}{e} \left( \frac{p'}{p_{ref}} \right)^{0.5}, \text{ in [MPa]}$$  \hspace{1cm} (3.46)
Table 3.16: Parameters for estimation of $G_0$ in clays using Eq.(3.45).

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>$I_p$</th>
<th>$A$</th>
<th>$f(e)$</th>
<th>$k$</th>
<th>$m_p$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quaternary Italian clays (see Fig.3.35)</td>
<td>-</td>
<td>60</td>
<td>$e^{-1.30}$</td>
<td>N/A</td>
<td>0.50</td>
<td>Jamiolkowski et al. (1995)</td>
</tr>
<tr>
<td>Avezzano clay (Holocene-Pleistocene)</td>
<td>10-30</td>
<td>74</td>
<td>$e^{-1.27}$</td>
<td>N/A</td>
<td>0.46</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Fucino clay (Holocene-Pleistocene)</td>
<td>45-75</td>
<td>64</td>
<td>$e^{-1.52}$</td>
<td>N/A</td>
<td>0.40</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Garigliano clay (Holocene)</td>
<td>10-40</td>
<td>44</td>
<td>$e^{-1.11}$</td>
<td>N/A</td>
<td>0.58</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Panigaglia clay (Holocene)</td>
<td>44</td>
<td>44</td>
<td>$e^{-1.30}$</td>
<td>N/A</td>
<td>0.50</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Montaldo di Castro clay (Pleistocene)</td>
<td>15-34</td>
<td>50</td>
<td>$e^{-1.33}$</td>
<td>N/A</td>
<td>0.40</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Reconstituted Valericca clay (Pleistocene)</td>
<td>27</td>
<td>44</td>
<td>1</td>
<td>N/A</td>
<td>0.85</td>
<td>Rampello et al. (1997)</td>
</tr>
<tr>
<td>Pisa clay (Pleistocene)</td>
<td>23-46</td>
<td>50</td>
<td>$e^{-1.43}$</td>
<td>N/A</td>
<td>0.44</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>London clay (reconstituted)</td>
<td>41</td>
<td>13</td>
<td>1</td>
<td>0.25*</td>
<td>0.76</td>
<td>Viggiani and Atkinson (1995)</td>
</tr>
<tr>
<td>Speswhite kaolin clay (reconstituted)</td>
<td>24</td>
<td>40</td>
<td>1</td>
<td>0.2*</td>
<td>0.65</td>
<td>Viggiani and Atkinson (1995)</td>
</tr>
<tr>
<td>Kaolin clay</td>
<td>35</td>
<td>45</td>
<td>$(2.97 - e)^2$</td>
<td>N/A</td>
<td>0.50</td>
<td>Marcuson and Wahls (1972)</td>
</tr>
<tr>
<td>Bentonite clay</td>
<td>60</td>
<td>4.5</td>
<td>$(4.40 - e)^2$</td>
<td>N/A</td>
<td>0.50</td>
<td>Marcuson and Wahls (1972)</td>
</tr>
</tbody>
</table>

*overconsolidation ratio OCR based on the mean stress ($\text{OCR} = \frac{p_c'}{p''}$)

Table 3.17: Overconsolidation ratio exponent $k$ used in Eq.(3.45).

<table>
<thead>
<tr>
<th>Plasticity Index $I_p$ [%]</th>
<th>Exponent $k$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>0.18</td>
</tr>
<tr>
<td>40</td>
<td>0.30</td>
</tr>
<tr>
<td>60</td>
<td>0.41</td>
</tr>
<tr>
<td>80</td>
<td>0.48</td>
</tr>
<tr>
<td>$\geq$ 100</td>
<td>0.50</td>
</tr>
</tbody>
</table>

source: Kramer (1996, after Hardin&Drnevich 1972)
CHAPTER 3. PARAMETER DETERMINATION

Figure 3.35: Typical void ratio-$G_0$ dependency using an empirical equation by Jamiolkowski et al. (1995) from Table 3.16.

Table 3.18: Parameters for estimation of $G_0$ in cohesive soils using Eq.(3.45). The relationships are illustrated in Figure 3.10.

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>$e_{min}$</th>
<th>$e_{max}$</th>
<th>$A$</th>
<th>$f(e)$</th>
<th>$m_p$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>All soils with $w_L &lt; 50%$</td>
<td>0.4</td>
<td>1.8</td>
<td>58*</td>
<td>$\frac{1}{e}$</td>
<td>0.5</td>
<td>Biarez and Hicher (1994)</td>
</tr>
<tr>
<td>Undisturbed clayey soils</td>
<td>0.6</td>
<td>1.5</td>
<td>33</td>
<td>$\frac{(2.97-e)^2}{1+e}$</td>
<td>0.5</td>
<td>Hardin and Black (1969)</td>
</tr>
<tr>
<td>Undisturbed cohesive soils</td>
<td>0.6</td>
<td>1.5</td>
<td>16</td>
<td>$\frac{(2.97-e)^2}{1+e}$</td>
<td>0.5</td>
<td>Kim and Novak (1981)</td>
</tr>
<tr>
<td>Loess</td>
<td>1.4</td>
<td>4.0</td>
<td>1.4</td>
<td>$\frac{(7.32-e)^2}{1+e}$</td>
<td>0.6</td>
<td>Kokusho et al. (1982)</td>
</tr>
</tbody>
</table>

*obtained from Eq. 3.14 assuming $\nu = 0.2$
Geotechnical evidence. In case of lack of test data at very small strains, $E_0$ for cohesive soils can be evaluated from an empirical relation proposed by Alpan (1970), see Figure 3.11. The chart relates so-called "static" modulus $E_s$ to the "dynamic" modulus $E_d$. For the sake of HS-SmallStrain model, $E_s$ can be considered as $E_{ur}$ obtained at engineering strain levels ($\varepsilon \approx 10^{-3}$), whereas $E_0$ can be considered as $\approx E_d$.

Geotechnical evidence. It can be observed in laboratory test that secant stiffness reduces with mobilization of the shear strength. Mayne (2007) provides a selection of secant modulus curves, represented by the ratio $G_s/G_0$ or $E_s/E_0$. The collected results were derived from monotonic laboratory shear tests performed on an sorted mix of clayey ans sandy materials, and they are presented in Figure 3.13. Geotechnical evidence and authors’ experience show that $E_0/E_{50}$ ratio varies in natural clays from 4.6 to 30 depending on soil aging and particle bonding. The higher values of $E_0/E_{50}$ ratio are suggested for aged, cemented and structured clay, whereas the lower ones for insensitive, unstructured and remoulded clays.

CPT. A relationship between $G_0$ and corrected tip resistance $q_t$ for clays has been proposed by Mayne and Rix (1993). The correlation also depends upon the inplace void ratio $e_0$ (cf. Figure 3.36):

$$G_0 = 49.4q_t^{0.695}e_0^{1.13} \text{ in MPa} \quad (3.47)$$

with $q_t$ in [MPa].

The effective vertical stress $\sigma'_v$ can used to recalculate the estimated modulus $E_0 = 2G_0(1 + \nu_{ur})$ to the reference one $E_{0\text{ref}}$ using the stiffness dependency power law assuming that minor stress $\sigma_3$ equal to min ($\sigma'_v; \sigma'_v; K_0$).

CPT. Based on a database for ten Norwegian marine soft clay sites, Long and Shane (2010) proposed an expression obtained by modifying the original expression by Simonini and Cola (2000). The relationship, apart of $q_t$, also accounts for pore pressure measurements (cf. Figure 3.36). The modification was related to replacing $\Delta u/\sigma_c$ ratio with pore pressure parameter $B_q = (u_2 - u_0)/(q_t - \sigma_v)$) and tunning empirical coefficients:

$$G_0 = 4.39q_t^{1.225}(1 + B_q)^{2.53} \pm 50\% \quad (3.48)$$

with $G_0$, $q_t$ in kPa and $B_q$ is a dimensionless pore pressure parameter.

DMT. $G_0$ can be estimated based on DMT data using a correlation reported by Hryciw (1990). The correlation was originally proposed for sands, silts and clays, however it seems to underestimate $G_0$ in sands (cf. Figure 3.37):

$$G_0 = \frac{530}{\sigma'_v/p_a^{0.25}} \frac{\gamma_{DMT}/\gamma_W - 1}{2.7 - \gamma_{DMT}/\gamma_W} K_0^{0.25} (p_a\sigma'_v)^{0.5} \quad (3.49)$$

with $G_0$, $p_a$, $\sigma'_v$ in same units; $\gamma_{DMT}/\gamma_W$ dilatometer-based unit weight ratio obtained using Marchetti’s chart for soil type and unit weight estimation.
CHAPTER 3. PARAMETER DETERMINATION

Figure 3.36: Comparison of empirical correlations for estimating $G_0$ ($G_{max}$) in clays from CPT data; plot for Eq. (3.47) obtained for $e_0 = 0.8$.

Figure 3.37: Estimating $G_0$ from DMT data using Eq. (3.49) proposed by Hryciw (1990) (plot obtained for $K_0 = 0.5$).
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

**Geotechnical evidence.** Experimental measurements reveal that in the case of fine plastic soils, the reference strain threshold $\gamma_{0.7}$ at which $G_s/G_0 = 0.722$ may be affected by many factors such as soil plasticity, stress history, confining pressure, number of cyclic loadings and others. A well known experimental database reported in Vucetic and Dobry (1991) (Figure 3.38) illustrates the relationship between $\gamma_{0.7}$ and plasticity index $I_P$. Based on this experimental data, $\gamma_{0.7}$ can be approximated by the following empirical correlation:

$$
\begin{align*}
\gamma_{0.7} &= \gamma_{0.7}^{\text{ref}} + 5 \cdot 10^{-6} I_P & \text{for } I_P < 15 \\
\gamma_{0.7} &= 10^{1.15 \log(I_P)-5.1} & \text{for } I_P \geq 15
\end{align*}
$$

(3.50)

with the reference strain threshold $\gamma_{0.7}^{\text{ref}} (I_P = 0) = 1 \cdot 10^{-4}$ and plasticity index $I_P$ in %. A fit of the above correlation to experimental data is illustrated in Figure 3.38. Since the $I_P$-dependent chart was compiled from the original data which showed a considerable scatter, a 50% error can be assumed in estimations giving max and min ranges. Results for $I_P < 100$ have been experimentally proved in many research, whereas extrapolation for soils which exhibit $I_P > 100$ should be treated carefully.

![Figure 3.38: Comparison of predictions for $\gamma_{0.7}$ from the plasticity index $I_P$ (PI) using Eq.(3.50) with experimental data reported for cohesive soils (after Vucetic and Dobry, 1991).](image)

Recently, Vardanega and Bolton (2011) reported a database of 20 clays and silts (OCR=1-17) for which a hyperbolic fit to the normalized reduction curve data has been proposed as follows:

$$
\frac{G}{G_0} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{0.5}}\right)^{0.74}}
$$

(3.51)

with the reference threshold parameter $\gamma_{0.5}$ which corresponds to $G_s/G_0 = 0.5$. They also confirmed that strain threshold for cohesive soils depends on the plasticity index $I_P$ (Figure 3.39), liquid limit $w_L$ (Figure 3.41) and plastic limit $w_P$. Based on these curves, they proposed a number of correlations for prediction of the reference strain threshold $\gamma_{0.5}$. Linear regression
analyzes were characterized by reasonable $R^2$ values although an error band of ±50% was observed. For the purpose of this report, the original coefficients obtained through regression analyzes for $\gamma_{0.5}$, where recalculated to the model parameter $\gamma_{0.7}$:

$$\gamma_{0.7} = \frac{0.5975I_P[-]}{1000} \pm 50\%, \quad R^2 = 0.75, \quad n = 61, \quad \text{for } I_P = 10 - 150\% \quad (3.52a)$$

$$\gamma_{0.7} = \frac{0.3442w_L[-]}{1000} \pm 50\%, \quad R^2 = 0.75, \quad n = 61, \quad \text{for } w_L = 25 - 240\% \quad (3.52b)$$

$$\gamma_{0.7} = \frac{0.7517w_P[-]}{1000} \pm 50\%, \quad R^2 = 0.57, \quad n = 61, \quad \text{for } w_P = 12 - 90\% \quad (3.52c)$$

Note that the above correlations account for no history stress nor confining pressure effects.

Figure 3.39: Comparison of predictions using Eq. (3.51) and (3.52a) with curves from Vucetic and Dobry (1991).
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

Figure 3.40: $I_p$-dependent predictions using Eq.(3.51) and (3.52a) with possible 50% error.

Figure 3.41: $w_L$-dependent predictions using Eq.(3.51) and (3.52b) with possible 50% error.
Note that the diagrams proposed in Vucetic and Dobry (1991) and Vardanega and Bolton (2011) are independent on stress history. In order to account an observed increase of $\gamma_{0.7}$ with the increasing OCR, Stokoe et al. (2004) proposed the following formula to predict $\gamma_{0.7}$ for plastic soils:

$$\gamma_{0.7} = \gamma_{0.7}^{ref} + 5 \cdot 10^{-6} I_P \text{OCR}^{0.3}$$

(3.53)

with the reference strain threshold $\gamma_{0.7}^{ref} (I_P = 0, \text{OCR} = 1) = 1 \cdot 10^{-4}$.

Darendeli (2001) proposed a correlation for the reference strain threshold which additionally accounts for the effect of confining pressure:

$$\gamma_{0.7} = (a_1 + a_2 I_P \cdot \text{OCR}^{a_3}) \sigma_0^{a_4}$$

(3.54)

Since the original correlation was developed for the reference strain threshold $\gamma_{0.5}$, for the purpose of the HS model, the empirical coefficients $a_1$, $a_2$, $a_3$ and $a_4$ have been adjusted in order to predict $\gamma_{0.7}$:

- $\gamma_{0.7}$ - reference strain threshold in [%]
- $I_P$ - plasticity index in [%]
- $\sigma_0 = \frac{p'_0}{p_{ref}}$ normalized confining pressure
- $p'_0 = \frac{(2K_0 + 1)\sigma'_v}{3}$ kPa *in situ* mean effective stress
- $p_{ref} \approx 100$ kPa (atmospheric pressure)
- $a_1 = 1.25e - 2$
- $a_2 = 3.7e - 4$
- $a_3 = 0.3$
- $a_4 = 0.35$

Note that in the current formulation of the HS model no stress dependency for $\gamma_{0.7}$ is considered. If needed, this parameter can be incorporated into boundary value problems through defining average mean effective stresses for defined sub-layers.
Figure 3.42: A graphical interpretation of Eq. (3.54): an example of estimation of shear strain threshold $\gamma_{0.7}$ for $I_P = 40\%$, OCR = 4 and mean effective stress $p'_0 = 400$ kPa.
3.3.2 Strength parameters

It is commonly known that the strength of clays in terms of effective stresses is mostly frictional and the effective cohesion \( c' \approx 0 \). Small values of cohesion which are observed during testing may appear in partially saturated clays where the meniscus effects (suction) draw soil particles together resulting in inter-particles stresses. Larger magnitudes of cohesion can be often observed in cemented soils due to bonding effects.

**Geotechnical evidence.** The values of the effective friction angle \( \phi' \) observed for fine soils fall in a wide range from \( 18^\circ \) to \( 42^\circ \). Some representative values of \( \phi' \) for compacted clays are provided in Table 3.19 after Carter and Bentley (1991).

*Table 3.19: Representative values of \( \phi' \) observed in compacted clays (after Carter and Bentley, 1991).*

<table>
<thead>
<tr>
<th>Soil type</th>
<th>UCS class</th>
<th>( \phi ) [(^\circ)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty clays, sand-silt mix</td>
<td>SM</td>
<td>34</td>
</tr>
<tr>
<td>Clayey sands, sandy-clay mix</td>
<td>SC</td>
<td>31</td>
</tr>
<tr>
<td>Silts and clayey silts</td>
<td>ML</td>
<td>32</td>
</tr>
<tr>
<td>Clays of low plasticity</td>
<td>CL</td>
<td>28</td>
</tr>
<tr>
<td>Clayey silts</td>
<td>MH</td>
<td>25</td>
</tr>
<tr>
<td>Clays of high plasticity</td>
<td>CH</td>
<td>19</td>
</tr>
</tbody>
</table>

**CPTU.** The estimation of effective stress parameters from the total stress analysis of undrained penetration is difficult. The solution needs to account for excess pore water pressure for which the distribution around the cone is highly complex and difficult to model analytically. Interpretation methods can be thus viewed as rather approximative.

The effective friction angle \( \phi' \) can be estimated using the solution which is based on the bearing capacity theory (Sandven et al., 1988):

\[
q_t - \sigma_{vo} = N_m(\sigma'_{vo} + a) \quad \text{with} \quad N_m = \frac{N_q - 1}{1 + N_u B_q}
\]  

(3.55)

where \( a' \) denotes the attraction \( (a' = c' \cot \phi') \), \( \beta \) is the angle of plastification, \( N_q \) and \( N_u \) are the bearing capacity factors \( (N_q = N_q(\phi', \beta) \text{ and } N_u \approx N_u(\phi)) \).

