

Parameter determination for advanced constitutive models in ZSoil

Part 2



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- ☐ Parameter identification
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Stress dependent stiffness: different formulations

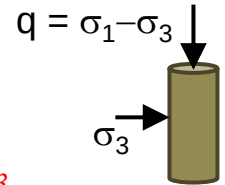
Default formulation in **HS SmallStrain** :

$$E = E^{ref} \left(\frac{\sigma'_3 + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^m$$

Advantage:

m and E_{50} , E_{ur}

easy to identify from triaxial test since σ_3 is constant during triaxial compression test



Drawback:

Stress rotation issue in:

- modeling of OC soils with initial $K_0 > 1$
- earthquake engineering

Use the formulation with mean stress p' as the reference stress instead (custom option)

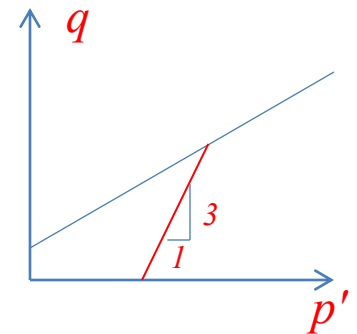
Custom option in **HS SmallStrain** and default formulation in **Densification model** :

$$E = E^{ref} \left(\frac{p'}{\sigma^{ref}} \right)^{m_p}$$

Drawback:

m and E_{50} , E_{ur}

not straightforward to identify from triaxial test p' varies during triaxial compression test



Stress dependent stiffness: different formulations

Custom option in HS SmallStrain: p – dependency

HS-small strain stiffness

Standard HS model setup

Young modulus unl./rel. at ref. stress E_{ur}^{ref} 60000 [kN/m²]

Reference stress for Young modulus σ_{ref} 100 [kN/m²]

Poisson ratio unl./rel. ν_{ur} 0.2

Exponent for power law m 0.46

Lower bound stiffness cut-off at stress σ_L 10 [kN/m²]

Small strain extension

☒ Active

Initial Young modulus at ref. stress E_o^{ref} 260000 [kN/m²]

Treshold shear strain $\gamma_{0.7}$ 0.0002

Stress dependency

☐ Sigma-3 ☒ p

☒ Advanced

Help OK Cancel

Selection of stress
dependency formulation
in *Advanced* mode

Stress dependent stiffness: different formulations

p' - formulation

$$E = E^{p.ref} \left(\frac{p'}{\sigma^{ref}} \right)^{m_p}$$

σ_3 - formulation

$$E = E^{ref} \left(\frac{\sigma_3' + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^m$$



For the same σ^{ref} the **initial maximal stiffness** may be different

$$E_0^{p.ref} \neq E_0^{\sigma.ref} \quad \text{if} \quad K_0 \neq 1$$

$$m_p \neq m \quad \text{if} \quad c \neq 0$$



Transformation required

Stress dependent stiffness

Transformation $E_0 \sigma^3 (\sigma^{ref})$ to $E_p^p(\sigma^{ref})$

1. Determine: E^{ref}, m for given σ^{ref}, ϕ, c
2. Specify: K_0 and n different σ'_v
3. Make initial guess:
4. Assumption: for each eff. vertical stress σ'_v

$$E(E^{ref}, \sigma^{ref}, m) \approx E(E^{p.ref}, \sigma^{ref}, m_p)$$

5. Find $E^{p.ref}, m_p$ which minimize the error for a system of nonlinear equations

$$\sigma_{ref} := 100 \quad E_{ref} := 260000 \quad m := 0.5$$

$$\phi := 30 \text{deg} \quad c := 5 \quad a := c \cdot (\tan(\phi))^{-1}$$

$$\sigma_{v_i} := \quad K_0 := 0.6 \quad \sigma_h := K_0 \cdot \sigma_v \quad \sigma_{3_i} := \min(\sigma_{h_i}, \sigma_{v_i})$$

50
100
200

$$\sigma_h = \begin{pmatrix} 30 \\ 60 \\ 120 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 30 \\ 60 \\ 120 \end{pmatrix}$$

$$E_{p.ref} := E_{ref} \quad m_p := m$$

Given

$$E_{ref} \left(\frac{\sigma_{3_1} + a}{\sigma_{ref} + a} \right)^m - E_{p.ref} \left[\frac{(1 + 2 \cdot K_0)}{3} \cdot \sigma_{v_1}}{\sigma_{ref}} \right]^{m_p} = 0$$

$$E_{ref} \left(\frac{\sigma_{3_2} + a}{\sigma_{ref} + a} \right)^m - E_{p.ref} \left[\frac{(1 + 2 \cdot K_0)}{3} \cdot \sigma_{v_2}}{\sigma_{ref}} \right]^{m_p} = 0$$

$$E_{ref} \left(\frac{\sigma_{3_3} + a}{\sigma_{ref} + a} \right)^m - E_{p.ref} \left[\frac{(1 + 2 \cdot K_0)}{3} \cdot \sigma_{v_3}}{\sigma_{ref}} \right]^{m_p} = 0$$

$$\text{solution} := \text{Minerr}(E_{p.ref}, m_p) = \left(\begin{pmatrix} 229937.611 \\ 0.458 \end{pmatrix} \right)$$

Stress dependent stiffness

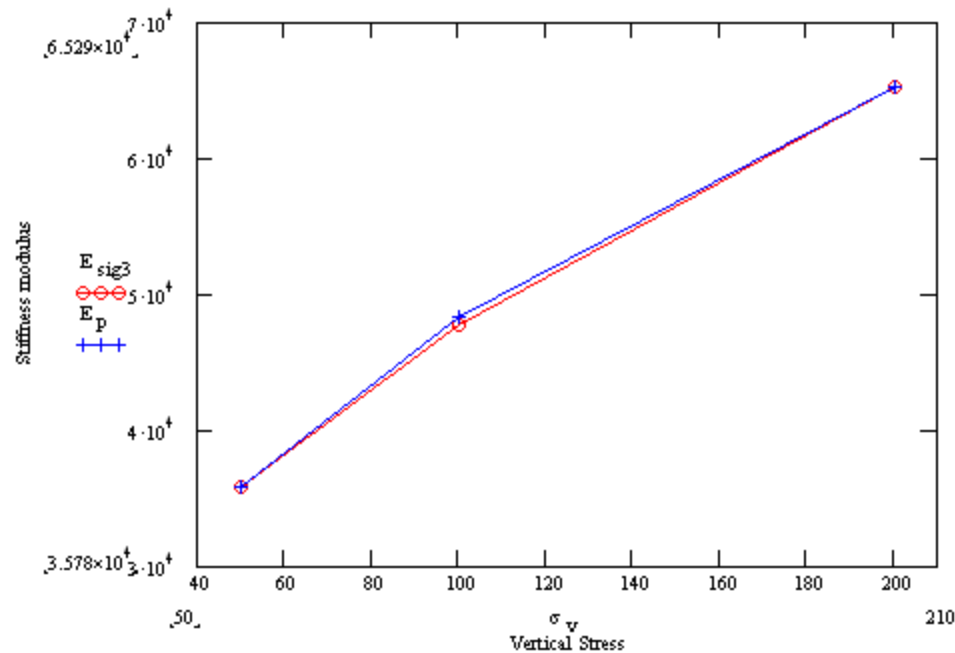
Transformation from $E_0^{\sigma^3}(\sigma^{ref})$ to $E_0^p(\sigma^{ref})$