Mayne (2005, 2007) proposed a simplified expression which is applicable to the ranges of \( 20^\circ \leq \phi' \leq 45^\circ \) and \( 0.1 \leq B_q \leq 1.0 \) (see Figure 3.43). By setting for the above method the effective cohesion intercept \( c' = 0 \) and plastification angle \( \beta = 0 \), the values of \( \phi' \) were evaluated line-by-line and the following approximate expression was obtained:

\[
\phi' \approx 29.5^\circ B_q^{0.121} (0.256 + 0.336 B_q + \log Q_t)
\]  

(3.56)

**Strength parameters vs. undrained shear strength.** Best-quality predictions of the strength parameters \( \phi' \) and \( c' \) for cohesive soils can be essentially derived from laboratory tests. In case of lack of laboratory data, these model parameters can be approximately calibrated based on the undrained shear strength \( s_u \). The magnitude of \( s_u \) can be determined

---

\(^7\)The approach proposed by researchers from the Norwegian University of Science and Technology (NTNU) is referred to NTNU method.
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

Friction angle \( \phi' \) and Resistance number \( N_M \)

Figure 3.43: Friction angle for sands, silts and clays based on approximation of NTNU original method (from Mayne, 2005).

from a variety of \textit{in situ} tests such as field vane tests (FVT), pressuremeter tests, cone penetration tests (CPT or CPTU), etc. Some interpretation formula for determining \( s_u \) from commonly used field tests are provided in Appendix A.

Considering that the undrained shear strength in the undrained triaxial conditions is defined as:

\[
s_u = \frac{1}{2} (\sigma_1 - \sigma_3) = \frac{1}{2} q_f
\]  

the model parameters \( \phi \) and \( c \) can be adjusted so that they satisfy the normalized condition:

\[
\frac{s_u^{\text{in situ}}}{p_0^{\text{in situ}}} \approx \frac{1/2q_f^{\text{sim}}}{p_0^{\text{sim}}}
\]  

where \( s_u^{\text{in situ}} \) and \( p_0^{\text{in situ}} \) denote field test results of the undrained shear strength and the effective mean stress respectively, whereas \( q_f^{\text{sim}} \) is the failure deviatoric stress obtained through a numerical simulation of the undrained triaxial test at given initial effective mean stress \( p_0^{\text{sim}} \). Note that the above relation should be considered as approximative since \( s_u \) is not a unique soil parameter as, it depends, among others, on the type of test, which involves particular strain paths related to dominant shear modes appearing during testing (cf. Wroth, 1984; Jamiolkowski et al., 1985).

Conceptually, normalization of data in terms of initial stress conditions removes the effect of depth. Although the mechanisms of particular field test are influenced by both \( \sigma_{h0}' \) and \( \sigma_{v0}' \), for practical reasons, the normalization can be carried out in terms of \( \sigma_{v0}' \) since there is often little information about \( \sigma_{h0}' \). Therefore, calibration of the strength parameters can be carried out to satisfy the following relation:

\[
\frac{s_u^{\text{in situ}}}{\sigma_{v0}^{\text{in situ}}} \approx \frac{1/2q_f^{\text{sim}}}{\sigma_{1,0}^{\text{sim}}}
\]  

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where $\sigma_{1,0}^{\text{sim}}$ denotes the axial effective stress at the beginning of simulation.

The calibration procedure can be summarized as follows:

1. Assess field values of $s_u$ for, at least, two different depths (different $\sigma_{1,0}^{\text{sim}}$) and plot the data on $\sigma_{1,0}^{\text{sim}} - s_u$ chart.

2. Run two simulations of the undrained triaxial compression test (preferably anisotropically-consolidated with the specified $K_0^{SR} \neq 1.0$) for different $\sigma_{1,0}^{\text{sim}}$ and corresponding OCRs with an initial guess of parameters $\phi$ and $c$, and an assumed failure ratio $R_f$ (note that an explicit ultimate deviatoric stress can be obtained for the dilatancy angle $\psi = 0$, or non-zero $\psi$ with the assumed dilatancy cut-off).

3. Plot numerical results of $1/2q_f$ on $\sigma_{1,0}^{\text{sim}} - s_u$ chart (as in Figure 3.44) and check the degree of fit for numerical and in situ trend lines.

4. Return to step 2 if the degree of fit is not satisfactory and modify parameters $\phi$ and $c$. Note that each modification of $\phi$ and $c$ requires updating $K_0^{NC}$ and evaluating of parameters $M$ and $H$ before the next calculation run.

![Figure 3.44: Undrained shear strength $s_u$ against normalizing effective stress $\sigma_{1,0}^{\text{sim}}$ ($p^*$ denotes the intersection between deviatoric and isotropic mechanisms, see Figure ??).](image)

In the case of overconsolidated material, if the initial mean effective stress $p_0'$ lies before the mean effective stress $p^*$ which value corresponds to the intersection between deviatoric and isotropic mechanisms (see Figure ??), $q_f$ can be estimated directly from the Mohr-Coulomb criterion (cf. Figure 3.1):

$$q_f = \alpha (p_0' + c \cot \phi)$$  \hspace{1cm} (3.60)

where $\alpha$ is related to the friction angle $\phi$ which depends on the dominating shear mode which is appropriate to a given in situ test:
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

\[ \alpha = \frac{6 \sin \phi}{3 - \sin \phi} \text{ for triaxial compression conditions} \quad (3.61a) \]

\[ \alpha = \frac{6 \sin \phi}{3 + \sin \phi} \text{ for triaxial extension conditions} \quad (3.61b) \]

\[ \alpha = \sqrt{3} \sin \phi \text{ for plane strain conditions} \quad (3.61c) \]

**Figure 3.45:** Effective stress paths derived from simulations of undrained compression test in normally- and overconsolidated soil.
### 3.3.3 Failure ratio

For most soils, the value of $R_f$ falls between 0.75 and 1 and an average value of the failure ratio can be taken as $R_f = 0.9$. Kempfert (2006) reported some $R_f$ values derived from triaxial compression tests for three lacustrine soft soils in southern Germany:

- **CICD test:** $R_f = 0.73 - 0.88$, with the average value of 0.82
- **CICU test:** $R_f = 0.70 - 0.99$, with the average value of 0.89
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

3.3.4 Dilatancy angle

While for granular soils the maximal dilatancy angle can be related to the relative density ($D_r$), an estimation of $\psi$ for clays relies on geotechnical experience. For instance, it can be observed in laboratory tests that normally consolidated clays may exhibit no dilatancy at all. Therefore, it is proposed that for cohesive soils, it can be assumed that dilatancy depends on the preconsolidation state and $\psi$ can be approximately taken as:

- $\psi = 0^\circ$ for normally- and lightly-overconsolidated soil
- $\psi = \phi' / 6$ for overconsolidated soil
- $\psi = \phi' / 3$ for heavily overconsolidated soil

Another reasonable approximation of $\psi$ can be obtained using the following equation:

$$\psi = \phi - 30^\circ$$

(3.62)

for the values of friction angle larger than $30^\circ$.

Notice that the above approximation gives $\psi \approx \phi' / 3$ for a dense gravel, whereas small values of $\psi$ will be obtained for the the friction angles that correspond to compacted silts or clays (typically $\phi \approx 30^\circ$ to $35^\circ$).
3.3.5 Stiffness moduli

In general, the relation between the stiffness moduli is as follows:

\[ E_{50} < E_s < E_{ur} \]  

(3.63)

where \( E_s \) denotes "static" modulus or secant modulus taken from the initial part of the \( \varepsilon_1 - q \) experimental curve at \( \varepsilon_1 = 0.1\% \).

In the case when one of the stiffness moduli cannot be directly determined, it may be relevant for many practical cases to set:

\[ \frac{E_{ur}^{ref}}{E_{50}^{ref}} = 3 \text{ to } 6 \quad \text{(an average can be assumed equal to 4)} \]

(3.64)

However, note that the following condition should be satisfied:

\[ \frac{E_{ur}^{ref}}{E_{50}^{ref}} > 2 \]

The secant modulus \( E_{50} \) can be approximated based on the known value of the "static" modulus \( E_s \) and according to the approach described in Section 3.2.2, Figure 3.22. A rough approximation of the order of magnitudes for the "static" modulus \( E_s \) is given in Table 3.20.

Table 3.20: Typical values for the "static" modulus \( E_s \) [MPa] (compiled from Kezdi, 1974; Prat et al., 1995).

<table>
<thead>
<tr>
<th>Soil Consistency</th>
<th>Soil Type</th>
<th>Very Soft</th>
<th>Soft</th>
<th>Medium</th>
<th>Stiff</th>
<th>Very Stiff</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Silts</td>
<td>slight plasticity</td>
<td>2.5</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>low plasticity</td>
<td>1.5</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Clays</td>
<td>low to medium plast.</td>
<td>0.5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>high plasticity</td>
<td>0.35</td>
<td>2</td>
<td>1.5</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Silt organic</td>
<td></td>
<td>0.5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay organic</td>
<td></td>
<td>0.5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assuming that during unloading/reloading soil behaves elastically, the modulus \( E_{ur} \) can be related with the constrained unloading/reloading oedometric modulus \( E_{oed,ur} \) through:

\[ E_{ur} = E_{oed,ur} \frac{(1 + \nu_{ur})(1 - 2\nu_{ur})}{1 - \nu_{ur}} \]

(3.65)

Note, however, that \( E_{ur} \) is not a unique value for a given soil in the oedometric test because \( E_{ur} \) depends on the previous maximal stress level \( \sigma_c' \) attained before the unloading and the corresponding void ratio \( e_c \), as shown in Figure 3.46. Therefore, assuming an infinitesimal change of the compression stress, i.e. \( \Delta \sigma_c' \to 0 \), the unloading/reloading oedometric modulus \( E_{oed,ur} \) should be approximated by similarity with Eq.(3.5) as:

\[ E_{oed,ur} = \frac{2.3(1 + e_c)}{C_s} \sigma_c' \]

(3.66)
where $C_s$ is the swelling index (Figure 3.46).

Since the $E_{oed,ur}$ was approximated for the stress point belonging to the primary loading line ($K_0^{NC}$ line), such a determined reference unloading/reloading modulus $E_{ur}^{ref}$ corresponds thus to the reference stress $\sigma^{ref}$ which can be estimated from:

$$\sigma^{ref} = K_0^{NC} \sigma'_c$$  \hspace{1cm} (3.67)

**Figure 3.46:** Idealized plot of one-dimensional oedometric compression test.

**DMT.** The "static" modulus corresponding to $E_{ur}$ can be evaluated for cohesive soils the vertical drained constrained modulus $M_{DMT}$ which is derived from three intermediate dilatometer parameters, i.e. the material index $I_D$, the horizontal stress index $K_D$, and the dilatometer modulus $E_D$. Note that $M_{DMT}$ may correspond to $E_{oed}^{ref}$ only for normally-consolidated soil. The unloading-reloading modulus $E_{ur}$ can be evaluated assuming that:

$$E_{ur} = E_s = aE_{oed} = aM_{DMT}$$  \hspace{1cm} (3.68)

where $a \approx 0.9$ as 

$$a = (1 + \nu_{ur})(1 - 2\nu_{ur})/(1 - \nu_{ur})$$

and the dilatometer vertical drained constrained modulus $M_{DMT}$:

$$M_{DMT} = R_M E_D$$  \hspace{1cm} (3.69)

with $E_D = 34.7(p_1 - p_0)$ and $R_M$ depending on soil type behavior, i.e.

- clayey silt to sandy silt ($0.6 < I_D < 1.8$): $R_M = R_{M,0} + (2.5 - R_{M,0}) \log K_D$ and $R_{M,0} = 0.14 + 0.15(I_D - 0.6)$
- clay to silty clays ($I_D \leq 0.6$): $R_M = 0.14 + 2.36 \log K_D$
- if $K_D > 10$: $R_M = 0.32 + 2.18 \log K_D$
- if $R_M < 0.85$: set $R_M = 0.85$

**Undrained vs drained moduli - theoretical relationship.** In case of lack of drained compression test data, the stiffness moduli can be calibrated based on the results derived from the undrained triaxial compression test (e.g. CIUC or CAUC). Since water filling skeleton
pores has no shear stiffness, the shear modulus is not affected by the drainage condition so one can write:

\[ \frac{E_u}{2(1 + \nu_u)} = G_u = G = \frac{E}{2(1 + \nu)} \]  

(3.70)

where \( \nu_u \) is the Poisson’s coefficient in undrained conditions. Considering that the undrained conditions imply \( \varepsilon_1 = \varepsilon_3 \), and therefore \( \nu_u = 0.5 \), the above equation can be rewritten as:

\[ \frac{E_u}{E} = \frac{3}{2(1 + \nu)} \]  

(3.71)

and for the drained Poisson’s coefficient ranging for most soils between 0.12 and 0.4:

\[ \frac{E_u}{E} \approx 1.07 \text{ to } 1.34 \]  

(3.72)

An assumption of \( \nu_u = 0.5 \) for undrained conditions can also be expressed with the condition of no volume change (\( \Delta \varepsilon_v = 0 \)). Since the undrained bulk modulus \( K_u \) tends to infinity in such conditions, \( \nu_u \rightarrow 0.5 \):

\[ K_u = \frac{\Delta \sigma}{\Delta \varepsilon_v} = \frac{E_u}{3(1 - 2\nu_u)} \]  

(3.73)

The undrained “static” modulus \( E_u^s \) can be estimated based on a value of undrained shear strength \( s_u \) using an empirical correlation:

\[ E_u = K_c s_u \]  

(3.74)

with an empirical correlation coefficient which depends on the plasticity index PI and OCR, and can be estimated from Figure 3.47.

Figure 3.47: Evaluating the undrained modulus from \( E_u^s \) from \( s_u \): chart for estimating the correlation coefficient \( K_c \) in Eq. (3.74) (from Duncan and Buchignani, 1976).
Undrained vs drained moduli - curve fitting. The "drained" model moduli can also be calibrated by means of curve-fitting. It is recommended because in the "undrained" test the effective stress $\sigma'_3$ (which corresponds to the reference stress) does not remain constant during compression due to a development of excess pore water pressure. The calibration of "drained" stiffness moduli ($E'_{ur}, E'_{50}$) from "undrained" test requires fitting laboratory data, i.e. curve $\varepsilon_1 - q$, with the results obtained through an axis-symmetric, one-element simulation of the undrained compression. The flowchart of the parameter calibration is presented in detail in Figure 3.48 and it is described below.

Considering that strength parameters $\phi'$ and $c'$ can be directly derived from undrained test data, they should be kept unmodified during curve-fitting. In order to avoid excessive gain in material resistance after reaching the failure stress point, the dilatancy angle can be set $\psi = 0^\circ$ during "undrained" simulations (cf. Truty and Obrzud (2015)). The soil unit weight should be set to $\gamma = 0$ in order to cancel body force loading. As regards the fluid weight the option $\checkmark$ Skip gravity term should be chosen. A non-zero value for the initial void ratio $e_0$ should be set.

In order to represent the undrained behavior a finite value of the fluid bulk modulus should be set, e.g. the bulk modulus of water is $2.26 \times 10^6$ kPa. The ratio between fluid and soil bulk moduli should be of order $K_f/K = 10^5 \div 10^6$. It corresponds to the penalty formulation which may fail to converge if the penalty factor is too large. In such a case computation will be terminated and some null pivots will be reported in the *log file. In order to remedy such a problem, the value of the fluid bulk modulus should be decreased. Note that the two-phase stabilization should not be activated in the single-element test as no pressure oscillation is observed (pore pressure is constant over the element). No initial pressure BC need to be introduced.

The simulation should be carried out using Axisymmetry analysis type, Deformation+Flow problem type and Consolidation driver should be set for when running compression of the element.
Figure 3.48: Flowchart for calibration of stiffness moduli $E_0$, $E_{ur}$, $E_{50}$ based on $\varepsilon_1 - q$ curve derived from the undrained compression triaxial test (CU) and a single-element test.
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

**Geotechnical evidence.** Kempfert (2006) have provided typical results for the ratios between stiffness moduli. These ratios are presented below in Tables 3.21 and 3.22.

**Table 3.21:** Relationship between triaxial stiffness moduli and oedometric moduli for three lacustrine clays in Germany, from Kempfert (2006).

<table>
<thead>
<tr>
<th>Soil</th>
<th>$E_i/E_{oed}$</th>
<th>$E_{50}/E_{oed}$</th>
<th>$E_{ur}/E_{oed,ur}$</th>
<th>$E_{oed,ur}/E_{oed}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil 1</td>
<td>2.08</td>
<td>1.03</td>
<td>2.33-2.52</td>
<td>2.60</td>
</tr>
<tr>
<td>Soil 2</td>
<td>1.63</td>
<td>0.77</td>
<td>1.29-2.09</td>
<td>3.63</td>
</tr>
<tr>
<td>Soil 3</td>
<td>2.82</td>
<td>1.45</td>
<td>1.32-2.51</td>
<td>6.65</td>
</tr>
<tr>
<td>Average</td>
<td>2.17</td>
<td>1.08</td>
<td></td>
<td>4.29</td>
</tr>
</tbody>
</table>

$E_i$ was derived from the initial slope of the triaxial curve $\varepsilon_1 - q$

$E_{oed,ur}$ denotes unloading/reloading oedometer modulus

**Table 3.22:** Relationship between stiffness moduli derived from drained and undrained triaxial tests for three lacustrine clays in Germany, from Kempfert (2006).

<table>
<thead>
<tr>
<th>Drainage conditions</th>
<th>$E_i/E_{50}$</th>
<th>$E_{ur}/E_i$</th>
<th>$E_{ur}/E_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil 1 undrained</td>
<td>1.48</td>
<td>1.94</td>
<td>3.02</td>
</tr>
<tr>
<td>Soil 2 drained</td>
<td>2.17</td>
<td>3.10</td>
<td>6.72</td>
</tr>
<tr>
<td>Soil 2 undrained</td>
<td>1.84</td>
<td>2.17</td>
<td>3.10</td>
</tr>
<tr>
<td>Soil 3 drained</td>
<td>1.94</td>
<td>6.55</td>
<td>12.66</td>
</tr>
<tr>
<td>Soil 3 undrained</td>
<td>3.02</td>
<td>3.02</td>
<td>3.02</td>
</tr>
<tr>
<td>Average drained</td>
<td>2.04</td>
<td>4.28</td>
<td>8.43</td>
</tr>
<tr>
<td>Average undrained</td>
<td>2.11</td>
<td>2.11</td>
<td>2.11</td>
</tr>
</tbody>
</table>

$E_i$ was derived from the initial slope of the triaxial curve $\varepsilon_1 - q$
CHAPTER 3. PARAMETER DETERMINATION

3.3.6 Oedometric modulus

The one-dimensional constrained tangent modulus $E_{\text{oed}}$ (which, in literature, is often assigned as $M_D$ when determined from in situ tests) is obtained for steady state measurements based on the oedometer test through the expression:

$$E_{\text{oed}} = M_D = \frac{\delta \sigma'_v}{\delta \varepsilon_v}$$ \hspace{1cm} (3.75)

which can also be expressed as:

$$E_{\text{oed}} = \frac{\sigma'}{C_c} = \frac{(1 + e) \sigma'}{\lambda}$$ \hspace{1cm} (3.76)

where $C_c$ is the compression index ($C_c = 2.3 \lambda$). A number of empirical correlations which can be used to evaluate $C_c$ are given in Appendix B.

In case of lack of relevant data the oedometric modulus can be approximately taken as:

$$E_{\text{oed}}^{\text{ref}} \approx E_{\text{oed}}^{50}$$ \hspace{1cm} (3.77)

In such a case, the oedometric vertical reference stress $\sigma_{\text{oed}}^{\text{ref}}$ should be matched to the reference minor stress $\sigma_{\text{ref}}$ since the latter typically corresponds to the confining (horizontal) pressure $\sigma_{\text{ref}} = \sigma_3 = \sigma'_h$:

$$\sigma_{\text{oed}}^{\text{ref}} = \sigma_{\text{ref}} / K_{\text{NC}}$$ \hspace{1cm} (3.78)

As an example, Kempfert (2006) reports $E_{\text{oed}}^{\text{ref}} / E_{\text{oed}}$ ratio for three lacustrine clays in Germany which varies from 0.77 to 1.45 with the average 1.08 (see Table 3.21).

**Important note.** In the case of the HS model, $E_{\text{oed}}^{\text{ref}}$ can be taken as equal to $M_D$ if the latter has been derived from CPT, CPTU or DMT but only for normally-consolidated soil. In such a case $\sigma_{\text{oed}}$ can be taken as $\sigma'_{v0}$ which corresponds to the testing depth for which $M_D$ has been evaluated.

**CPT.** The constrained modulus for clays can be interpreted from the CPT or CPTU test using the measured cone resistance $q_c$ and an empirical coefficient $\alpha_m$. Lunne et al. (1997) quote the values of $\alpha_m$ for different types of soils proposed by Sanglerat (1972).

**CPTU.** The constrained modulus can be interpreted from the CPTU using the net cone resistance $q_t - \sigma_{v0}$ ($q_t$ denotes the corrected cone resistance):

$$M_D = \alpha_n (q_t - \sigma_{v0})$$ \hspace{1cm} (3.79)

where $\alpha_n$ is observed for most clays between 5 and 15 while for normally consolidated clays, it is between 4 to 8 (Sandven et al., 1988; Senneset et al., 1989). A more general correlation was suggested by Kulhawy and Mayne (1990) (cf. Figure 3.49):

$$M_D = 8.25 (q_t - \sigma_{v0})$$ \hspace{1cm} (3.80)

As discussed by Lunne et al. (1997), the estimation of "drained" parameter $M_D$ from an undrained penetration test using general empirical correlations may suffer from errors as large as $\pm 100\%$. An individual site-specific calibration is thus recommended for $\alpha_n$. They also concluded that it is difficult to correlate "drained" parameters without accounting for the pore pressure measurements as the cone resistance is measured in total stress.

**DMT.** The constrained modulus $M_D$ can be interpreted from three intermediate dilatometer parameters, i.e. the material index $I_D$, the horizontal stress index $K_D$, and the dilatometer modulus $E_D$, by applying the correlation presented in Eq.(3.69).
Table 3.23: Estimation of constrained modulus $M_D$ for clays (after Lunne et al. (1997)).

$$M_D = \alpha_m \cdot q_c$$

<table>
<thead>
<tr>
<th>$q_c$ range</th>
<th>$\alpha_m$ range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_c &lt; 0.7$ MPa</td>
<td>$3 &lt; \alpha_m &lt; 8$</td>
<td>Clay of low plasticity (CL)</td>
</tr>
<tr>
<td>$0.7 &lt; q_c &lt; 2.0$ MPa</td>
<td>$2 &lt; \alpha_m &lt; 5$</td>
<td></td>
</tr>
<tr>
<td>$q_c &gt; 2.0$ MPa</td>
<td>$1 &lt; \alpha_m &lt; 2.5$</td>
<td></td>
</tr>
<tr>
<td>$q_c &lt; 2.0$ MPa</td>
<td>$1 &lt; \alpha_m &lt; 3$</td>
<td>Silts of low plasticity (ML)</td>
</tr>
<tr>
<td>$q_c &gt; 2.0$ MPa</td>
<td>$3 &lt; \alpha_m &lt; 6$</td>
<td></td>
</tr>
<tr>
<td>$q_c &lt; 2.0$ MPa</td>
<td>$2 &lt; \alpha_m &lt; 6$</td>
<td>Highly plastic silts and clays (MH, CH)</td>
</tr>
<tr>
<td>$q_c &lt; 1.2$ MPa</td>
<td>$2 &lt; \alpha_m &lt; 8$</td>
<td>Organic silts (OL)</td>
</tr>
<tr>
<td>$q_c &lt; 0.7$ MPa</td>
<td>$1.5 &lt; \alpha_m &lt; 4$</td>
<td>Peat and organic clay ($P_t$, $OH$)</td>
</tr>
<tr>
<td>$50 &lt; w &lt; 100$</td>
<td>$1 &lt; \alpha_m &lt; 1.5$</td>
<td>($w$-water content [%])</td>
</tr>
<tr>
<td>$100 &lt; w &lt; 200$</td>
<td>$0.4 &lt; \alpha_m &lt; 1$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.49: General $M_D$ correlation for CPTU data proposed by Kulhawy and Mayne (1990) (from Lunne et al., 1997).
3.3.7 Stiffness exponent

Geotechnical evidence. The formulation of HS models assumes the same exponent $m$ for four different stiffness moduli, i.e. $E_0$, $E_{50}$, $E_{ur}$ and $E_{oed}$. Kempfert (2006) demonstrated that in reality, the following relation may appear $m_0 < m_{50} < m_{ur}$; they also provide some typical values derived from drained tests (oedometer loading and triaxial tests) for three lacustrine soft soils:

<table>
<thead>
<tr>
<th>Soil 1</th>
<th>$m_{oed}$</th>
<th>$m_{oed}^{avg}$ (no. of tests)</th>
<th>$m_0$</th>
<th>$m_0^{avg}$ (no. of tests)</th>
<th>$m_{50}$</th>
<th>$m_{50}^{avg}$ (no. of tests)</th>
<th>$m_{ur}$</th>
<th>$m_{ur}^{avg}$ (no. of tests)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil 1</td>
<td>0.73-0.76</td>
<td>0.75 (2)</td>
<td>0.3-0.42</td>
<td>0.34 (3)</td>
<td>0.39-0.51</td>
<td>0.45 (3)</td>
<td>0.74</td>
<td>0.74 (1)</td>
</tr>
<tr>
<td>Soil 2</td>
<td>0.58-0.69</td>
<td>0.64 (2)</td>
<td>0.52-0.79</td>
<td>0.68 (4)</td>
<td>0.66-0.84</td>
<td>0.72 (4)</td>
<td>0.61-0.67</td>
<td>0.64 (2)</td>
</tr>
<tr>
<td>Soil 3</td>
<td>0.58</td>
<td>0.58 (1)</td>
<td>0.42-0.56</td>
<td>0.51 (3)</td>
<td>0.38-0.54</td>
<td>0.48 (3)</td>
<td>0.79-0.89</td>
<td>0.84 (3)</td>
</tr>
</tbody>
</table>

Kempfert (2006) also highlighted that the exponent $m$ for undrained tests can be generally higher than for drained tests.

Viggiani and Atkinson (1995) reports the exponent numbers $m$ for different clays at very small strains as a function of the plasticity index $I_P$ (see Figure 3.50(a)) whereas Hicher (1996) presents them as a function of the liquid limit $w_L$ (see Figure 3.50(b)).

Figure 3.50: Power law exponent $m$ related to (a) plasticity index $I_P$ (Viggiani and Atkinson, 1995), and (b) liquid limit $w_L$ (Hicher, 1996).
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

Table 3.24: Typical values for $m_p$ observed in clays for the shear modulus $G_0$ (from Benz, 2007).

<table>
<thead>
<tr>
<th>Soil tested</th>
<th>$I_P$ [%]</th>
<th>$m_p$ [-]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avezzano clay (Holocene-Pleistocene)</td>
<td>10-30</td>
<td>0.46</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Fucino clay (Holocene-Pleistocene)</td>
<td>45-75</td>
<td>0.40</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Garigliano clay (Holocene)</td>
<td>10-40</td>
<td>0.58</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Panigaglia clay (Holocene)</td>
<td>44</td>
<td>0.50</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Montaldo di Castro clay (Pleistocene)</td>
<td>15-34</td>
<td>0.40</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>Recon. Valericca clay (Pleistocene)</td>
<td>27</td>
<td>0.85</td>
<td>Rampello et al. (1997)</td>
</tr>
<tr>
<td>Pisa clay (Pleistocene)</td>
<td>23-46</td>
<td>0.44</td>
<td>Lo Presti and Jamiolkowski (1998)</td>
</tr>
<tr>
<td>London clay (reconstituted)</td>
<td>41</td>
<td>0.76</td>
<td>Viggiani and Atkinson (1995)</td>
</tr>
<tr>
<td>Speswhite kaolin clay (reconstituted)</td>
<td>24</td>
<td>0.65</td>
<td>Viggiani and Atkinson (1995)</td>
</tr>
<tr>
<td>Kaolin clay</td>
<td>35</td>
<td>0.50</td>
<td>Marcuson and Wahls (1972)</td>
</tr>
<tr>
<td>Bentonite clay</td>
<td>60</td>
<td>0.50</td>
<td>Marcuson and Wahls (1972)</td>
</tr>
</tbody>
</table>

The number $m_p$ was obtained for the relation $G_0 \propto \left(\frac{p^l}{\sigma_{ref}}\right)^{m_p}$

Table 3.25: Suggested ranges of stiffness exponent $m$ observed for oedometric modulus $E_{oed}$ (von Soos, 1991).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$m$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt: low plasticity</td>
<td>0.6 $\div$ 0.8</td>
</tr>
<tr>
<td>Silt: medium and high plasticity</td>
<td>0.7 $\div$ 0.9</td>
</tr>
<tr>
<td>Clay: low plasticity</td>
<td>0.9 $\div$ 1.0</td>
</tr>
<tr>
<td>Clay: medium plasticity</td>
<td>0.95 $\div$ 1.0</td>
</tr>
<tr>
<td>Clay: high plasticity</td>
<td>1.0</td>
</tr>
<tr>
<td>Silt or clay: organic</td>
<td>0.85 $\div$ 1.0</td>
</tr>
<tr>
<td>Peat</td>
<td>1.0</td>
</tr>
<tr>
<td>Mud</td>
<td>0.9 $\div$ 1.0</td>
</tr>
</tbody>
</table>
3.3.8 Overconsolidation ratio

In the case of a soil which is located below the ground water table, a qualitative estimation of the overconsolidation ratio can be done based on the Atterberg limits and the natural moisture content. Assuming that the soil is saturated, it can be expected that smaller void ratios have less water space and \( w_n \) would be smaller \((\text{Bowles (1997)})\). From this observation, the following may be deduced:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>if ( w_n ) is close to ( w_L )</td>
<td>soil is normally consolidated</td>
</tr>
<tr>
<td>if ( w_n ) is between ( w_P ) and ( w_L )</td>
<td>soil is lightly consolidated</td>
</tr>
<tr>
<td>if ( w_n ) is close to ( w_L )</td>
<td>soil is lightly- to heavily overconsolidated</td>
</tr>
<tr>
<td>if ( w_n ) is larger than ( w_L )</td>
<td>soil is on verge of being a viscous liquid</td>
</tr>
</tbody>
</table>

In the latter case (\( w_n > w_L \)), stability of soil in \textit{in situ} conditions may be ensured by overburden pressure and interparticle bonds, unless visual inspection indicates a liquid mass.

**Geotechnical evidence.** \(\text{Mayne (1988)}\) provides empirical upper and lower limits derived from laboratory tests:

- undrained shear strength \( s_u \) determined under isotropically consolidated-undrained triaxial conditions (CIUC):
  \[
  \left( 1.82 \frac{s_u}{\sigma'_{v0}} \right)^{1.43} \leq OCR \leq \left( 4 \frac{s_u}{\sigma'_{v0}} \right)^{1.43}
  \]  
  \( (3.81) \)

- \( s_u \) determined under isotropically consolidated-undrained triaxial conditions (CAUC):
  \[
  \left( 3.70 \frac{s_u}{\sigma'_{v0}} \right)^{1.25} \leq OCR \leq \left( 5.26 \frac{s_u}{\sigma'_{v0}} \right)^{1.25}
  \]  
  \( (3.82) \)

**CPTU.** One of the best working approaches relates the overconsolidation ratio OCR to the net cone resistance \( q_t - \sigma_{v0} \):

\[
OCR = k_{\sigma_{et}} \frac{q_t - \sigma_{v0}}{\sigma'_{v0}}
\]  
\( (3.83) \)

where \( k_{\sigma_{et}} \) is an empirical coefficient which falls in the interval from 0.1 to 0.5 for non-fissured clays \((\text{Larsson and Mulabdić, 1991; Hight and Leroueil, 2003})\). The higher values are suggested for cemented, aged and heavily consolidated soils (between 0.9 and 2.2). For good-quality interpretation, this coefficient needs to be calibrated for specific site conditions based on the benchmark values derived from oedometer test. However, the first-order approximates of OCR can be obtained using the values of \( k_{\sigma_{et}} \) from multiple regression analyzes which are based on historical syntheses from many characterization sites (see Table 3.26). \(\text{Mayne (2006b)}\) suggests assuming \( k_{\sigma_{et}} = 0.30 \) for first-order estimates.

Another approach combines measurements of cone resistance \( q_t \) and pore pressure \( u_2 \) measured behind the cone:

\[
OCR = k_{\sigma_e} \frac{q_t - u_2}{\sigma'_{v0}} = k_{\sigma_e} \frac{q_e}{\sigma'_{v0}}
\]  
\( (3.84) \)

with \( k_{\sigma_e} \) being obtained through site-specific correlations. By analogy to the previous approach, the first-order approximates of OCR can be obtained using the values of \( k_{\sigma_e} \) through
Table 3.26: Comparison of the empirical coefficients obtained from multiple regression analyzes for non-fissured clays.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Geographical region</th>
<th>Number of sites/points</th>
<th>Results of regression analysis</th>
<th>Number of sites/points</th>
<th>$k_{\sigma e}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sweden</td>
<td>9/110</td>
<td>$k_{\sigma t}$</td>
<td>0.292</td>
<td>9/110</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>Canada</td>
<td>31/153</td>
<td>$k_{\sigma t}$</td>
<td>0.294</td>
<td>31/153</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>Worldwide</td>
<td>123/1121</td>
<td>$k_{\sigma t}$</td>
<td>0.305</td>
<td>84/811</td>
<td>0.50</td>
</tr>
</tbody>
</table>


regression analyzes (see Table 3.26). Mayne (2006b, 2007) suggested assuming $k_{\sigma e} = 0.60$ for the first-order estimates. This approach is often used as a comparative to the previous one and local correlations are strongly recommended. The formula is also viewed as less reliable in soft, lightly overconsolidated clays the $q_t$ results accompanied by large values of $u_2$ yield in a small number for $q_t - u_2$ (Houlsby and Hitchman, 1988; Lunne et al., 1997).
DMT. Based on dilatometer measurements, estimation of OCR for clays can be carried out with the formula proposed by Marchetti which relates the horizontal stress index\(^8\) \(K_D\) to OCR from oedometer tests with the following correlation:

\[
OCR = (0.5K_D)^{1.56}
\]  

(3.85)

The application of this correlation is restricted to materials with \(I_D < 1.2\), free of cementation which have experienced simple one-dimensional stress histories (Totani et al., 2001).

An improved relationship which takes into account a large range of soil plasticity in the exponent was proposed by Lacasse and Lunne (1988):

\[
OCR = 0.225K_D^{1.35÷1.67}
\]  

(3.86)

where the exponent varies from 1.35 for plastic clays, up to 1.67 for low plasticity materials.

\[\text{Figure 3.51: Various correlations } K_D - OCR \text{ for cohesive soils from various geographical areas (from Totani et al., 2001).}\]

\(^8\)Horizontal stress index which is calculated based on the first dilatometer reading \(p_0\), i.e. \(K_D = (p_0 - u_0)/\sigma'_{v0}\).
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

3.3.9 Coefficient of earth pressure "at rest"

The coefficient of earth pressure "at rest" for normally-consolidated clays can be estimated through Eq.(3.37a) or the similar expression suggested in Brooker & Ireland (1965):

\[ K_{0}^{NC} = 0.95 - \sin \phi' \] (3.87)

For cohesive soils, \( K_{0}^{NC} \) can also be related through empirical correlations with soil plasticity:

\[ K_{0}^{NC} = 0.19 + 0.233 \log I_P \quad \text{(Alpan, 1967)} \] (3.88a)

and similar

\[ K_{0}^{NC} = 0.44 + 0.0042 I_P \quad \text{(Holtz & Kovacs, 1981)} \] (3.88b)

where \( I_P \) is the plasticity index in \%.