$$E_0 = E_0^{ref} \left(\frac{\sigma_3 + c \cot \phi}{\sigma^{ref} + c \cot \phi} \right)^m$$

$$\sigma_{ref} := 100 \quad E_{ref} := 260000 \quad m := 0.5$$

$$E_0 = E_0^{p.ref} \left(\frac{p}{\sigma^{ref}} \right)^{m_p}$$

$$\sigma_{ref} := 100 \quad E_{p.ref} = 229938 \quad m_p = 0.458$$

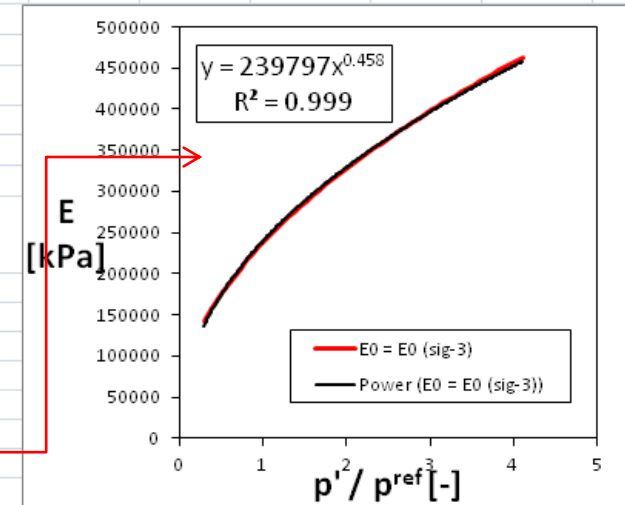


Stress dependent stiffness

Transformation from $E_0^{\sigma^3}(\sigma^{ref})$ to $E_0^p(\sigma^{ref})$ using SPREADSHEET

	A	B	C	D	E	F	G	H	I
1	σ ₃ - dependent stiffness								
2	Input:								
3		E ₀ ^{ref}	260000	kPa					
4		m	0.5	-					
5		σ ₃ ^{ref}	100	kPa					
6		c	5	kPa					
7		φ	30	deg					
8		cot(φ)	1.73	-					
9		a = c * cot φ	8.66	kPa					
10		K ₀	0.6	-					
11									
12					$E = E^{ref} \left(\frac{\sigma_3 + a}{\sigma_{ref} + a} \right)^m$				
13		Input	Computed variables					Plotted variables	
14		σ' _v	σ' _h = K ₀ σ' _v	σ ₃ = min(σ' _h , σ' _v)	E ₀ (sig-3)	p' = (σ' _v +2σ _h)/3		p' / σ _{ref}	E ₀ = E ₀ (sig-3)
15		10	6	6	95501.2	7.33			
16		20	12	12	113372.0	14.67			
17		30	18	18	128786.4	22.00			
18		40	24	24	142543.6	29.33		0.293	142543.6
19		50	30	30	155085.2	36.67		0.367	155085.2
20		60	36	36	166885.8	44.00		0.440	166885.8
21		70	42	42	177530.0	51.33		0.513	177530.0
22		80	48	48	187748.8	58.67		0.587	187748.8
23		90	54	54	197439.5	66.00		0.660	197439.5
24		100	60	60	206676.3	73.33		0.733	206676.3
25		110	66	66	215517.6	80.67		0.807	215517.6
26		120	72	72	224010.2	88.00		0.880	224010.2
27		130	78	78	232192.4	95.33		0.953	232192.4
28		140	84	84	240095.9	102.67		1.027	240095.9
29		150	90	90	247747.4	110.00		1.100	247747.4
30		160	96	96	255169.6	117.33		1.173	255169.6
31		170	102	102	262381.9	124.67		1.247	262381.9
32		180	108	108	269401.2	132.00		1.320	269401.2

p' - dependent stiffness					
Output (trendline coefficients):					
E ₀ ^{p.ref}	239797 kPa				
m _p	0.458				
E ₀ ^{p.ref} / E ₀ ^{σ₃.ref}	0.922				



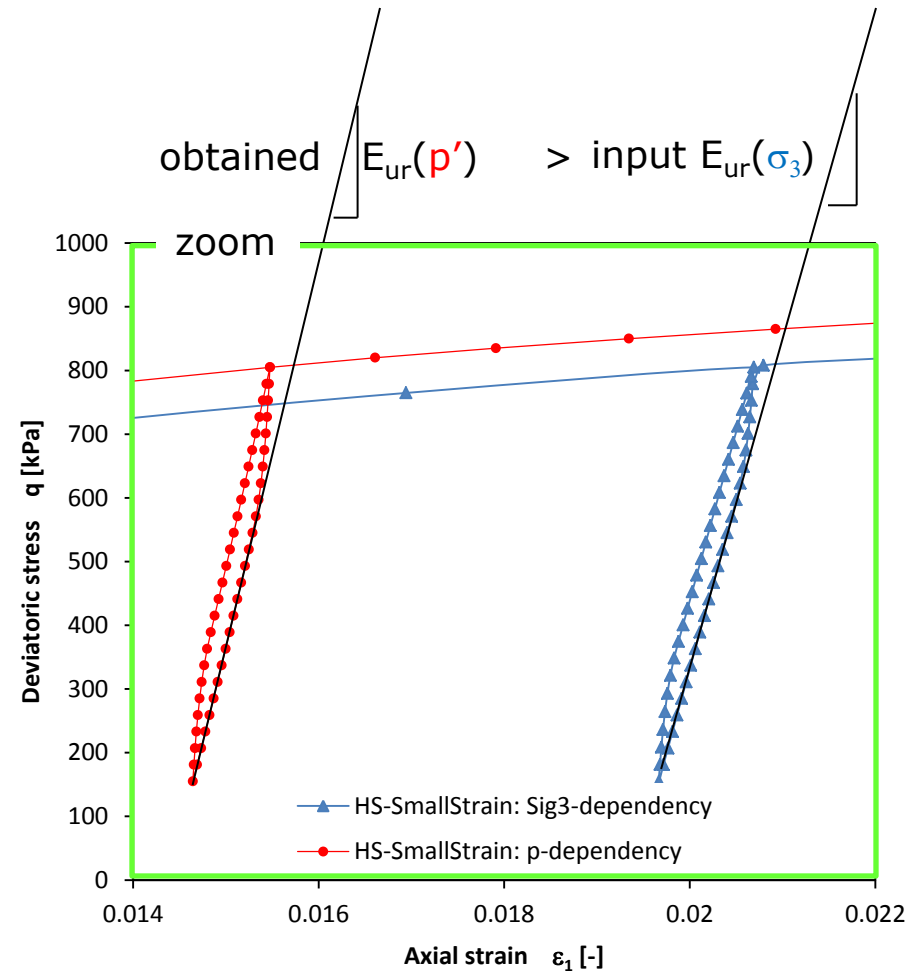
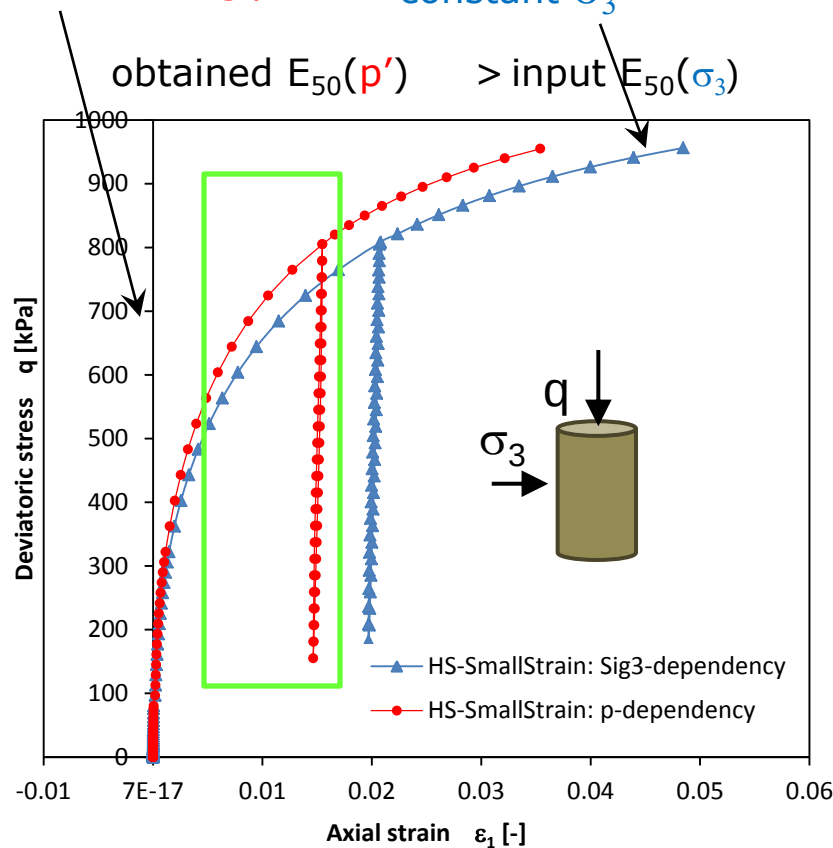
1. If $c = 0$ then $m_p = m$
2. With increasing c , larger discrepancies at lower stresses

Stress dependent stiffness: E_{50} and E_{ur} transformation

Drained triaxial test with the unloading/reloading cycle with $K_0=1$ and $c=0$.
 In such a case using the previous transformation: $E^p(\sigma^{ref}) = E^{\sigma^3}(\sigma^{ref})$ and $m_p = m$
 however **what occurs** ...

Increase of stiffness
 due to **increasing p'**

Reference curve for
 constant σ_3



Stress dependent stiffness

Transformation $E_{50}^{\sigma.ref}(\sigma^{ref})$ to $E_{50}^{p.ref}(\sigma^{ref})$

1. Determine: $q_f = \frac{2 \sin(\phi)}{1 - \sin(\phi)} (\sigma_3 + c \cot(\phi))$ for given ϕ, c and $\sigma_3 = \sigma^{ref}$

2. Take p-stress dependency law: $E_{50}^p = E_{50}^{p.ref} \left(\frac{p'}{\sigma^{ref}} \right)^{m_p}$

3. Assumptions: $E_{50}^{p.ref}(\sigma^{ref}) = E_{50}^{\sigma.ref}(\sigma^{ref})$ at $0.5q_f$

4. Introduce assumptions to the p-stress dependency law: $E_{50}^{p.ref} \approx \frac{E_{50}^{\sigma.ref}}{\left(\frac{0.5q_f}{\sigma^{ref}} \right)^{m_p}}$



The same approach can be applied to transformation of the unloading-reloading modulus

$$E_{ur}^{p.ref}(\sigma^{ref})$$

Stress dependent stiffness

Transformation $E_{50}^{\sigma.ref}(\sigma^{ref})$ to $E_{50}^{p.ref}(\sigma^{ref})$ using SPREADSHEET

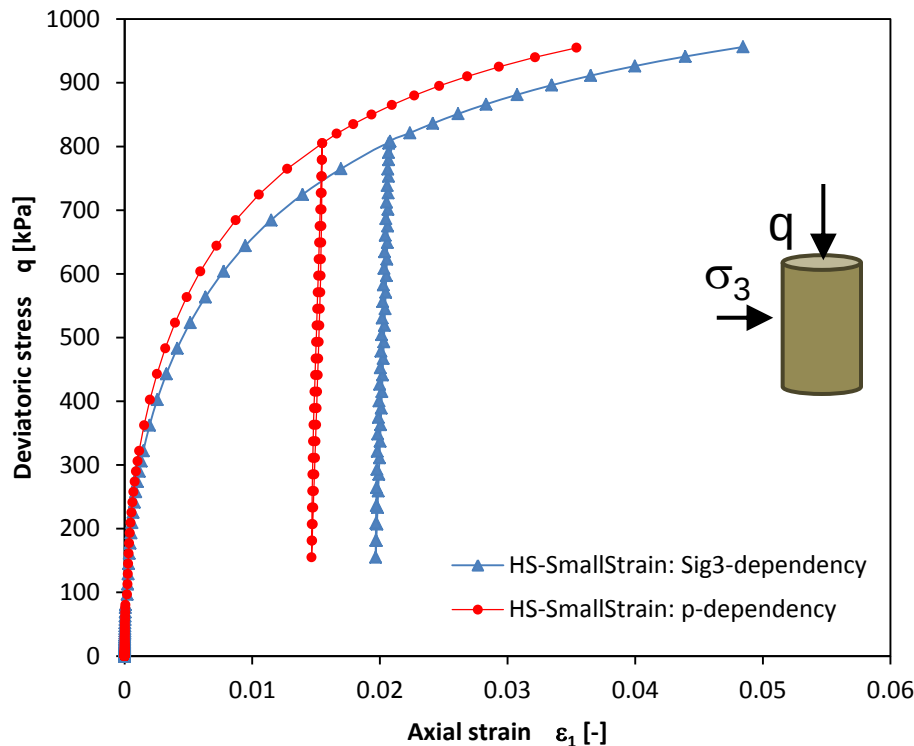
	A	B	C	D	E	F	G
1		σ_3 - dependent stiffness					
2		Input:					
3		E_{50}^{ref}	14000 kPa				
4		E_{ur}^{ref}	60000 kPa				
5		σ^{ref}	100 kPa				
6		c	0 kPa				
7		ϕ	37 deg				
8		m_p	0.46 -	*obtained for transformation of E_0			
9		$q_f(\sigma_3=\sigma^{ref})$	302.28 kPa				
10				$q_f = \frac{2\sin\phi}{1-\sin\phi}(\sigma_3 + c\cot\phi)$			
11							
12							