It is commonly known that in cohesive soils the preconsolidation plays an important role and \( K_0 \) typically increases with the overconsolidation ratio OCR. Estimations of the initial stress state for overconsolidated soil take a general form:

\[ K_0 = K_{0}^{NC} OCR^m \] (3.89)

where \( m \) is a coefficient which for estimation of \( K_{0}^{NC} \) for most practical purposes can be taken as:

\[ m = 0.5 \quad \text{suggested by Meyerhof (1976)} \] (3.90a)

\[ m = \sin \phi' \quad \text{suggested in Mayne & Kulhawy (1982)} \] (3.90b)

However, the upper bound value for \( K_0 \) should be limited by the passive lateral earth pressure coefficient:

\[ K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \] (3.91)

The equations are presented graphically in Fig.3.52 (\( K_{0}^{NC} \) was calculated using Eq. (3.37a)).

SBPT. Approximation of \( K_0 \) from the self-boring pressuremeter test requires determination of the horizontal effective stress \( \sigma_{h0} \) since the vertical effective stress \( \sigma_{v0} \) can be estimated based on depth, unit weight and groundwater information. In the case of SBPT, the in situ total horizontal stress can be directly estimated from the "lift-off" pressure (Jamiolkowski et al., 1985; Clough et al., 1990; Amar et al., 1991). The "lift-off" pressure corresponds to the internal cavity pressure \( \psi_0 \) when the membrane starts to deform the wall of a borehole, therefore \( \psi_0 \cong \sigma_{h0} + u_0 \) (Figure 3.53). The "lift-off" is typically estimated based on the averaging procedure including the measurements of three feeler arms spaced at 120° around the instrument (Dalton and Hawkins, 1982; Mair and Wood, 1987). In general, the lateral stress measurements can be considered as fairly accurate in clays, particularly in soft deposits (Jamiolkowski et al., 1985).
CHAPTER 3. PARAMETER DETERMINATION

Figure 3.52: Typical relationships between $K_0$ and OCR observed for clays based on the correlation proposed by Mayne and Kulhawy (1982).

Figure 3.53: Example of the total horizontal stress estimation from the lift-off pressure in soft clay at Panigaglia site (after Jamiołkowski et al., 1985).
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

**CPTU.** At present, no reliable method exists for interpretation $K_0$ from CPT data. Rough evaluations related directly to CPTU measurements can be made using various approximative methods.

Observing that the pore pressure distribution around the cone is a function of $\sigma'_{ho}$, Sully and Campanella (1991) proposed to approximate $K_0$ based on a linear regression analysis using the normalized difference between pore pressure measured at the cone tip $u_1$ and behind the tip at the sleeve shoulder $u_2$:

$$K_0 = 0.11 \cdot \text{PPSV} + 0.5$$  \hspace{1cm} (3.92)

where PPSV = $(u_1 - u_2)/\sigma'_{vo}$ and the empirical coefficient $a_K$ was obtained equal to 0.11, see Figure 3.54. Note that the regression analysis reveals a considerable scatter and this identification approach should be used carefully.

Masood and Mitchell (1993) proposed the estimation of $K_0$ based on measurements at the friction sleeve $f_s$. In this method, $K_0$ is a function of the normalized sleeve friction $f_s/\sigma'_{vo}$ and the overconsolidation ratio OCR, as presented in Figure 3.55. Thus the approach requires prior evaluation of OCR and reliable measurements of $f_s$.

The most common technique for estimating $K_0$ employs an empirical formula which is based on the normalized cone resistance:

$$K_0 = k_K \left( \frac{q_t - \sigma'_{vo}}{\sigma'_{vo}} \right)$$  \hspace{1cm} (3.93)
where $k_K$ is an empirical coefficient. Using the regression analysis, Kulhawy and Mayne (1990) obtained the value of $k_K = 0.1$ for several $K_0$ values estimated from the self-boring pressuremeter test (SBPT), see Figure 3.56.

**DMT.** The original correlation for $K_0$ based on the dilatometer data, relative to uncemented clays is (Marchetti, 1980):

$$K_0 = \left( \frac{K_D}{1.5} \right)^{0.47} - 0.6 \quad (3.94)$$

In highly cemented clays, the above equation may significantly overestimate $K_0$, since part of $K_D$ is due to the cementation.
3.3. ALTERNATIVE PARAMETER ESTIMATION FOR COHESIVE MATERIALS

**DMT.** $K_0$ can also be interpreted from dilatometer test data. Since the original Marchetti relationship tends to overestimate $K_0$, its estimation can be carried out through the correlation suggested in *Lacasse and Lunne (1988)*:

$$K_0 = 0.34K_D^{0.44-0.64}$$

(3.95)

where the lower exponent value is associated with highly plastic clays, whereas higher values are suggested for low plasticity materials.
3.3.10 Void ratio

Typical values of voids ratio and dry unit weights observed for cohesive soils are provided in Table 3.27 and 3.28.

Table 3.27: Typical values of void ratios and dry unit weights observed in cohesive soils (from Hough, 1969).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$e$ [-]</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty or sandy clay</td>
<td>0.25</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Gap-graded silty clay w. gravel or larger</td>
<td>0.2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Well-graded gravel/sand/silt/clay</td>
<td>0.13</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Clay (30 to 50% of 2microns size)</td>
<td>0.5</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>Colloidal clay (over 50% of 2microns size)</td>
<td>0.6</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Organic silt</td>
<td>0.55</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Uniform, inorganic silt</td>
<td>0.4</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>Organic clay (30 to 50% of 2microns size)</td>
<td>0.7</td>
<td>4.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.28: Typical values of void ratios and unit weights observed in granular soils (from Terzaghi et al., 1996).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$e$ [-]</th>
<th>Dry unit weights [kN/m$^3$]</th>
<th>Wet weights [kN/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glacial till, very mixed grained</td>
<td>0.25</td>
<td>20.8</td>
<td>22.7</td>
</tr>
<tr>
<td>Soft glacial clay</td>
<td>1.2</td>
<td>12</td>
<td>17.4</td>
</tr>
<tr>
<td>Stiff glacial clay</td>
<td>0.6</td>
<td>16.7</td>
<td>20.3</td>
</tr>
<tr>
<td>Soft slightly organic clay</td>
<td>1.9</td>
<td>9.1</td>
<td>15.5</td>
</tr>
<tr>
<td>Soft very organic clay</td>
<td>3</td>
<td>6.7</td>
<td>14.4</td>
</tr>
<tr>
<td>Soft bentonite</td>
<td>5.2</td>
<td>4.2</td>
<td>12.5</td>
</tr>
</tbody>
</table>
3.4 Automated assistance in parameter determination

**Virtual Lab v2018** is a highly-interactive module which provides users with:

- assistance in selecting a relevant constitutive law with regards to the general behavior of the real material
- first-guess parameter estimation based on field test records
- automated parameter selection (first-guess values of model parameters for soil for any incomplete or complete specimen data)
- user-engaged parameter selection (interactive parameter selection which involves browsing different parameter correlations including field tests data)
- ranges of parameter values which can be considered in parametric studies
- automated parameter identification from laboratory experimental data
- possibility of running numerical simulations of elementary laboratory tests in order to visualize the constitutive model response for the defined model parameters
- possibility of comparing numerical simulations of elementary laboratory tests with curves obtained in the laboratory

A separated report provides Help to **Virtual Lab**.

The toolbox is initialized by clicking on **Virtual Lab** which is visible once one of the following continuum models has been chosen as the material definition (Figure 3.57):

- Mohr-Coulomb
- Hardening-Soil Small Strain
- Cam-Clay
- Cap
Figure 3.57: Initializing the Virtual Lab from the Materials window
Chapter 4

Benchmarks

4.1 Triaxial drained compression test on dense Hostun sand

Files:
HS-std-dh-sand-100kPa.inp, HS-small-dh-sand-100kPa.inp,
HS-std-dh-sand-300kPa.inp, HS-small-dh-sand-300kPa.inp,
HS-std-dh-sand-600kPa.inp, HS-small-dh-sand-600kPa.inp

The following section presents a validation of both HS and HS-small models on a triaxial drained compression test for Hostun sand. Material properties are taken from PhD thesis by Benz (2007) and are given in the following table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{ref}}$</td>
<td>[kPa]</td>
<td>90000</td>
<td>$\psi$</td>
<td>[$^\circ$]</td>
<td>16.0</td>
</tr>
<tr>
<td>$E_{\text{ref}}^{50}$</td>
<td>[kPa]</td>
<td>30000</td>
<td>$f_t$</td>
<td>[kPa]</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_{\text{ref}}$</td>
<td>[kPa]</td>
<td>100</td>
<td>$D$</td>
<td>[-]</td>
<td>0.0/0.25</td>
</tr>
<tr>
<td>$m$</td>
<td>[-]</td>
<td>0.55</td>
<td>$M$</td>
<td>[-]</td>
<td>1.727/1.723</td>
</tr>
<tr>
<td>$\nu_{ur}$</td>
<td>[-]</td>
<td>0.25</td>
<td>$H$</td>
<td>[kPa]</td>
<td>55967/57055</td>
</tr>
<tr>
<td>$R_f$</td>
<td>[-]</td>
<td>0.9</td>
<td>OCR</td>
<td>[-]</td>
<td>1.0</td>
</tr>
<tr>
<td>$c$</td>
<td>[kPa]</td>
<td>0.0</td>
<td>$E_{\text{ref}}^{0}$</td>
<td>[kPa]</td>
<td>270000</td>
</tr>
<tr>
<td>$\phi$</td>
<td>[$^\circ$]</td>
<td>42.0</td>
<td>$\gamma_{0.7}$</td>
<td>[-]</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

The parameters $M$ and $H$ were estimated automatically by the code assuming $K_0^{NC} = 0.4$ and $E_{\text{oed}} = 30000$ kPa at reference stress equal to 100 kPa. It must be emphasized that $M$ and $H$ values will not be equal to the ones given by Benz because of the different hardening law applied to the preconsolidation pressure $p_c$. All results obtained with Z_Soil match very well results published by Benz. The only differences appear in $G - \gamma$ plots where Z_Soil results for standard HS-model are lower than the reference ones. In our opinion, $G - \gamma$ curves for standard HS model which were published by Benz, begin at $G_{ur}$ value and are not obtained from the elasto-plastic solution.
Figure 4.1: Results for the confining pressure $\sigma_3 = 100$ kPa
4.1. TRIAXIAL DRAINED COMPRESSION TEST ON DENSE HOSTUN SAND

Figure 4.2: Results for the confining pressure $\sigma_3 = 300$ kPa
(a) $\frac{\sigma_1}{\sigma_3} (\varepsilon_1) \ (Z_{\text{Soil}})$

(b) $G(\gamma) \ (Z_{\text{Soil}})$

(c) $\frac{\sigma_1}{\sigma_3} (\varepsilon_1) \ (\text{zoom}) \ (Z_{\text{Soil}})$

(d) $\varepsilon_v(\varepsilon_1) \ (Z_{\text{Soil}})$

(e) Solution by Benz benz (2007)

Figure 4.3: Results for the confining pressure $\sigma_3 = 600 \ \text{kPa}$
4.2 Isotropic compression of dense Hostun sand

File: HS-isotropicCompr.inp

This benchmark is solved analytically for the HS-Std model. The decomposed total strain increments for the elastic and plastic part in isotropic compression conditions \((p = p_c)\) are presented in the following expression (NB. the increment of total volumetric strain is measured from the initial configuration of equilibrium \(p_0 = p_c\) to the current one):

\[
\Delta \varepsilon^p_v = \frac{\sigma^{ref} + c \cot \phi}{1 - m} \left[ \left( \frac{p_c + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^{1-m} - \left( \frac{p_c + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right) \right]
\]

(4.1)

\[
\Delta \varepsilon^e_v = \frac{3(1 - 2\nu_{ur})}{E^{ur}_{ref}} \left( \frac{\sigma^{ref} + c \cot \phi}{1 - m} \right) \left[ \left( \frac{p_c + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^{1-m} - \left( \frac{p_c + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^{1-m} \right]
\]

(4.2)

\[
\Delta \varepsilon_v = \Delta \varepsilon^e_v + \Delta \varepsilon^p_v
\]

(4.3)

Verification was carried out on a single axisymmetric finite element which is subject to an external uniformly distributed load varying from \(q = 50\, \text{kN/m}^2\) to \(q = 250\, \text{kN/m}^2\). The initial effective stresses are \(\sigma_{o} = \{-50, -50, 0, -50\}^T\) kPa. Material data for the dense Hostun sand (see section (4.1)) is used in the simulation. Numerical and analytical solutions are compared in the following figure.

ISOTROPIC COMPRESSION TEST
4.3 Oedometric compression test

File: HS-oedometer.inp, HS-oedometer-1.inp

This single-element benchmark demonstrates that using the HS-Std model an assumed $E_{oed} = 30000$ kPa at the reference stress $\sigma_{oed}^{ref}$ kPa and $K_{0NC} = 0.4$ are correctly reproduced. This benchmark uses material data for dense Hostun sand presented in Section 4.1.

The first test is modeled in axisymmetry, with an element subject to an external, uniformly distributed load which varied from $q = 75$ kN/m² to $q = 275$ kN/m². In the second test, a strain driven program is applied in one step with vertical strain amplitude $\Delta \varepsilon_y = -1e^{-5}$.

The initial effective stress state is described as $\sigma_o = \{-30, -75, 0, -30\}^T$ kPa.

The results which are derived the first test are shown in the following two figures. The result of the second test yields the tangent oedometric modulus which were computed with the forward difference scheme, equal to $E_{oed} = \frac{\Delta \sigma_y}{\Delta \varepsilon_y} = 30000$ kPa, which is an exact value.

\[
\begin{array}{c}
-0.005 & -0.004 & -0.003 & -0.002 & -0.001 & 0 \\
-50 & -100 & -150 & -200 & -250 & -300 \\
\end{array}
\]

$\sigma_y - \varepsilon_y$ plot for different number of load increments

\[
\begin{array}{c}
0.38 & 0.39 & 0.4 & 0.41 & 0.42 \\
2 & 4 & 6 & 8 & 10 \\
\end{array}
\]

Estimated $K_{0NC}^{NC}$
4.4 Oedometric compression test - $K_{0}^{NC}$-path test

This single-element benchmark that HS-Std model is able to correctly reproduce $K_{0}^{NC}$-path for the oedometric test for different values of the friction angle. The results are compared with the data obtained with other constitutive models.

The oedometric test on the normally-consolidated soil ($q^{POP} = 0.1$ kPa) is modeled in axisymmetry, with a single element subject to an external, uniformly distributed load which varied from $q = 0$ kN/m$^2$ to $q = 800$ kN/m$^2$. The results which are presented in Figure 4.4 show that HS-Std model correctly reproduces $K_{0}^{NC}$ stress paths which obey the empirical expression $K_{0}^{NC} = 1 - \sin \phi$ for three different values of the friction angle.

The same model is used to compare the results from the oedometric test using different constitutive models: the Modified Cam clay (MCC), the standard Mohr-Coulomb (M-C) and the Cap model (CAP). Simulations for different models were carried out for the same value of the friction angle $\phi = 30^\circ$ (and equivalent $M = 1.2$ for the MCC model). Hence, the value of cohesion was assumed as $c = 0$ kPa. The value of Poisson’s ratio was assumed $\nu_{ur} = 0.2$ for the HS model and $\nu = 0.3$ for other models. Figure 4.5 shows that starting from the initial “zero” stress setup and gradually increasing the vertical stress, only the HS model is able to reproduce expected $\sigma'_{h}/\sigma'_{v}$ path. It is so because the parameter $M$ which defines the shape of the cap surface in the HS-model, is optimized so that the tangent to the cap surface at the stress reversal point is perpendicular to $K_{0}^{NC}$ stress path.

Figure 4.4: $K_{0}^{NC}$-path test for the HS-Std model and different values of the friction angle $\phi = 20^\circ, 30^\circ, 40^\circ$: a) stress paths in $p' - q$ plane, b) $K_{0}^{NC}$ with increasing loading.
Figure 4.5: $K_0^{\text{NC}}$-path test using different models for $\phi = 30^\circ$ ($K_0^{\text{NC}} = 0.5$): Hardening Soil (HS), Modified Cam clay (MCC), standard Mohr-Coulomb (M-C) and Cap model (CAP).
Chapter 5

Case studies

5.1 Excavation in Berlin Sand

File: HS-std-Exc-Berlin-Sand-2phase.inp
File: HS-small-Exc-Berlin-Sand-2phase.inp
File: MC-Exc-Berlin-Sand-2phase.inp

This example demonstrates the importance of modeling excavation problems with the use of the Hardening Soil model. The study case presents an analysis of main differences between HS-Std, HS-Small and standard Mohr-Coulomb (MC) models based on a numerical simulation of a deep excavation in Berlin Sand.