	J	K	L	M	N	O	P
	p' - dependent stiffness						
	Output:						
	$E_{50}^{p.ref}$	11577.5 kPa					
	$E_{ur}^{p.ref}$	49617.8 kPa					
	$E_{50}^{p.ref} / E_{50}^{\sigma.ref}$	0.827					
	$E_{50}^{p.ref} \approx \frac{E_{50}^{\sigma.ref}}{\left(\frac{0.5 q_f}{\sigma^{ref}}\right)^{m_p}}$						

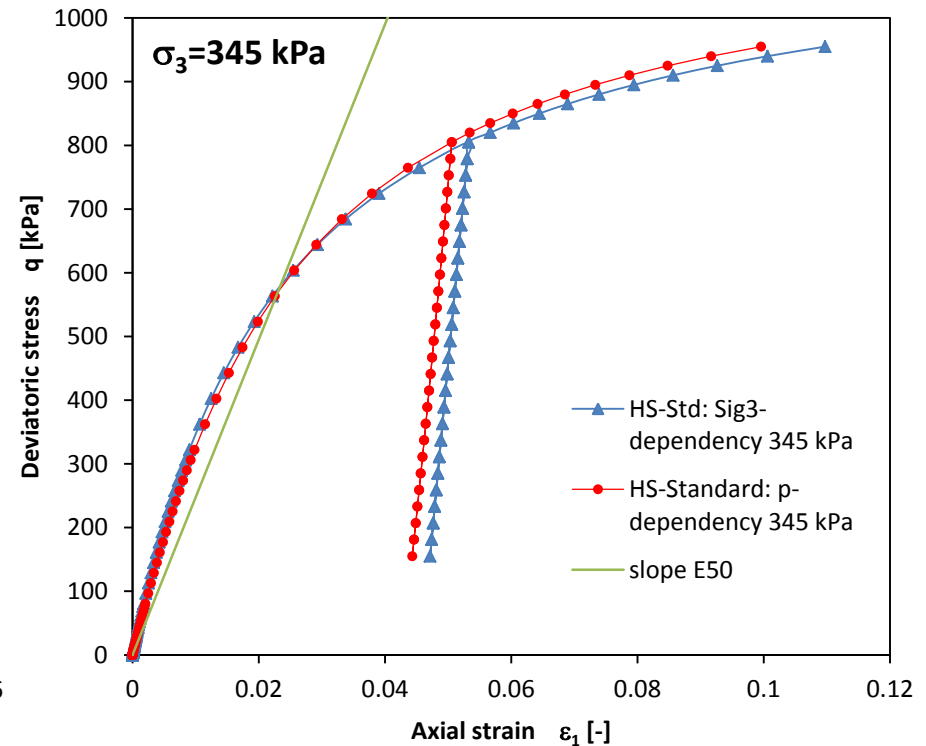
Stress dependent stiffness

Simulation of a **drained triaxial test** with the unloading/reloading cycle with $K_0=1$ and $c=0$

Without transformation
for the same input value of E_{50}



With transformation
for the same input value of E_{50}



Discrepancies may arise with the increasing cohesion

Stress dependent stiffness

Transformation $E_{50}^{\sigma.ref}(\sigma^{ref})$ to $E_{50}^{p.ref}(\sigma^{ref})$

If $0.5q_f > \sigma_3$ e.g. for $(\phi > 30^\circ, c = 0)$

$$E_{50}^{p.ref} \approx \frac{E_{50}^{\sigma.ref}}{\left(\frac{0.5q_f}{\sigma^{ref}}\right)^{m_p}} \quad \text{gives} \quad \frac{E_{50}^{p.ref}}{E_{50}^{\sigma.ref}} < 1$$

else it would give $\frac{E_{50}^{p.ref}}{E_{50}^{\sigma.ref}} > 1$



so if

$$0.5q_f < \sigma_3$$

apply

$$E_{50}^{p.ref} \approx \frac{E_{50}^{\sigma.ref}}{\left(\frac{\sigma^{ref}}{0.5q_f}\right)^{m_p}}$$

Stress dependent stiffness

Transformation from $E^{\sigma^3}(\sigma^{ref})$ to $E^p(\sigma^{ref})$

$$E = E^{p.ref} \left(\frac{p^*}{\sigma^{ref}} \right)^{m_p} \quad \text{applies only to } E_0, E_{ur}, E_{50}$$

In the current version (v2012), the evolution of the preconsolidation parameter remains as follows:

$$dp_c = -H \left(\frac{p_c + c \cot \phi}{\sigma_{ref} + c \cot \phi} \right)^m$$

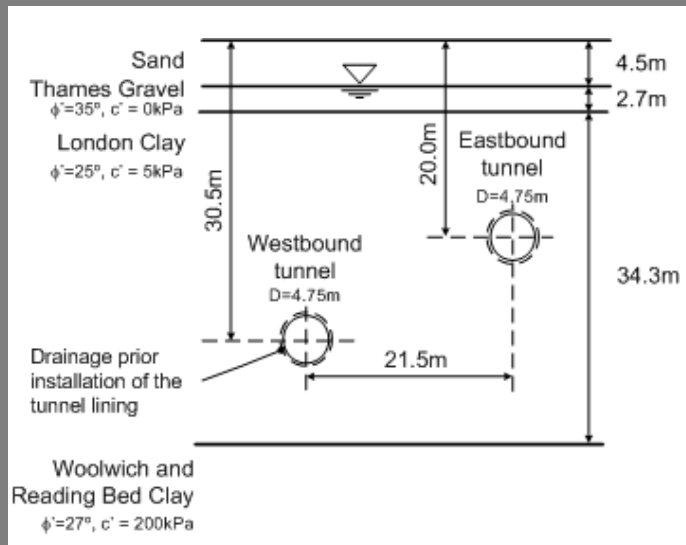
Note that parameter **transformation should not apply to E_{oed}** because this input parameter is not a model parameter and it only serves as the reference value for calibrating parameters M and H .

NB. E_{oed} is referred to the vertical stress in the oedometric test.

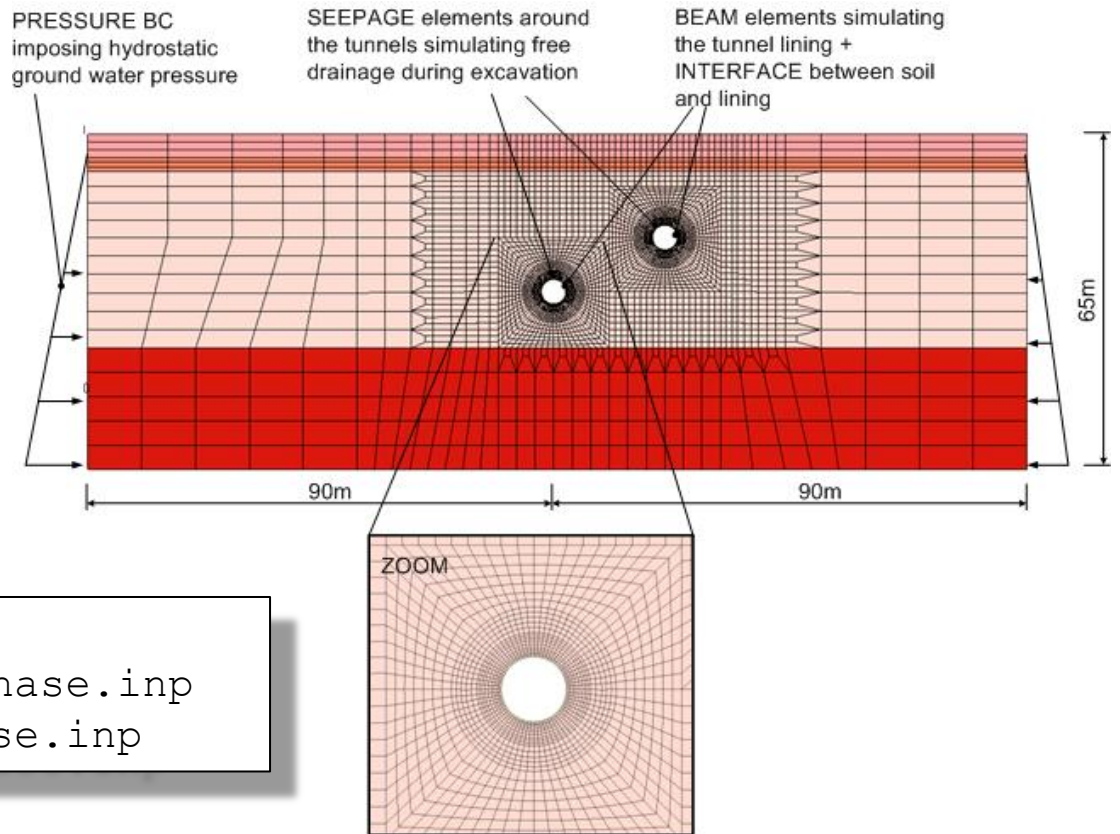
Stress dependent stiffness: Comparison of different formulations

London Clay - tunnel excavation (Addenbrooke et al., 1997)

Problem statement



FE mesh

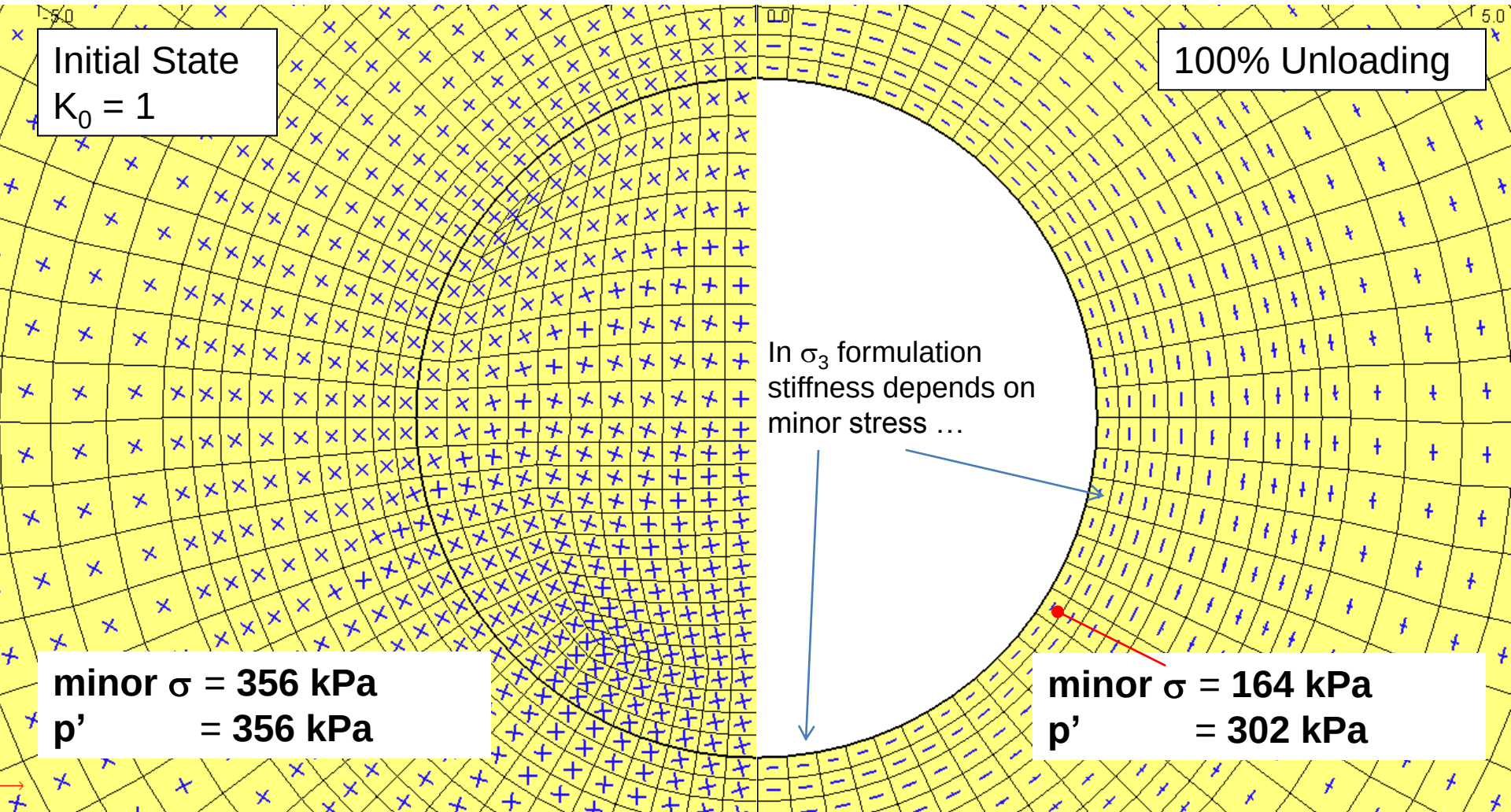


Input files:

HS-small-Exc-London-Clay-2phase.inp

HS-std-Exc-London-Clay-2phase.inp

Stress dependent stiffness: Evolution of principal stresses



STRESSES

License : GEOMOD2011 Project : HS-small-Exc-London-Clay-2phase Date : 25.8.2012 0:40

Stress dependent stiffness

Parameter migration for London clay

σ_3 - dependent stiffness		
Input:		
E_0^{ref}	390000	kPa
m	0.75	-
σ_3^{ref}	360	kPa
c	5	kPa
ϕ	25	deg
$\cot(\phi)$	2.14	-
$a = c * \cot \phi$	10.72	kPa
K_0	1	-

p' - dependent stiffness		
Output (trendline coefficients):		
$E_0^{p.ref}$	390000	kPa
m_p	0.706	-
$E_0^{p.ref} / E_0^{\sigma_3.ref}$	1	

The same
 E_0^{ref}

σ_3 - dependent stiffness		
Input:		
E_{50}^{ref}	35000	kPa
E_{ur}^{ref}	75000	kPa
σ^{ref}	360	kPa
c	5	kPa
ϕ	25	deg
m_p	0.706	-
$q_f (\sigma_3 = \sigma^{ref})$	542.71	kPa