An engineering draft of the problem and the sequence of both excavation and construction steps, are given in Figure 5.1. Material data for calibration of sand was taken from Benz (2007) and Schweiger (2002). The data with standard MC model was generated assuming that stiffness of sand varies according to the power law:

\[
E = \begin{cases} 
20000 \sqrt{y} \text{ kPa} & \text{for } y \leq 20m \\
60000 \sqrt{y} \text{ kPa} & \text{for } y > 20m 
\end{cases}
\]

where \( y \) is the depth expressed in [m]. The same strength parameters apply to both the MC model and HS models.
Sequence of stages:

1. Generating an initial stress state for an assumed $K_0^{\text{in situ}}$ in sand layers
2. Installation of the diaphragm wall
3. Lowering the ground water level in the excavated zone up to the elevation -17.90m
4. Excavation step 1 (up to -4.80m)
5. Introducing the first row of anchors (distance 2.30 m) and applying the prestress $P_0 = 768$ kN
6. Excavation step 2 (up to -9.30m)
7. Introducing of the second row of anchors (distance 1.35 m) and applying the prestress $P_0 = 945$ kN
8. Excavation step 3 (up to -14.35m)
9. Introducing of the third row of anchors (distance 1.35 m) and applying the prestress $P_0 = 980$ kN
10. Excavation step 4 (up to -16.80m)
A finite element model of the problem is shown in the figure below. The mesh represents:

- deposits consisting of two sand layers which are described with two different groups of stiffness characteristics
- three rows of prestressed anchors
- diaphragm wall
- contact interfaces between sand and the wall
- zone of artificial contact elements which are used to model a hydraulic barrier (preserving continuity of displacement field and discontinuity of pore pressure)
- external displacement boundary conditions (BC) (box type)
- pressure BC which are applied via fluid head and set up along the right hand side boundary, as well as along the left hand boundary up to the level of impermeable barrier (pressure fluid head BC is applied with the aid of seepage elements)
### Table 5.1: Excavation in Berlin Sand: material properties for soils

<table>
<thead>
<tr>
<th>Material</th>
<th>Model</th>
<th>Data group</th>
<th>Properties</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand (-20m↑)</td>
<td>HS-small</td>
<td>Elastic</td>
<td>$E_{ur}$</td>
<td>[kN/m$^2$]</td>
<td>180000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{ref}$</td>
<td>[kN/m$^2$]</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\nu_{ur}$</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$m$</td>
<td>-</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{L}$</td>
<td>-</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$E_{0}^{ref}$</td>
<td>[kN/m$^2$]</td>
<td>405000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\gamma_{0.7}$</td>
<td>-</td>
<td>0.0002</td>
</tr>
<tr>
<td>Density</td>
<td></td>
<td></td>
<td>$\gamma_D$</td>
<td>[kN/m$^3$]</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\gamma_F$</td>
<td>[kN/m$^3$]</td>
<td>10</td>
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<td></td>
<td></td>
<td></td>
<td>$e_0$</td>
<td>-</td>
<td>0.66</td>
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<tr>
<td>Non-linear</td>
<td></td>
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<td>$\phi$</td>
<td>[°]</td>
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<td></td>
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<td>$\psi$</td>
<td>[°]</td>
<td>5</td>
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<td></td>
<td></td>
<td></td>
<td>$c$</td>
<td>[kN/m$^2$]</td>
<td>1</td>
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<tr>
<td></td>
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<td></td>
<td>$E_{0}^{ref}$</td>
<td>[kN/m$^2$]</td>
<td>450000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$R_f$</td>
<td>-</td>
<td>0.9</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>$D$</td>
<td>-</td>
<td>0.25/0.0(HS-std)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_t$</td>
<td>-</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M$</td>
<td>-</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>$H$</td>
<td>[kN/m$^2$]</td>
<td>129305</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_0^{NC}$</td>
<td>-</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$p_{min}^{(c)}$</td>
<td>[kN/m$^2$]</td>
<td>10.0</td>
</tr>
<tr>
<td>Initial $K_0$ state</td>
<td></td>
<td></td>
<td>$K_{oz}^{(c)}/K_{nz}^{(c)}$</td>
<td>-</td>
<td>0.43</td>
</tr>
</tbody>
</table>

| Sand (-20m↓) | HS-small | Elastic   | $E_{ur}$   | [kN/m$^2$] | 300000    |
|              |          |            | $\sigma_{ref}$ | [kN/m$^2$] | 100       |
|              |          |            | $\nu_{ur}$  | -          | 0.2       |
|              |          |            | $m$         | -          | 0.55      |
|              |          |            | $\sigma_{L}$ | -          | 10.0      |
|              |          |            | $E_{0}^{ref}$ | [kN/m$^2$] | 675000    |
|              |          |            | $\gamma_{0.7}$ | -          | 0.0002    |
| Density  |       |            | $\gamma_D$  | [kN/m$^3$] | 16        |
|          |       |            | $\gamma_F$  | [kN/m$^3$] | 10        |
|          |       |            | $e_0$       | -          | 0.66      |
| Non-linear |       |            | $\phi$      | [°]        | 38        |
|          |       |            | $\psi$      | [°]        | 6         |
|          |       |            | $c$         | [kN/m$^2$] | 1         |
|          |       |            | $E_{0}^{ref}$ | [kN/m$^2$] | 75000     |
|          |       |            | $R_f$       | -          | 0.9       |
|          |       |            | $D$         | -          | 0.25/0.0(HS-std) |
|          |       |            | $f_t$       | -          | 0.0       |
|          |       |            | $M$         | -          | 2.955     |
|          |       |            | $H$         | [kN/m$^2$] | 128964    |
|          |       |            | $K_0^{NC}$  | -          | 0.38      |
|          |       |            | $p_{min}^{(c)}$ | [kN/m$^2$] | 10.0      |
| Initial $K_0$ state |   |            | $K_{oz}^{(c)}/K_{nz}^{(c)}$ | - | 0.38 |
### Table 5.2: Excavation in Berlin Sand: material properties for the diaphragm wall, anchors and interfaces

<table>
<thead>
<tr>
<th>Material</th>
<th>Model</th>
<th>Data group</th>
<th>Properties</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Wall</td>
<td>Beams</td>
<td>Elastic</td>
<td>$E$</td>
<td>[kN/m$^2$]</td>
<td>30000000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\nu$</td>
<td>–</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Density</td>
<td>Unit weight</td>
<td>[kN/m$^3$]</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Geometry</td>
<td>Interval</td>
<td>[m]</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A$</td>
<td>[m$^2$]</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$I_z$</td>
<td>[m$^4$]</td>
<td>0.0426667</td>
</tr>
<tr>
<td>4 Anchors</td>
<td>Truss</td>
<td>Elastic</td>
<td>$E$</td>
<td>[kN/m$^2$]</td>
<td>210000000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Density</td>
<td>Unit weight</td>
<td>[kN/m$^3$]</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Geometry</td>
<td>Interval</td>
<td>[m]</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A$</td>
<td>[m$^2$]</td>
<td>0.0015</td>
</tr>
<tr>
<td>5 Anchors</td>
<td>Truss</td>
<td>Elastic</td>
<td>$E$</td>
<td>[kN/m$^2$]</td>
<td>210000000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Density</td>
<td>Unit weight</td>
<td>[kN/m$^3$]</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Geometry</td>
<td>Interval</td>
<td>[m]</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A$</td>
<td>[m$^2$]</td>
<td>0.0015</td>
</tr>
<tr>
<td>6 Anchors</td>
<td>Truss</td>
<td>Elastic</td>
<td>$E$</td>
<td>[kN/m$^2$]</td>
<td>210000000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Density</td>
<td>Unit weight</td>
<td>[kN/m$^3$]</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Geometry</td>
<td>Interval</td>
<td>[m]</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A$</td>
<td>[m$^2$]</td>
<td>0.0015</td>
</tr>
<tr>
<td>7 Interface</td>
<td>Contact</td>
<td>Non-linear</td>
<td>$\phi$</td>
<td>[°]</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\psi$</td>
<td>[°]</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c$</td>
<td>[kN/m$^2$]</td>
<td>0.0</td>
</tr>
</tbody>
</table>
CHAPTER 5. CASE STUDIES

The chart below presents four unloading functions which are defined and associated with the excavated elements in order to gradually unload each excavated region. Note that the same unloading functions must be applied to interface elements adjacent to the excavated continuum. All existence functions and unloading functions which are applied for excavated zones of sand are shown in the chart below.

Figure 5.3: Excavation in Berlin Sand: unloading functions
5.1. EXCAVATION IN BERLIN SAND

Figure 5.4: Excavation in Berlin Sand: Bending moments and wall deflections at the last stage of excavation

Remarks:

1. The largest bending moments are generated by HS-Std model due to excessive plastic soil deformation as the result of lack of small strain stiffness. The shape of the $M$ diagrams is similar for all models.

2. The most significant overshoot is observed in the bottom part of the wall. In the basic MC model elastic stiffness remains unchanged and insensitive to the current stress state while HS-Std and HS-Small models exhibit strong stress dependency (cf. Eq.(2.5)).

3. Prediction of wall deflection by the HS-Small model matches in situ measurements.
Figure 5.5: Excavation in Berlin Sand: Soil deformations at last stage of excavation
(a) vertical heaving of subsoil, (b) settlements of the ground behind the wall ($y = 0$ m)

Remarks:

1. The HS-Std and MC models with variable stiffness generate similar heavings.

2. Vertical heaving generated by the HS-Small model is significantly reduced with respect to results which are generated by HS-Std and MC models.

3. The MC model results in an unrealistic lifting of the retaining wall associated with unloading of the bottom of an excavation. Settlements behind the wall are realistically generated with HS-Std and HS-Small models.
5.2 Twin tunnels excavation in London Clay

This example demonstrates the importance of modeling tunnel construction problems with the use of advanced constitutive models such as Hardening Soil models which allows modeling pre-failure non-linear stiffness. The study highlights the differences in predictions of subsurface displacements during tunnel excavations in the stiff, heavily overconsolidated London Clay modeled with:

- Linear-elastic, perfectly plastic Mohr-Coulomb model.
- Non-linear elastic, perfectly plastic models: HS-Std and HS-SmallStrain.

This study reanalyzes the excavation model of the twin Jubilee Line Extension Project tunnels beneath St James’s Park (London, UK) which has been reported in the original paper by Addenbrooke et al. (1997). The predictions of displacements obtained with the Hardening Soil models are additionally compared with the results obtained by Addenbrooke et al. (1997) for the isotropic non-linear elastic model J4 and field data.

The problem statement, i.e. subsurface stratigraphy and the orientation of tunnels is presented in Figure 5.6. The following paragraphs present the analysis details, the excavation/construction stages and the material data assumed in the analyzes.
CHAPTER 5. CASE STUDIES

Analysis details

• analysis type: plain strain deformation + flow
• driver type: consolidation
• mesh: Figure 5.7
• constitutive models:
  * Sand: linear elastic
  * Thames Gravel: linear elastic Mohr-Coulomb (M-C)
  * Woolwich and Reading Bed Clay: M-C

Excavation/construction stages

1. Generating the initial state in substrata for the assumed $K_{inr}^{\text{insitu}}$ across the FE mesh presented in Figure 5.7
2. Installation of seepage elements around the tunnel which permits free flow simulating drains and excavation of the westbound tunnel with gradual unloading - 100% unloading after 8 hours
3. Installation of the 1st tunnel lining at 75% of unloading; parameter for tunnel lining are given in Table 5.7 and removing seepage elements
4. Consolidation during 8.5 months
5. Installation of seepage elements around the tunnel which permits free flow simulating drains and excavation of the eastbound tunnel during 8 hours with gradual unloading
6. Installation of the 2nd tunnel lining at 70% of unloading and removing seepage elements
7. Consolidation
5.2. TWIN TUNNELS EXCAVATION IN LONDON CLAY

Figure 5.6: Soil stratigraphy and diagonally oriented tunnels at St James's Park, London, UK.

Figure 5.7: Finite element mesh
CHAPTER 5. CASE STUDIES

Material data

- Unit weights - see Table 5.3

- Stiffness parameters
  - for HS models - $E_0$, $E_{ur}$, $E_{50}$ and $\gamma_{0.7}$ calibrated using laboratory $\varepsilon_1 - q$ data points for the isotropically consolidated undrained extension triaxial test (CIEU) at $p'_0 = 750$ kPa, as shown in Figure 5.10. The constant $m$ was assumed for London Clay equal to 0.75 as reported in Viggiani and Atkinson (1995). The stiffness parameters are given in Table 5.5. A similar value of $E_0$ to the calibrated $E_0 = 390$ MPa has been also reported by Gasparre (2005).
  - for the M-C model: two variants for $E$ have been considered (see Table 5.4)
    Set 1: $E$ varying with depth $z$ in meters ($E = 6000z$) as in the original paper by Addenbrooke et al. (1997),
    Set 2: profile for $E$ adapted to the $E_{ur}$ profile assumed for HS models (i.e. $E = 3600z$), as graphically presented in Figure 5.8.

- Strength and plastic potential parameters - typical values for London Clay (see Table 5.3) have been adapted from the original paper (Addenbrooke et al., 1997) for all considered models.

- Initial state parameters (see Table 5.6)
  - the value of the overconsolidation ratio OCR for London Clay was assumed equal to 15 as it is typically observed for depths around 20-30 meters.
  - although the estimates of $K_{0}^{in situ}$ for the London Clay are typically reported of around 1.5, the value $K_0 = 1.0$ has been adopted in the analysis. It was observed that the isotropic Hardening Soil models may give incorrect predictions for $K_0 >> 1.0$ since stiffness in the model depends on the minor principal stress (maximal settlements were not observed in the tunnel axis but on its sides for $K_0 = 1.5$). The comparative results produced by model J4 in Addenbrooke et al. (1997) were obtained for $K_0 = 1.5$.

- Permeability - sand and gravel were modelled as highly permeable materials, whereas clayey soils were attributed with an anisotropic permeability decreasing with depth, as shown in Figure 5.9. The fluid bulk modulus was assumed equal to $\beta_f = 2.2$ GPa.

- Characteristics for the tunnel lining which were adopted after the original paper are summarized in Table 5.7.
5.2. TWIN TUNNELS EXCAVATION IN LONDON CLAY

Figure 5.8: Variation of the unloading-reloading modulus $E \left( E_{ur} \right)$ with depth for different models considered in the study.

Figure 5.9: Permeability profile assumed in the analysis.
Comments:

- Small strain nonlinear models, HS-SmallStrain and J4, well match laboratory data points (in red) both in the very small strain (up to 0.01%) and in the small strains (between 0.01 and 0.4%).
- HS-Std acceptably fits laboratory data at small strains, i.e. from 0.1%.
- M-C model fitted to laboratory data at 0.1% of axial strain strongly overestimates soil stiffness with the increasing axial strain.

Figure 5.10: Stress-strain curves: comparison between non-linear models (HS-Std, HS-SmallStrain, and J-4 model), linear Mohr-Coulomb model and laboratory test data points obtained in the isotropically consolidated undrained extension test ($p'_0 = 750$ kPa).

Figure 5.11: Variation of the undrained secant stiffness-strain curve $\varepsilon_1 - E_{ud}$: comparison between numerical models and laboratory data points obtained in the isotropically (CIEU) and anisotropically (CAEU) consolidated undrained extension tests ($p'_0 = 750$ kPa).
Comments:

- In practical applications, linear models may underestimate excess pore water pressure at very small strains.

- In consolidation analyzes, M-C model may overestimate excess pore water pressure in the zones of small strain (in this example beyond the axial strain of 0.1%).

Figure 5.12: Pore pressure-strain curves: comparison between non-linear models (HS-Std, HS-SmallStrain, and J-4 model), linear Mohr-Coulomb model and laboratory test data points obtained in the isotropically consolidated undrained extension test ($p_0' = 750$ kPa).
Comments:

- Predictions from M-C model are strongly underestimated in contrast to the field data.
- Non-linear pre-failure analyzes predict deeper and narrower profiles.
- HS-SmallStrain model gives a narrower shape of surface settlements than HS-Std. The higher stiffness of the HS-SmallStrain concentrates the strain levels at the unloading boundary giving slightly deeper profile than HS-Std, and therefore the displacements from 10m-offset from the tunnel axis are reduced further away to the mesh sides.
- The family of HS models gives a similar settlement profile to J4 model used in the original paper.

*Figure 5.13:* Surface settlement profiles after excavation of 1st tunnel: comparison for different models.
Comments:

• Decreasing $K_0$ to 0.8 pronounces the effect of smaller horizontal stresses by bringing closer the numerical results to the field data.

• The results obtained for $K_0 = 1.2$ resemble the settlement profile which was obtained with J4 model for $K_0 = 1.5$ (cf. Figure 5.13).

Figure 5.14: Surface settlement profiles after excavation of 1st tunnel: HS-SmallStrain response for different $K_0$. 
Figure 5.15: Pore pressure in the 1st tunnel axis: comparison of different models.
Comments:

- A slight asymmetry in the field data is accordingly reproduced by all models.
- The family of HS models significantly better reproduces the shape of the surface settlement profile in contrast to the M-C model.

Figure 5.16: Surface settlement profiles after excavation of 2\textsuperscript{nd} tunnel: comparison for different models.
Figure 5.17: Surface settlement profiles after excavation of 2\textsuperscript{nd} tunnel: HS-SmallStrain response for different $K_0$. 
Figure 5.18: Excavation of the westbound tunnel (a) vertical settlement in the tunnel axis; (b) horizontal displacements along tunnel axis level of the westbound tunnel.
Comments:

- In general, M-C produces smaller vertical and horizontal displacements in the tunnel axes than non-linear models.

- Unlike to HS models, the model J4 does not automatically return high stiffness behavior on loading reversal occurring during consolidation after $1^{st}$ excavation, the displacements profiles for $2^{nd}$ excavation are smaller than those obtained with HS models. However, the results by HS models reveal accordance with the J4 model for which the strains were zeroed across the entire mesh prior to $2^{nd}$ excavation (J40).

![Excavation of Eastbound Tunnel](image)

**Figure 5.19:** Excavation of the eastbound tunnel (a) vertical settlement in the tunnel axis; (b) horizontal displacements along tunnel axis level of the eastbound tunnel.
### 5.2. TWIN TUNNELS EXCAVATION IN LONDON CLAY

#### Table 5.3: Unit weight, permeability, yield surface and plastic potential parameters for Mohr-Coulomb and HS-models.