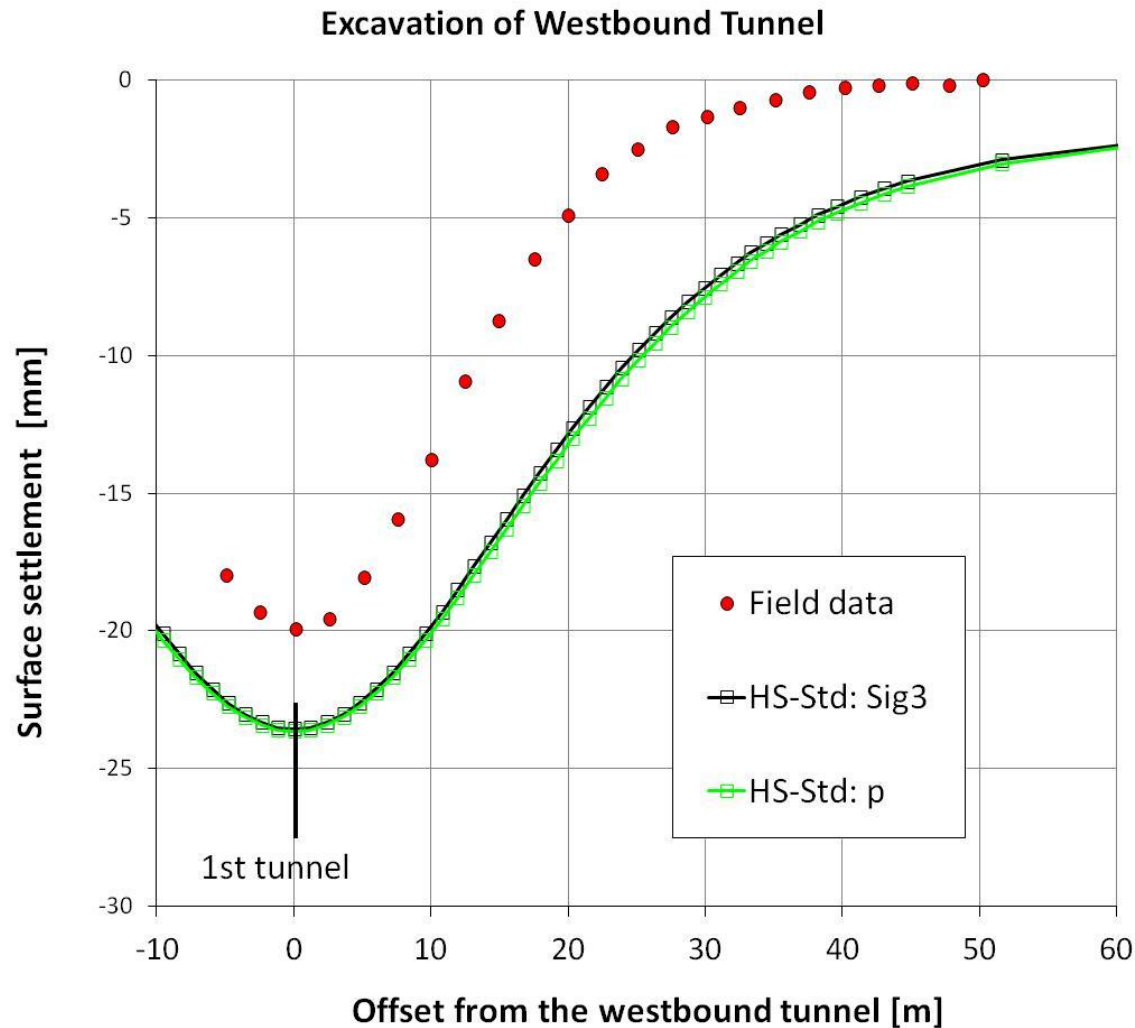
*obtained for transformation of E_0

$$q_f = \frac{2\sin\phi}{1-\sin\phi} (\sigma_3 + c\cot\phi)$$

p' - dependent stiffness		
Output:		
$E_{50}^{p.ref}$	28668	kPa
$E_{ur}^{p.ref}$	61431	kPa
$E_{50}^{p.ref} / E_{50}^{\sigma_3.ref}$	0.819	

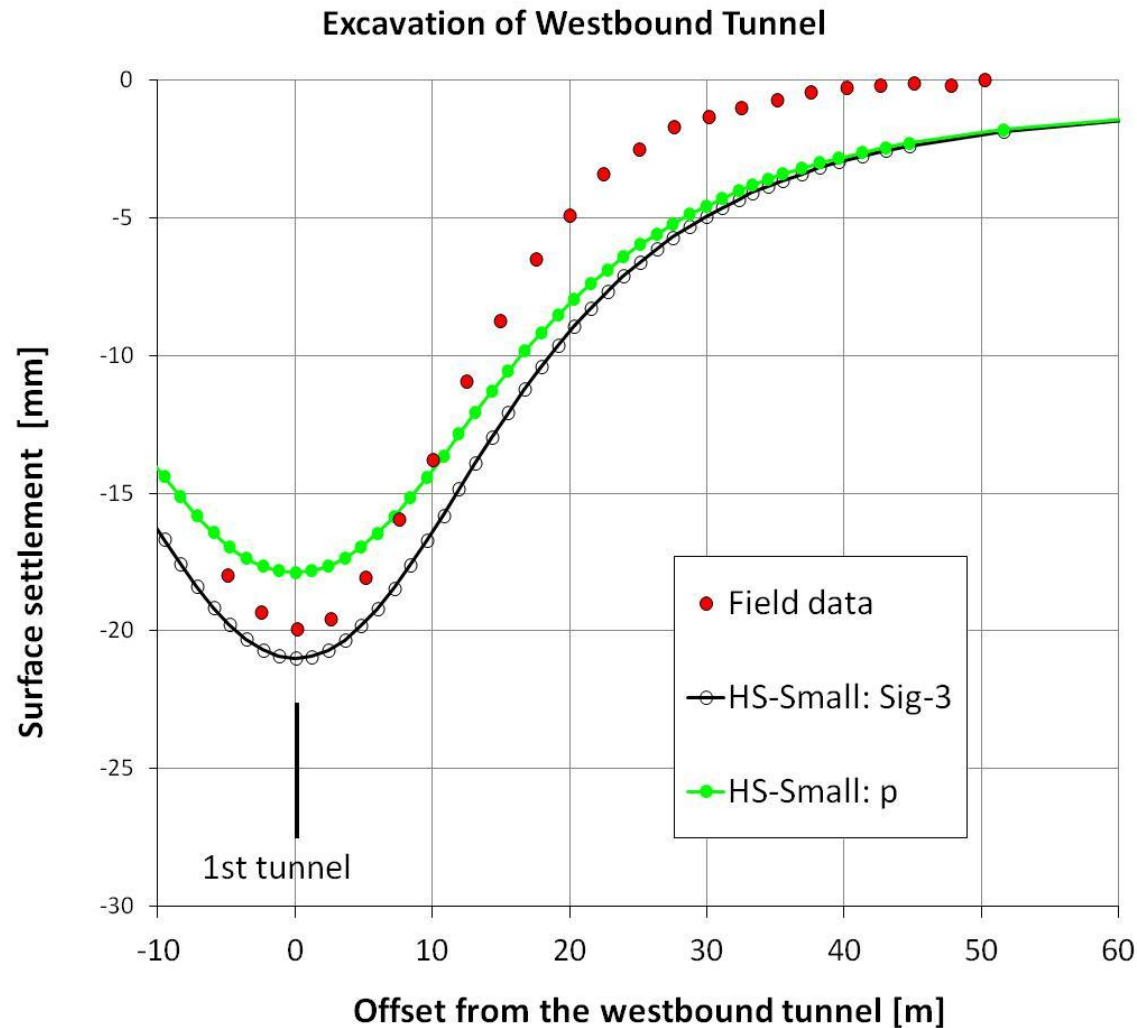
Different
 E_{50}^{ref} and E_{ur}^{ref}