<table>
<thead>
<tr>
<th></th>
<th>Strength parameters</th>
<th>Angle of dilatation</th>
<th>Bulk unit weight</th>
<th>Permeability coefficient m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sand</td>
<td>Gravel</td>
<td>London Clay</td>
<td>Woolwich and Reading Clay</td>
</tr>
<tr>
<td>Linear elastic</td>
<td>$c' = 0$ kPa</td>
<td>$\phi' = 35^\circ$</td>
<td>$c' = 5.0$ kPa</td>
<td>$c' = 200$ kPa</td>
</tr>
<tr>
<td>elastic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi' = 35^\circ$</td>
<td>$\psi = 17.5^\circ$</td>
<td>$\psi = 12.5^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi' = 25^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi' = 27^\circ$</td>
<td>$\psi = 13.5^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{dry}$</td>
<td>18</td>
<td></td>
<td></td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>$\gamma_{sat}$</td>
<td>20</td>
<td></td>
<td></td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

#### Table 5.4: Elastic parameters of soil for linear elastic models, varying with depth below ground surface, $z$ in meters.

<table>
<thead>
<tr>
<th></th>
<th>Poisson’s ratio, $\nu$</th>
<th>Young’s modulus, $E$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sand</td>
<td>Gravel</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

$E$ moduli of London Clay for HS models are provided in Table 5.5.

** set 1: values of $E$ modulus from the reference paper; set 2: profile of $E$ modulus for London Clay similar to the profile of $E_{ur}$ obtained for the HS models as shown in Figure 5.8.

#### Table 5.5: Stiffness parameters of London Clay for HS models at the reference stress $\sigma_{ref} = 360$ kPa.

<table>
<thead>
<tr>
<th>$E_0$ [MPa]</th>
<th>$\gamma_{0.7}$ [-]</th>
<th>$E_{ur}$ [MPa]</th>
<th>$E_{50}$ [MPa]</th>
<th>$m$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>390</td>
<td>$3e - 004$</td>
<td>75</td>
<td>35</td>
<td>0.75</td>
</tr>
</tbody>
</table>

#### Table 5.6: Initial state parameters assumed in this study.

<table>
<thead>
<tr>
<th>$K_0^{NC}$ [-]</th>
<th>$K_0^{SR}$ [-]</th>
<th>$K_0$ [-]</th>
<th>OCR [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.5</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.5</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>London Clay</td>
<td>0.58</td>
<td>0.58</td>
<td>1.0</td>
</tr>
<tr>
<td>Woolwich and Reading Clay</td>
<td>0.58</td>
<td>1.0</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 5.7: Tunnel lining characteristics.

<table>
<thead>
<tr>
<th>Young's modulus $E$</th>
<th>Poisson's ratio $\nu$</th>
<th>Cross sectional area $A$</th>
<th>Moment of inertia $I_z$</th>
<th>Lining-soil friction angle $\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 GPa</td>
<td>0.15</td>
<td>0.168 m$^2$/m</td>
<td>3.95136 m$^4$/m</td>
<td>20$^\circ$</td>
</tr>
</tbody>
</table>
5.3 Spread footing on overconsolidated Sand

File: HS-small-Foot-Berlin-Sand-2phase.inp
File: MC-Footing-Texas-Sand-2phase_EurVar.inp

A verification of the HS-SmallStrain model for a quadratic footing load test is demonstrated in this section. This load test examines the model in terms of its sensitivity to loading and unloading modes applied to overconsolidated sand. This simple boundary value problem also illustrates:

- Combined parameter identification from triaxial tests and dilatometer tests
- Interpretation of laboratory and in situ measurements
- Imposing soil stress history through the $q^{\text{POP}}$ (obtaining a variable OCR profile)
- Imposing a variable $K_0$ profile
- Sensitivity of numerical prediction to small-strain stiffness parameters
- Sensitivity of numerical prediction to initial stress setup

The study case demonstrates main differences between HS-Std, HS-Small and standard Mohr-Coulomb (MC) for the analyzes of footing.

Problem statement.

A number of load tests on quadratic footing were performed at Texas A&M University’s National Geotechnical Site (Briaud and Gibbens, 1997). In this example, the measurements derived from a load test of 3x3m "North" footing are compared to numerical predictions. The test setup and soil stratigraphy is presented in Figure 5.20. The subsoil mainly consists of sandy clay to silty sand layers and it has been confirmed by the interpreted DMT data (see Figure 5.26(a)). Vertical displacements were measured at the corners of the quadratic footing.
CHAPTER 5. CASE STUDIES

Figure 5.20: A draft of the test setup and soil stratigraphy at A&M University’s National Geotechnical Site in Texas.

Parameter identification from a triaxial test. The parameters for the HS model have been determined from available triaxial compression tests for two sampling depths 0.6m and 3.0m, and three confining pressures 34.5, 138 and 345kPa. The results from several resonant column tests with the confining pressure 100kPa have been used to evaluate the order of magnitude of small strain characteristics. The reader can analyze parameter identification in the spreadsheet presented in Table 5.8. Note that the determination of $\phi'$ and $c'$ was carried out only for 34.5kPa and 138kPa tests since the ultimate state (failure deviatoric stress $q_f$) was not achieved at 345kPa.

A comparison of numerical results and experimental data derived from triaxial compression tests is presented in Figure 5.22. Note that the preconsolidated state of soil specimens was taken into account in the triaxial test simulations by applying the initial value of the minimal preconsolidation pressure $p_{c0} = 250\, \text{kPa}$. The value was evaluated assuming that:

$$\sigma'_{v0} = 14.9\, \text{kN/m}^3 \times 3.0\, \text{m} = 44.7\, \text{kPa}$$

and applying an estimated $q^{\text{POP}}$ (see Figure 5.25):

$$\sigma^{\text{SR}}_y = \sigma'_{vc} = \sigma'_{v0} + q^{\text{POP}} = 44.7\, \text{kPa} + 350\, \text{kPa} = 394.7\, \text{kPa} \quad \text{cf. Eq. (2.26a)}$$

$$\sigma^{\text{SR}}_x = \sigma'_{v0} \cdot K_{SR}^{\text{SR}} = 394.7\, \text{kPa} \times 0.41 = 161.8\, \text{kPa}$$
5.3. SPREAD FOOTING ON OVERCONSOLIDATED SAND

\[ p_{c0} = \frac{(2\sigma_{x}^{SR} + \sigma_{y}^{SR})}{3} \approx 250 \text{kPa} \]

Note that prescribing the initial preconsolidation state may have an influence on the results of the numerical simulation of the triaxial compression test. In the presented example, the volumetric mechanism is not activated during the triaxial compression test for the confining pressures 34.5kPa and 138kPa because the specimens are preconsolidated, i.e. the initial stress state \( p_{c0}' \) is largely inferior with respect to \( p_{c0}' \). On the other hand, the simulation for 345kPa is affected by both shear and cap mechanisms as the specimen is normally consolidated. In the other words, one may expect less stiffness at 50% of \( q_f \), where the volumetric straining also occurs. Note that in the case of the full scale simulation of the footing load test, the initial preconsolidation state will be prescribed through the preoverburden pressure \( q_{POP} \) which imposes the preconsolidation pressure with respect to the initial effective vertical stress \( \sigma_{e0} \) (see Figure 5.25).

Since the results derived from resonant columns present a considerable scatter (even for the same confining pressure 100kPa, see Figure 5.22 in the right hand bottom corner), they were used to evaluate the first guess of small-strain stiffness parameters. It is commonly recognized that the small-strain stiffness derived from \textit{in situ} seismic probes is typically larger compared to that measured with laboratory devices. This is typically attributed to specimen disturbances during soil sampling.

The results derived from numerical simulations presented in Figure 5.22 illustrate the sensitivity analysis of the HS-SmallStrain model to two parameters \( E_0 \) and \( \gamma_{0.7} \) which define the model behavior for very small amplitudes of shear strain. It can be noticed that a considerable scatter of the results derived from resonant columns does not allow performing a precise parameter calibration, especially for parameter \( \gamma_{0.7} \). Considering probable disturbances due to sampling, the simulations of triaxial tests were conducted for lower small-strain stiffness parameters, whereas higher values of parameters were assumed to simulate \textit{in situ} load test.

![Figure 5.21: Interpretation of triaxial compression test data from A&M test site (sampling depths: 0.6m and 3.0m, refer to Table 5.8).](image)
Table 5.8: Parameter identification spreadsheet for the triaxial test - spread footing benchmark.

<table>
<thead>
<tr>
<th>Sample</th>
<th>0.6m</th>
<th>3.0m</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confining pressure $\sigma_3$ [kPa]</td>
<td>34.5</td>
<td>138</td>
<td>345</td>
</tr>
<tr>
<td>Ultimate dev. stress $q_f$ [kPa]</td>
<td>112.3</td>
<td>417.7</td>
<td>994.4$^#$</td>
</tr>
<tr>
<td>Vertical stress $\sigma_1$ [kPa]</td>
<td>146.8</td>
<td>555.7</td>
<td>1339.4</td>
</tr>
<tr>
<td>Mean stress at failure [kPa]</td>
<td>71.9</td>
<td>277.2</td>
<td>676.5</td>
</tr>
</tbody>
</table>

$^\#$ $q_f$ not achieved, identification of $\phi$ and $c$ for 34.5kPa and 138kPa

Identification of $\phi$ and $c$ | Mean |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope $M^*$ [-]</td>
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</tr>
<tr>
<td>Intercept in $p^<em>-q$ plane $c^</em>$ [kPa]</td>
<td>5.262</td>
</tr>
<tr>
<td>Friction angle $\phi$ [deg]</td>
<td>36.6</td>
</tr>
<tr>
<td>Cohesion $c$ [kPa]</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Identification of $R_f$ and $E_{50}(\sigma_3)$ | Mean |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope $a$ [-]</td>
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</tr>
<tr>
<td>Intercept $b$ [kPa]</td>
<td>5.01e-05</td>
</tr>
<tr>
<td>Failure ratio $R_f(= b \cdot q_f)$ [-]</td>
<td>0.95</td>
</tr>
<tr>
<td>Secant modulus $E_{50}(= 1/2b)$ [kPa]</td>
<td>9988</td>
</tr>
</tbody>
</table>

Identification of $E_{50}^{\text{ref}}$ and $m$ | Mean |
<table>
<thead>
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<th></th>
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<tbody>
<tr>
<td>In $E_{50}$</td>
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<tr>
<td>Reference stress $\sigma_3$ [kPa]</td>
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<tr>
<td>$a = (\sigma_3 + c \cot \phi)/(\sigma_{\text{ref}} + c \cot \phi)$</td>
<td>0.367</td>
</tr>
<tr>
<td>In $a$</td>
<td>-1.001</td>
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<tr>
<td>Stiffness exponent $m$ [-]</td>
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Identification of $\psi$ | Mean |
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<tbody>
<tr>
<td>Slope $d$</td>
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</tr>
<tr>
<td>Dilatancy angle $\psi$ [deg]</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Identification of $\psi$ | Mean |
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Slope $d$</td>
<td>4.88e-02</td>
</tr>
<tr>
<td>Dilatancy angle $\psi$ [deg]</td>
<td>1.37</td>
</tr>
</tbody>
</table>
Figure 5.22: Triaxial compression test data from A&M test site (sampling depths: 0.6m and 3.0m) and numerical results for HS-SmallStrain and HS-Std. Parameters for HS-Std model were derived from parameter identification presented in Table 5.8; the results for different small-strain parameters are compared to the results from a couple of resonant column tests (right hand bottom corner: $\gamma_s = \varepsilon_1 - \varepsilon_3$ and $G_{50} = \Delta q/2\Delta \gamma_s$). The other parameters used for numerical simulations are summarized in Table 5.9.
Figure 5.23: Interpretation of triaxial test data in $\varepsilon_1 - \varepsilon_1/q$ plane: samples 0.6m and 3.0m. Note that slope $a$ is used to calculate $R_f$ whereas the intercept $b$ is used to compute $E_{50}$ as shown in Figure 3.3 (sampling depths: 0.6m and 3.0m, refer to Table 5.8).
5.3. SPREAD FOOTING ON OVERCONSOLIDATED SAND

Parameter estimation from the dilatometer test.

Records derived from two dilatometer tests (DMT-1 and DMT-2) were used for profiling silty sand. The field tests, i.e. CPT, pressuremeter and Marchetti’s dilatometer, showed that the silty sand site is overconsolidated due to removal of an overburden surcharge and soil aging. Estimation of two initial state variables OCR and $K_0$ was carried out based on the interpreted results of the horizontal stress index $K_D$ (see Figure 5.24(a)). It can be noticed that the OCR decreases with increasing depth which is characteristic to superficial layers of subsoil which may be directly subject to mechanical unloading such as erosion, excavations, changes in ground water level, or due to other phenomena such as dessication or formation of particle bonds.

In addition, the profile of the effective friction angle $\phi'$ which has been obtained applying empirical correlations for DMT data, was compared to $\phi'$ derived from the triaxial test (see Figure 5.24(b)). The following correlations were used for parameter profiling:

- OCR: Eq.(3.86) with the exponent value for low plastic materials equal to 1.67,
- $K_0$: Eq.(3.95) with the exponent value for low plastic materials equal to 0.64,
- $E_{ur}$: Eq.(3.25),
- $\phi'$: upper bound Eq.(3.32), lower bound Eq.(3.33), and the mean value as the average of both.

While the profile $\phi'$ derived from DMT was used to verify the values obtained with triaxial tests, OCR and $K_0$ profiles assumed in the model were directly interpreted from in situ data. As regards OCR, the variable profile which is illustrated in Figure 5.25(a) was obtained based on the effective vertical stress by applying preoverburden pressure $q^{POP}$, see Figure 5.25(b):

$$\sigma_{y^{SR}} = \sigma_{vc} = \sigma_{v0} + q^{POP} \quad \text{cf. Eq.(2.26a)}$$

and

$$\text{OCR} = \frac{\sigma_{vc}'}{\sigma_{v0}'}$$

where:

- $\sigma_{vc}'$ - preconsolidation pressure
- $\sigma_{v0}'$ - effective vertical stress

Clearly, the stress history (soil overconsolidation) could be also obtained by applying and removing a surcharge before the installation of the footing. However, such manipulation leaves behind a strain history. Typically, a strain history in natural deposits may be erased relatively fast due to stress relaxation and soil aging effects such as cementing of soil particles. Therefore, most boundary value problems should be started from zero initial strains which is a default setting in ZSoil®. Applying the stress history through $q^{POP}$ option allows the user to account for the overconsolidation effect (variable OCR typically observed at superficial layers of subsoil) with zero strains at the beginning of the analysis.

As regards $K_0$, the profile assumed in the model was obtained by fitting DMT interpretation, as shown in Figure 5.27.
Figure 5.24: Interpretation of dilatometer test data: a) profile of horizontal stress index $K_D$, b) DMT-based profiling of the effective friction angle compared to $\phi'$ derived from the triaxial test.
Figure 5.25: Initial stress state profiling: a) OCR profile derived from DMT data and OCR profiles by applying different $q^{\text{POP}}$ (in the reference simulation $q^{\text{POP}} = 350\text{kPa}$ has been considered), b) the vertical effective stress and the preconsolidation stress profiles assumed in the analysis.
Figure 5.26: Dilatometer test data: a) profile of material index $I_D$, b) profile of dilatometer modulus $E_D$ (both profiles have been used to determine $E_{ur}$ profile based on an empirical solution given in Eq.3.25).
Figure 5.27: Profiles interpreted from DMT data and assumed profiles for in FE model: a) coefficient $K_0$, b) stiffness modulus interpreted from DMT data, stiffness moduli assumed for HS-Small ($E_{50}, E_{ur}$ and $E_0$) and the Young modulus $E$ assumed for the Mohr-Coulomb model.
Material data. Material data which was assumed for numerical simulation of footing test loading are summarized in Table 5.9.

- **Physical properties** - taken from the original report (Briaud and Gibbens, 1997)
- **Small strain properties** - evaluated from the resonant column tests
- **Deformation characteristics** - $E_{50}$ derived from triaxial compression test data, and $E_{ur}$ double-checked with DMT data
- **Strength characteristics** - derived from triaxial compression test data
- **Initial state variables** - evaluated from DMT data

Table 5.9: Model parameters used in simulations of the spread footing at Texas site.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Physical properties</th>
<th>Unit</th>
<th>Sand (triaxial test)</th>
<th>Sand (in situ simulation)</th>
<th>Mohr-Coulomb Sand (in situ simulation)</th>
</tr>
</thead>
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<tr>
<td>$\gamma_D$</td>
<td>[-]</td>
<td>[kN/m$^3$]</td>
<td>-</td>
<td>14.9</td>
<td>14.9</td>
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<tr>
<td>$\gamma'$</td>
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<td>[kN/m$^3$]</td>
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<td>0.75</td>
<td>0.75</td>
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<tr>
<td>$\gamma_{max}$</td>
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<td>$\gamma_0$</td>
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<td>0.0006</td>
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<tr>
<td>$E_0$</td>
<td>[kPa]</td>
<td></td>
<td>260000</td>
<td>380000</td>
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<td>$E_{50}$</td>
<td>[kPa]</td>
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<td>70000</td>
<td>70000</td>
<td>imposed profile for $E$: Fig.5.27(b)</td>
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<td>0.2</td>
<td>0.3</td>
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<tr>
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<td>Oedometric test characteristics</td>
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<td>$E_{oed}$</td>
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<tr>
<td>$\sigma_{ref}$</td>
<td>[kPa]</td>
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<td>Initial state variables</td>
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<td>$q_{POP}$</td>
<td>[kPa]</td>
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<td>-</td>
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<tr>
<td>$p_{c0}$</td>
<td>[kPa]</td>
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<td>250</td>
<td>-</td>
<td>-</td>
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<td>$K_d$ (in situ)</td>
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<td>1</td>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>$K_{SR}$</td>
<td>[-]</td>
<td></td>
<td>1</td>
<td>0.41</td>
<td>-</td>
</tr>
</tbody>
</table>
5.3. SPREAD FOOTING ON OVERCONSOLIDATED SAND

Numerical simulation of the footing problem. A 3D model of the footing problem which is shown in Figure 5.28 was considered in the analysis. The 3D mesh can represent a quarter of the real setup thanks to the two symmetry planes. The mesh includes:

- simplified stratigraphy of the subsoil deposits consisting of one sand layer; mesh dimension 19.8m x 19.8m x 17m
- quarter of 3x3m footing which is embedded in the soil at 0.76m and its thickness is equal to 1.22m
- interface between footing faces and soil
- external displacement boundary conditions (BC) (box type)
- pressure BCs which are applied via fluid head and set up along the external boundaries; ground water level is set to 4.9m below soil surface
- nodal force representing a hydraulic jack (due to two symmetry planes the applied nodal is equal to 0.25 x $F$)

In addition, Figure 5.29 illustrates how to impose a variable profile of $K_0$ by means of the Initial Stresses option. Note that an existence function is attributed to the superelements describing the initial stresses. It means that the effect of imposed initial stress applies only to the first analysis step, i.e. during the generation of the initial state. This intervention is needed to avoid imposing soil’s initial stress onto the material which replaces excavated soil.

Analysis details

- analysis type: 3D deformation+flow
- driver type: steady state
- constitutive models:
  - Sand: three variants: (i) HS-Standard, (ii) HS-SmallStrain, (iii) Standard Mohr-Coulomb
  - Footing: elastic material $E = 20$GPa, $\nu = 0.2$

The following stages are considered in the model:

1. Generating an initial stress state for an assumed $K_0^{\text{in situ}}$ in sand layer and initial pore pressure BC
2. Installation of the quadratic footing with simultaneous replacement of soil’s material in the embedded part of footing
3. Gradual application of test load at the quadratic footing
Figure 5.28: 3D mesh representing a quarter of 3x3m footing.

Figure 5.29: Imposing a variable $K_0$ profile by means of the Initial Stresses option.
Analysis of results. The following paragraph presents the results derived from numerical simulations.

Figure 5.30(a) shows that the HS-SmallStrain satisfactorily reproduces the overall load-settlement curve. The initial parts of the unloading branches are also well simulated. Certain discrepancies between experimental data and numerical results can be observed for the completely unloaded stage which may be attributed to the adopted stiffness dependency law which depends on the minor stress $\sigma'_3$ and not on the mean stress $p'$. The charts also show that failing to account for small strain stiffness by using the HS-Std model may lead to severe overestimation of settlements in overconsolidated sand.

Figure 5.30(b) demonstrates the incapability of the standard Mohr-Coulomb model to realistically reproduce the evolution of settlements under mixed loading conditions. In this analysis, an imposed profile of Young modulus $E$ corresponding to $E_{ur}$ has been chosen to describe stiffness of the M-C model (see Figure 5.27(b)).
Figure 5.31: Experimental and computed load-settlement results for 3x3m footing test on sand at A&M site: sensitivity to small-strain stiffness parameters.

Figure 5.31(a) demonstrates sensitivity of HS-SmallStrain to $E_0$. It can be noticed that underestimating small-strain stiffness may lead to an overestimation of settlements, affecting especially the initial part of the load-settlement curve. Hence, by comparing the results for underestimated small-stiffness and the curve derived for HS-Std (small-strain stiffness inactive), it can be concluded that it is better to run a simulation with underestimated small-strain parameters than not to account for the small-strain stiffness at all.

Figure 5.31(b) illustrates the sensitivity of HS-SmallStrain to the small-strain threshold parameter $\gamma_{0.7}$. It can be noticed that the lower value of this parameter allows an earlier degradation of small-strain stiffness.
5.3. SPREAD FOOTING ON OVERCONSOLIDATED SAND

(a) HS-SmallStrain: Overconsolidated vs normally-consolidated sand

(b) HS-SmallStrain: Overconsolidated sand through $q_{\text{POP}}$ (variable) vs OCR = 10 (constant)

Figure 5.32: Experimental and computed load-settlement results for 3x3m footing test on sand at A&M site: sensitivity to the initial stress history setup.

Figure 5.32(a) shows an example of settlement overestimation if the simulation is carried out not accounting for the preconsolidation effect in the overconsolidated sand. In this particular example, assuming that the soil is normally-consolidated leads to a severe settlement overestimation of 270% for the first portion of loading (up to 4500kN) and maximal settlements achieved during the load test are doubled compared to the reference simulation for the overconsolidated material.

Figure 5.32(b) compares two analyzes for the overconsolidated sand (i) with variable OCR profile obtained with $q_{\text{POP}} = 350$ kPa, and (ii) with an equivalent OCR = 10 as an “averaged” value between 0.5m and 5m. No difference between numerical curves can be observed between 0kN and 2500kN as small-strain stiffness has not been degraded yet. The difference starts to occur at the magnitude of load from 2500kN to 8000kN. This is due to an underestimation of the preconsolidation pressure in the upper part of subsoil for a case with constant OCR = 10. In this case, an earlier activation of the cap mechanism yields in larger volumetric strains below the footing. The curves start to overlap each other for loads from 8000kN to 10000kN as the zone of the load influence expands below the footing. A further increase of load would show larger settlements for the case with the variable OCR as the zone of influence includes depth levels of lower OCR values.
CHAPTER 5. CASE STUDIES

Figure 5.33: Experimental and computed load-settlement results for 3x3m footing test on sand at A&M site: sensitivity to the initial stress setup.

Figure 5.33(a) illustrates the sensitivity of numerical predictions to the initial setup of the coefficient of earth pressure "at rest". In the reference simulation, the profile of $K_0$ for overconsolidated soil has been evaluated by means of the Marchetti’s dilatometer test (DMT), whereas in the second simulation a constant profile of $K_0 = K_0^{NC} = 1 - \sin(\phi)$ has been assumed. This analysis reveals slightly larger settlements for $K_0^{NC} = 0.41$ which is attributed to lower initial soil stiffness. Note that soil stiffness in the HS model depends on the minor effective stress so in the case where $K_0 = K_0^{NC}$ the initial stiffness is defined by $\sigma'_h$, whereas for the reference simulation, the initial soil stiffness is defined by $\sigma'_{h0}$ up to around 2.2m as $K_0 > 1$. The user should be aware that soil stiffness in the HS model evolves during simulation with amplitudes of stress level.

Finally, Figure 5.33(b) reveals the sensitivity of a numerical simulation to the initial preconsolidation setup. The chart presents numerical predictions for three $q^{POP}$ values: 350kPa (reference simulation), 300kPa and 400kPa. The sensitivity of the OCR profile to the specified values of $q^{POP}$ is presented in Figure 5.25(a).
Appendix A

Determination of undrained shear strength

A.1 Non-uniqueness of undrained shear strength

It has been widely recognized that the in situ behavior of soils may be significantly different from that of laboratory samples. This can be mainly attributed to the quality of the intact specimens which may depend on drilling and sampling methods and sample geometry (DeGroot and Sandven, 2004). The disturbance of samples may increase during their insertion into a sampling tube, transportation, relaxation of stresses, drying, temperature changes, trimming and, finally, their installation in the testing cells, etc. (Hight et al., 1992). Different sampling devices such as piston samplers, thin walled tubes or downhole block samplers can provide specimens for which different magnitudes of preconsolidation pressure or undrained shear strength $s_u$ are measured (e.g. Hight et al., 1992; Tanaka and Tanaka, 1999). Experience shows that sample disturbance may lead to underestimation of the apparent preconsolidation pressure or undrained shear strength (Karlsrud, 1999; Fioravante, 2004).

Inconsistencies in $s_u$ values measured by laboratory and field tests may also stem from the non-uniqueness of this property. The undrained shear strength $s_u$ is not a unique soil parameter (Wroth, 1984; Jamiolkowski et al., 1985), as it depends on the type of test, which involves particular strain paths (cf. Figure A.1). The differences in interpreted results also stems from the time-dependent behavior of soils (e.g. Vaid and Campanella, 1977; Leroueil, 1988; Sheahan et al., 1996; Penumadu et al., 1998). The undrained shear strength increases linearly with the logarithm of the shear strain (Bjerrum, 1972; Nakase and Kamei, 1986). For instance, the testing speed for SBPT or CPTU can be one or more orders of magnitude greater than that used in the triaxial compression test TC ($\dot{\varepsilon} = 0.01\%$/min), see Figure A.2. Laboratory tests with the use of a model pressuremeter in clays have revealed an increase of $s_u$ of about 10% for every tenfold increase of strain rate (Prapaharan et al., 1989). While the overestimation of $s_u$ derived from pressuremeter test due to the strain rate effect can be reasonably small, in the order of 10-20%, the differences for CPTU can be much larger. The extrapolated results of Bjerrum (1972) (Figure A.2) can indicate that neglecting the strain effect in the analysis of penetration may lead to the considerable overestimation of $s_u$ of about 40% with respect to the value obtained for the conventional triaxial compression. Since the undrained shear strength is a function of the stress history, the similar effects can be observed for the derived values of preconsolidation pressure $\sigma'_p$. The study carried out by Leroueil et al. (1983b) revealed the increase of
APPENDIX A. DETERMINATION OF UNDRAINED SHEAR STRENGTH

Figure A.1: Undrained shear strength in normally consolidated soil (a) as a function of shear modes for various tests: triaxial undrained compression tests (CIUC and CK₀UC), plain strain compression test (PSC), direct simple shear test (DSS) and field vane test (FVT) (after Wroth, 1984), (b) profiles for field and laboratory tests on Onsøy clay (from Lacasse et al., 1981).

about 10-14% for $\sigma'_p$ per log cycle of volumetric strain rate $\dot{\varepsilon}_v$ in the constant rate of strain oedometer test (CRS).

Figure A.2: The strain rate effect on undrained shear strength $s_u$ for different shear modes and a schematic comparison of test strain rates for triaxial compression (TC), self-boring pressuremeter (SBPT) and piezocone (CPTU).
A.2  Determination of $s_u$ from field tests

**FVT.** The interpretation of $s_u$ from the standardly used field vane test (see Figure A.3) can be carried out with the conventional formula:

$$s_u = \frac{6M}{7\pi D^3} = 0.2728 \frac{M}{D^3} \quad (A.1)$$

in which $M$ is the maximum recorded torque, and $D$ is the diameter of vane. Since the values of $s_u$ obtained with the above equation can be too conservative, Chandler (1988) suggested increasing the factor 0.2728 to 0.2897.

![Figure A.3: Standard dimensions of the most commonly used field vane test (from Chandler, 1988).](image)

**DMT.** The values of $s_u$ can be correlated with the Marchetti’s dilatometer data through the original formula suggested by Marchetti (1980):

$$s_u = 0.22\sigma'_{v0}(0.5K_D)^{1.25} \quad (A.2)$$

where $K_D$ is the horizontal stress index which is calculated based on the first dilatometer reading $p_0$ ($K_D = (p_0 - u_0)/\sigma'_{v0}$).
Appendix B

Estimation of compression index

Table B.1: Some correlation equations for estimating consolidation parameters (after Holtz et al., 1986; Bowles, 1997; Kempfert, 2006).

<table>
<thead>
<tr>
<th>Compression index</th>
<th>Applicability</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_c = 0.009(w_L - 10) )</td>
<td>Normally consolidated clays of low to medium sensitivity and ( w_L &lt; 100(\pm30% ) error)</td>
<td>Terzaghi and Peck (1967)</td>
</tr>
<tr>
<td>( C_c = 0.141G_s(\gamma_{SAT}/\gamma_D) )</td>
<td>All clays</td>
<td>Rendon-Herrero (1983)</td>
</tr>
<tr>
<td>( C_c = 0.141G_s^{1.2}[(1 + e_0)/G_s]^{2.38} )</td>
<td>All clays (94 data points)</td>
<td>Rendon-Herrero (1983)</td>
</tr>
<tr>
<td>( C_c = 1.15(e_0 - 0.35) )</td>
<td>All clays, uniformly packed</td>
<td>Nishida (1956)</td>
</tr>
<tr>
<td>( C_c = 1.15(e - 0.91) )</td>
<td>All clays, loosely packed ( e &gt; 0.9 )</td>
<td>Nishida (1956)</td>
</tr>
<tr>
<td>( C_c = 0.009w_n + 0.005w_L )</td>
<td>All clays</td>
<td>Koppula (1986)</td>
</tr>
<tr>
<td>( C_c = 0.00234w_LG_s )</td>
<td>All inorganic clays</td>
<td>Nagaraj and Murthy (1985)</td>
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<tr>
<td>( C_c = 0.37(e_0 + 0.003w_L + 0.0004w_n - 0.34) )</td>
<td>678 data points</td>
<td>Azzouz et al. (1976)</td>
</tr>
<tr>
<td>( C_c = 0.00917(w_L - 13) )</td>
<td>56 data points</td>
<td>Mayne (1980)</td>
</tr>
<tr>
<td>( C_c = -0.156 + 0.411e + 0.00058w_L )</td>
<td>72 data points</td>
<td>Al-Khafaji and Andersland (1992)</td>
</tr>
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<td>( C_c = 0.156e + 0.0107 )</td>
<td>Remoulded all clays</td>
<td>Azzouz et al. (1976)</td>
</tr>
<tr>
<td>( C_c = 0.007(w_L - 10) )</td>
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<td>Azzouz et al. (1976)</td>
</tr>
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<td>( C_c = 0.007(w_L - 7) )</td>
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<td>Skempton (1944)</td>
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<td>Azzouz et al. (1976)</td>
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<tr>
<td>( C_c = 0.5(I_P/100)G_s )</td>
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<td>Wroth and Wood (1978)</td>
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<td>( C_c = 0.046 + 0.0104I_P )</td>
<td>Best for ( I_P &lt; 50% )</td>
<td>Nakase (1988)</td>
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<td>( C_c = 1.15e )</td>
<td>Deformable but incompressible soil</td>
<td>Nishida (1956)</td>
</tr>
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<td>( C_c = 0.0115w_n )</td>
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<td>Hough (1969)</td>
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<tr>
<td>( C_c = PI/74 )</td>
<td>Data from different soils</td>
<td>Kulhawy and Mayne (1990)</td>
</tr>
</tbody>
</table>
### APPENDIX B. ESTIMATION OF COMPRESSION INDEX

\[ C_c = 0.0093w_n \quad \text{Chicago clays and Alberta Province in Canada (109 data points)} \]
\[ C_c = 17.66/100000w_n^2 + 0.00593w_n - 0.135 \quad \text{Chicago clays} \]
\[ C_c = 0.015(w_n - 8) \quad \text{Soils in Taipei} \]
\[ C_c = 0.54(\gamma - 0.23) \quad \text{Soils in Taipei} \]
\[ C_c = 0.0046(w_L - 9) \quad \text{Brazilian clays} \]
\[ C_c = 1.21 + 1.005(\gamma - 1.87) \quad \text{Motley clays from Sao Paulo} \]
\[ C_c = 0.0037(w_L + 25.5) \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = 0.0135w_n - 0.1169 \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = 0.0042I_P + 0.165 \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = 0.46\gamma - 0.049G_s + 0.0023 \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = 0.4965\gamma - 0.0014w_n - 0.123 \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = -0.247\gamma + 0.004w_L + 0.01w_n + 0.021 \quad \text{Cincinnati and Northern Kentucky} \]
\[ C_c = 0.0018/(1 - 0.0109n) \quad \text{Remoulded clays in Japan (Ariake)} \]
\[ C_c = 0.00269/(1 - 0.0115n) \quad \text{Undisturbed clays in Japan (Ariake)} \]
\[ C_c = 0.02 + 0.014I_P \quad \text{North Atlantic clays} \]
\[ C_c = 0.02 + 0.014I_P \quad \text{North Atlantic clays} \]
\[ C_c = 0.011(w_L - 6.36) \quad \text{East coast of Korea} \]
\[ C_c = 0.01(w_n + 2.83) \quad \text{East coast of Korea} \]
\[ C_c = 0.39(\gamma - 0.13) \quad \text{East coast of Korea} \]
\[ C_c = -0.16\gamma + 2.4 \quad \text{East coast of Korea} \]
\[ C_c = 0.0098w_n + 0.194e - 0.0025I_P - 0.256 \quad \text{East coast of Korea} \]
\[ C_c = 0.012(w_L + 16.4) \quad \text{South coast of Korea} \]
\[ C_c = 0.013(w_n - 3.85) \quad \text{South coast of Korea} \]
\[ C_c = 0.54(\gamma - 0.37) \quad \text{South coast of Korea} \]
\[ C_c = -0.0003w_n + 0.538e + 0.002w_L - 0.3 \quad \text{South coast of Korea} \]
\[ C_c = 0.165 + 0.01I_P \quad \text{South coast of Korea} \]
\[ C_c = 0.01(w_L - 10.9) \quad \text{West coast of Korea} \]
\[ C_c = 0.011(w_n - 11.22) \quad \text{West coast of Korea} \]
\[ C_c = 0.37(\gamma - 0.28) \quad \text{West coast of Korea} \]
\[ C_c = -0.066\gamma + 1.15 \quad \text{West coast of Korea} \]
\[ C_c = 0.0038w_n + 0.12e + 0.0065w_L - 0.248 \quad \text{West coast of Korea} \]

References:
- Koppula (1981)
- Azzouz et al. (1976)
- Moh et al. (1989)
- Azzouz et al. (1976)
- Dayal (2006)
- Park and Koumoto (2004)
Table B.2: Some correlation equations for estimating recompression index (after Holtz et al., 1986; Bowles, 1997; Kempfert, 2006).