Stress dependent stiffness: σ_3 vs p' formulation



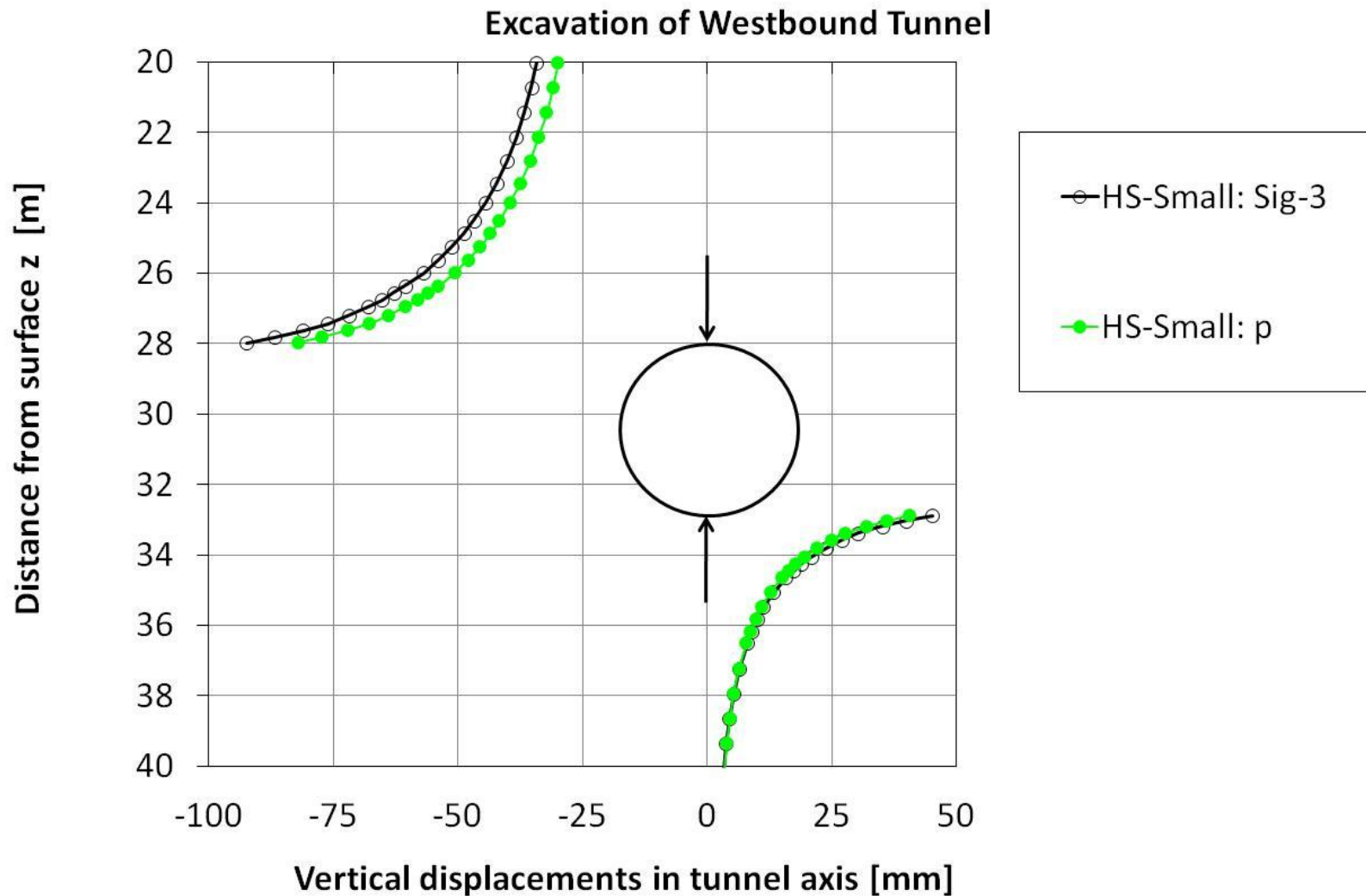
Even if
 $E_{50}^{\sigma_3} > E_{50}^p$
 $E_{ur}^{\sigma_3} > E_{ur}^p$