<table>
<thead>
<tr>
<th>Recompression index</th>
<th>Regions of applicability</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s = 0.000463w_LG_s$</td>
<td></td>
<td>Nagaraj and Murthy (1985)</td>
</tr>
<tr>
<td>$C_s = 0.00194(\text{PI} - 4.6)$</td>
<td>Best for PI &lt; 50%</td>
<td>Nakase (1988)</td>
</tr>
<tr>
<td>$C_s = \text{PI}/370$</td>
<td>Data from different soils</td>
<td>Kulhawy and Mayne (1990)</td>
</tr>
</tbody>
</table>

$C_r/C_c$ is observed between 0.05 and 0.5, with typical values between 0.1-0.25 and the average 0.2 (cf. Mayne, 1980, 2007; Kempfert, 2006); lower values are observed for cemented soils.

PI - plastic index in [%], $w_L$ - liquid limit in [%], $w_n$ natural water content in [%].
## Appendix C

### Estimation of shear wave velocity

*Table C.1: Typical values of shear wave velocity and density for different geomaterials (after Lavergne, 1986).*

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Shear wave velocity $V_s$ [m/s]</th>
<th>Dry unit weight $\gamma$ [g/cm$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Screes, organic topsoil</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Dry sands</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Wet sands</td>
<td>400</td>
<td>1200</td>
</tr>
<tr>
<td>Clays</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>Marls</td>
<td>750</td>
<td>1500</td>
</tr>
<tr>
<td>Sandstones</td>
<td>1200</td>
<td>2800</td>
</tr>
<tr>
<td>Lime stones</td>
<td>2000</td>
<td>3300</td>
</tr>
<tr>
<td>Chalk</td>
<td>1100</td>
<td>1300</td>
</tr>
<tr>
<td>Salt</td>
<td>2500</td>
<td>3100</td>
</tr>
<tr>
<td>Anhydrite</td>
<td>2200</td>
<td>3100</td>
</tr>
<tr>
<td>Dolomite</td>
<td>1900</td>
<td>3600</td>
</tr>
<tr>
<td>Granite</td>
<td>2500</td>
<td>3300</td>
</tr>
<tr>
<td>Basalte</td>
<td>2800</td>
<td>3400</td>
</tr>
<tr>
<td>Carbon</td>
<td>1000</td>
<td>1400</td>
</tr>
<tr>
<td>Ice</td>
<td>1700</td>
<td>1900</td>
</tr>
</tbody>
</table>
**Table C.2:** Typical values of shear wave velocity for different geomaterials (after FOWG, 2003).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top soil layers (3 to 6m), lightly compact., desagregated, unsat.</td>
<td>110</td>
<td>480</td>
</tr>
<tr>
<td>Ballast (gravelly or sandy), unsaturated</td>
<td>220</td>
<td>450</td>
</tr>
<tr>
<td>Ballast, saturated</td>
<td>400</td>
<td>600</td>
</tr>
<tr>
<td>Ballast cemented</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>Silt from the lake bottom, not completely saturated</td>
<td>290</td>
<td>540</td>
</tr>
<tr>
<td>Silt from the lake bottom, saturated</td>
<td>390</td>
<td>530</td>
</tr>
<tr>
<td>Silt at banks, unsaturated</td>
<td>120</td>
<td>400</td>
</tr>
<tr>
<td>Moraine</td>
<td>500</td>
<td>1150</td>
</tr>
<tr>
<td>Loess</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>Marl and mollase sandstone, soft, desagregated</td>
<td>520</td>
<td>1050</td>
</tr>
<tr>
<td>Marl , not desagregated</td>
<td>1000</td>
<td>1900</td>
</tr>
<tr>
<td>Sandstone, hard</td>
<td>1100</td>
<td>2200</td>
</tr>
<tr>
<td>Molasse at plateau</td>
<td>600</td>
<td>2500</td>
</tr>
<tr>
<td>Schist</td>
<td>1100</td>
<td>3100</td>
</tr>
<tr>
<td>Limestone</td>
<td>1800</td>
<td>3700</td>
</tr>
<tr>
<td>Gneiss</td>
<td>1900</td>
<td>3500</td>
</tr>
<tr>
<td>Granite</td>
<td>2500</td>
<td>3900</td>
</tr>
</tbody>
</table>

**Table C.3:** Typical values of shear wave velocity for different geomaterials (after Lindeburg, 2001).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard rocks</td>
<td>&gt;1400</td>
<td></td>
</tr>
<tr>
<td>Firm to hard rocks</td>
<td>700</td>
<td>1400</td>
</tr>
<tr>
<td>Gravelly soils and soft to firm rocks</td>
<td>375</td>
<td>700</td>
</tr>
<tr>
<td>Stiff clays and sandy soils</td>
<td>200</td>
<td>375</td>
</tr>
<tr>
<td>Soft soils</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Very soft soils</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

**Table C.4:** Typical values of shear wave velocity for different geomaterials (after NAVFAC, 1986).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard rock</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>760</td>
<td>1500</td>
</tr>
<tr>
<td>Very dense soil and soft rock (N_{60} &gt; 50, s_u &gt; 100kPa)</td>
<td>360</td>
<td>760</td>
</tr>
<tr>
<td>Stiff soil (15 &gt; N_{60} &lt; 50, 50 &lt; s_u &lt; 100kPa)</td>
<td>180</td>
<td>360</td>
</tr>
<tr>
<td>Soft soil (N_{60} &lt; 15, s_u &lt; 50kPa)</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>Any soil with PI&gt; 20%,w&gt; 40%, (s_u &lt; 25kPa)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Most published correlations for SPT are based on uncorrected N-values. These correlations are given in:

- for all soil types: Table C.5 (insight provided in C.1),
- for sands: Table C.6 (Figure C.2),
- for silts: Table C.7 (Figure C.3),
- for clays: Table C.8 (Figure C.4).

**Table C.5: SPT-Based correlations for all type of soils.**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Soil Type</th>
<th>Equation</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohba &amp; Toriumi (1970)</td>
<td>Japan</td>
<td>Alluvium</td>
<td>$V_s = 84N_{60}^{0.31}$</td>
<td>–</td>
</tr>
<tr>
<td>Fujiwara (1972)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 92.1N_{60}^{0.337}$</td>
<td>–</td>
</tr>
<tr>
<td>Ohsaki &amp; Iwasaki (1973)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 81.3N_{60}^{0.39}$</td>
<td>0.886</td>
</tr>
<tr>
<td>Imai and Yoshimura (1975)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 76N_{60}^{0.151}$</td>
<td>–</td>
</tr>
<tr>
<td>Imai (1977)</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>$V_s = 91N_{60}^{0.337}$</td>
<td>–</td>
</tr>
<tr>
<td>Ohta and Goto (1978)</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>$V_s = 85.34N_{60}^{0.348}$</td>
<td>0.719</td>
</tr>
<tr>
<td>Imai &amp; Tonouchi (1982)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 97N_{60}^{0.314}$</td>
<td>0.868</td>
</tr>
<tr>
<td>Seed et al. (1983)</td>
<td>–</td>
<td>–</td>
<td>$V_s = 56N_{60}^{0.5}$</td>
<td>–</td>
</tr>
<tr>
<td>Jinan (1987)</td>
<td>Shanghai</td>
<td>Soft Holocene Deposits</td>
<td>$V_s = 116.1(N_{60}^{0.3185} + 0.7}$</td>
<td>–</td>
</tr>
<tr>
<td>Athanasopoulos (1995)</td>
<td>Greece</td>
<td>–</td>
<td>$V_s = 107.6N_{60}^{0.36}$</td>
<td>–</td>
</tr>
<tr>
<td>Sisman (1995)</td>
<td>–</td>
<td>–</td>
<td>$V_s = 32.8N_{60}^{0.51}$</td>
<td>–</td>
</tr>
<tr>
<td>Iyisan (1996)</td>
<td>–</td>
<td>–</td>
<td>$V_s = 51.5N_{60}^{1.516}$</td>
<td>–</td>
</tr>
<tr>
<td>Kiku et al. (2001)</td>
<td>Turkey</td>
<td>–</td>
<td>$V_s = 68.3N_{60}^{2.92}$</td>
<td>–</td>
</tr>
<tr>
<td>Hasancebi and Ulusay (2007)</td>
<td>Turkey</td>
<td>Quaternary Alluvium and Detritus</td>
<td>$V_s = 90N_{60}^{0.308}$</td>
<td>0.73</td>
</tr>
</tbody>
</table>

1 Referenced by Sykora (1987)
2 Referenced by Hasancebi and Ulusay (2007)
## APPENDIX C. ESTIMATION OF SHEAR WAVE VELOCITY

### Table C.6: SPT-Based correlations for sands.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Soil Type</th>
<th>Equation</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shibata (1970)</td>
<td>–</td>
<td>–</td>
<td>$V_s = 31.7N_{60}^{0.54}$</td>
<td>–</td>
</tr>
<tr>
<td>Ohta et al. (1972)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 87.2N_{60}^{0.36}$</td>
<td>–</td>
</tr>
<tr>
<td>Imai (1977)²</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>$V_s = 80.6N_{60}^{0.331}$</td>
<td>–</td>
</tr>
<tr>
<td>Ohta &amp; Goto (1978b)¹</td>
<td>Japan</td>
<td>Quaternary Alluvium</td>
<td>$V_s = 88.4N_{60}^{0.333}$</td>
<td>0.719</td>
</tr>
<tr>
<td>Imai &amp; Tonouchi (1982)¹</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>$V_s = 87.8N_{60}^{0.314}$</td>
<td>0.69</td>
</tr>
<tr>
<td>Sykora &amp; Stokoe (1983)¹</td>
<td>–</td>
<td>–</td>
<td>$V_s = 100.5N_{60}^{0.29}$</td>
<td>0.84</td>
</tr>
<tr>
<td>Okamoto et al. (1989)</td>
<td>Japan</td>
<td>–</td>
<td>$V_s = 125N_{60}^{0.3}$</td>
<td>–</td>
</tr>
<tr>
<td>Lee (1990)</td>
<td>Taiwan</td>
<td>–</td>
<td>$V_s = 57.4N_{60}^{0.49}$</td>
<td>0.62</td>
</tr>
<tr>
<td>Lee (1992)</td>
<td>Taiwan</td>
<td>–</td>
<td>$V_s = 157.13 + 4.74N_{60}$</td>
<td>0.691</td>
</tr>
<tr>
<td>Pitilakis et al. (1999)</td>
<td>Greece</td>
<td>Alluvium</td>
<td>$V_s = 145N_{60}^{0.178}$</td>
<td>0.70</td>
</tr>
<tr>
<td>Hasancebi and Ulusay (2007)</td>
<td>Turkey</td>
<td>Quaternary Alluvium and Detritus</td>
<td>$V_s = 90.82N_{60}^{0.319}$</td>
<td>0.65</td>
</tr>
<tr>
<td>Dikmen (2009)</td>
<td>Western central Anatolia, Turkey</td>
<td>–</td>
<td>$V_s = 73N_{60}^{0.33}$</td>
<td>–</td>
</tr>
</tbody>
</table>

¹ Referred by Sykora (1987)
² Referred by Hasancebi and Ulusay (2007)

### Table C.7: SPT-Based correlations for silts.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Soil Type</th>
<th>Equation</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lee (1990)</td>
<td>Taiwan</td>
<td>–</td>
<td>$V_s = 105.64N_{60}^{0.32}$</td>
<td>0.73</td>
</tr>
<tr>
<td>Lee (1992)</td>
<td>Taiwan</td>
<td>–</td>
<td>$V_s = 103.99(N_{60} + 1)^{0.334}$</td>
<td>0.798</td>
</tr>
<tr>
<td>Pitilakis et al. (1999)</td>
<td>Greece</td>
<td>Alluvium</td>
<td>$V_s = 145N_{60}^{0.178}$</td>
<td>0.70</td>
</tr>
<tr>
<td>Dikmen (2009)</td>
<td>Western central Anatolia, Turkey</td>
<td>–</td>
<td>$V_s = 60N_{60}^{0.36}$</td>
<td>–</td>
</tr>
</tbody>
</table>
Table C.8: SPT-Based correlations for clays.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Soil Type</th>
<th>Equation</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imai (1977)</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>(V_s = 80.2N_{60}^{0.292})</td>
<td>–</td>
</tr>
<tr>
<td>Ohta &amp; Goto (1978b)</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>(V_s = 86.9N_{60}^{0.333})</td>
<td>0.719</td>
</tr>
<tr>
<td>Imai &amp; Tonouchi (1982)</td>
<td>Japan</td>
<td>Quaternary and Pleistocene Alluvium</td>
<td>(V_s = 107N_{60}^{0.274})</td>
<td>0.721</td>
</tr>
<tr>
<td>Lee (1990)</td>
<td>Taiwan</td>
<td>–</td>
<td>(V_s = 114.43N_{60}^{0.31})</td>
<td>0.62</td>
</tr>
<tr>
<td>Lee (1992)</td>
<td>Taiwan</td>
<td>–</td>
<td>(V_s = 138.36(N_{60} + 1)^{0.242})</td>
<td>0.695</td>
</tr>
<tr>
<td>Athanasopoulos (1995)</td>
<td>Greece</td>
<td>Alluvium</td>
<td>(V_s = 76.55N_{60}^{0.445})</td>
<td>–</td>
</tr>
<tr>
<td>Pitilakis et al. (1999)</td>
<td>Greece</td>
<td>Alluvium</td>
<td>(V_s = 132N_{60}^{0.271})</td>
<td>0.75</td>
</tr>
<tr>
<td>Hasancebi and Ulusay (2007)</td>
<td>Turkey</td>
<td>Quaternary and Detritus</td>
<td>(V_s = 97.80N_{60}^{0.269})</td>
<td>0.75</td>
</tr>
<tr>
<td>Dikmen (2009)</td>
<td>Western central Anatolia, Turkey</td>
<td>–</td>
<td>(V_s = 44N_{60}^{0.48})</td>
<td>–</td>
</tr>
</tbody>
</table>

1 Referenced by Sykora (1987)
2 Referenced by Hasancebi and Ulusay (2007)

Figure C.1: Comparison of "all soil type"-correlations for estimating \(V_s\) from SPT data (correlations are given in Table C.5).
Figure C.2: Comparison of -correlations for estimating $V_s$ in sands from SPT data (correlations are given in Table C.6).

Figure C.3: Comparison of -correlations for estimating $V_s$ in silts from SPT data (correlations are given in Table C.7).
**Figure C.4:** Comparison of -correlations for estimating \( V_s \) in clays from SPT data (correlations are given in Table C.8).

**CPT.** A number of correlations can be used to estimate shear wave velocity based on CPT data. Correlations were developed for a generic soil types: Table C.9, for sands: Table C.10, for clays: Table C.11. Graphical comparisons of these correlations are presented in Figure for sands.

**Table C.9:** CPT-based correlations for all soil types.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Soil Type</th>
<th>( V_s [\text{m/s}] )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hegazy and Mayne (1995)</td>
<td>sands, silts, clays, mixed soil types</td>
<td>(10.1 ( \log q_t - 11.4 ))(^{1.67}(f_s/q_t \cdot 100)^{0.3} )</td>
<td>( q_t, f_s ) in kPa</td>
</tr>
<tr>
<td>Mayne (2006a)</td>
<td>saturated clays, silts, and sands</td>
<td>118.8 ( \log f_s + 18.5 )</td>
<td>( f_s ) in kPa</td>
</tr>
</tbody>
</table>

**Table C.10:** CPT-based correlations for sands.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Soil Type</th>
<th>( V_s [\text{m/s}] )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baldi et al. (1989)</td>
<td>Giola Taura Sand w/Gravel, Po River sands</td>
<td>( 277 q_t^{0.13} \sigma_{vo}^{0.27} )</td>
<td>( q_t, \sigma_{vo} ) in MPa</td>
</tr>
<tr>
<td>Hegazy and Mayne (1995)</td>
<td>–</td>
<td>13.18( q_t^{0.192} \sigma_{vo}^{0.179} )</td>
<td>( q_t, \sigma_{vo} ) in kPa</td>
</tr>
<tr>
<td>Hegazy and Mayne (1995)</td>
<td>–</td>
<td>12.02( q_t^{0.319} f_s^{-0.0466} )</td>
<td>( q_t, f_s ) in kPa</td>
</tr>
</tbody>
</table>
### Table C.11: CPT-based correlations for clays.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Soil Type</th>
<th>( V_s [\text{m/s}] )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mayne and Rix (1993)</td>
<td>soft to firm to stiff intact clays to fissured clays, Piedmont silts</td>
<td>( 9.44 q_t^{0.435} e_0^{-0.532} )</td>
<td>( q_t ) in kPa</td>
</tr>
<tr>
<td>Mayne and Rix (1993)</td>
<td>–</td>
<td>( 1.75 q_t^{0.627} )</td>
<td>( q_t ) in kPa, ( n = 481, R = 0.86 )</td>
</tr>
<tr>
<td>Hegazy and Mayne (1995)</td>
<td>–</td>
<td>( 14.13 q_t^{0.359} e_0^{-0.473} )</td>
<td>( q_t ) in kPa</td>
</tr>
<tr>
<td>Hegazy and Mayne (1995)</td>
<td>–</td>
<td>( 3.18 q_t^{0.549} f_s^{0.025} )</td>
<td>( q_t, f_s ) in kPa</td>
</tr>
</tbody>
</table>

### Figure C.5: Comparison of empirical correlations for estimating \( V_s \) from CPT data: \( q_t \) and \( f_s \), plotted for different values of friction ratio \( R_f = f_s/q_t \times 100 \) (correlations are given in Table C.9 and C.10).
Figure C.6: Comparison of empirical correlations for estimating $V_s$ in sands from CPT data (correlations are given in Table C.10).

Figure C.7: Comparison of empirical correlations for estimating $V_s$ in sands from CPT data (correlations are given in Table C.11).


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