Stress dependent stiffness: σ_3 vs p' formulation

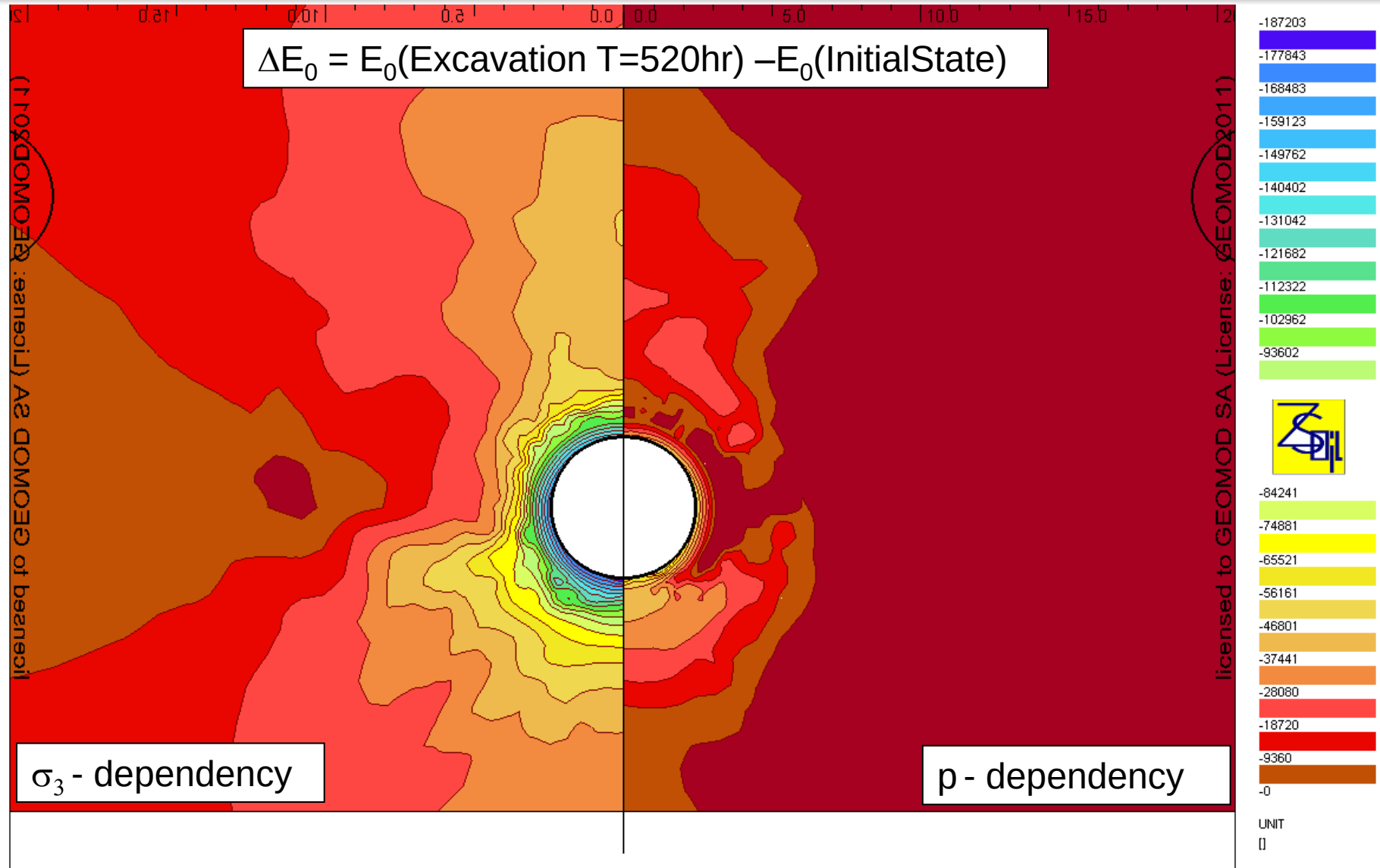


Even if
 $E_0^{\sigma_3} = E_0^p$

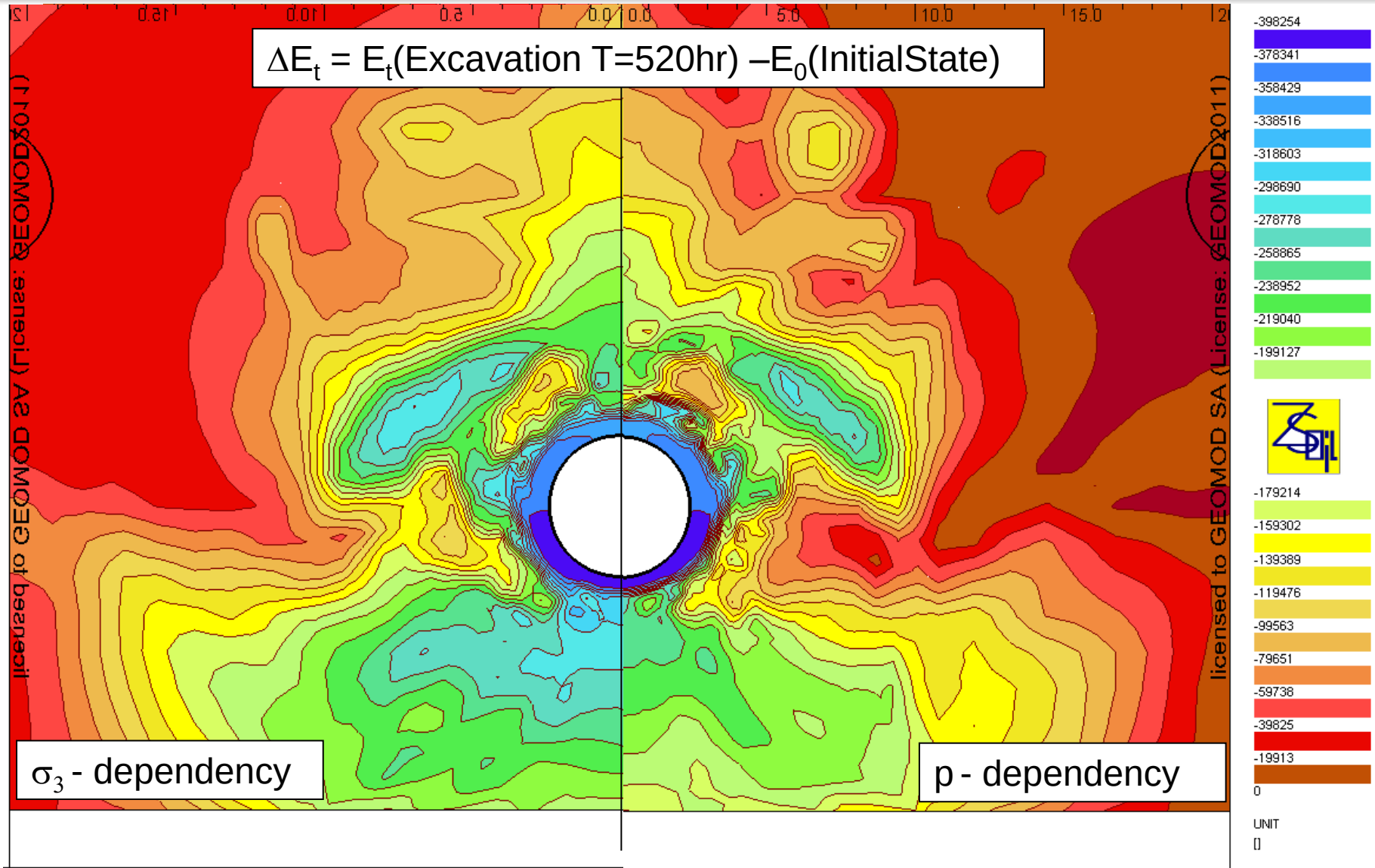
Stress dependent stiffness: σ_3 vs p' formulation



Initial stiffness degradation



Tangent stiffness degradation



Content

- ❑ Introduction

Part 1

- ❑ Parameter estimation toolbox v2012
- ❑ Combined parameter determination based on laboratory and field test results

Part 2

- ❑ Parameter migration from σ'_3 to p'-stress stiffness dependency
- ❑ **Parameter identification**
- ❑ Enhanced parameter identification supported by the gradient-based optimization

Definitions used in this presentation

Parameter determination refers to a reliable **assessment of design soil properties** which allow us to reproduce soil behavior by means of a numerical analysis assuming that an adequate constitutive model has been chosen.

- **Parameter estimation** refers to evaluation of values of model parameters by using **empirical correlations** obtained through statistical regression analyses. Such correlations relate test measurements with a constitutive parameter.
- **Parameter identification** refers to deterministic algorithms which are developed for given constitutive model and test geometry. These **analytical solutions** allow us to directly identify model parameters from test measurements.
- **Parameter optimization** refers to minimizing the difference between results obtained from a numerical simulation and experimental curves (i.e. **curve-fitting**). This procedure may consist of an iterative minimization of the defined objective function using a gradient-based optimization algorithm.

Parameter identification

1. **Parameter identification** from laboratory curves

A. TXC: ε_1 - q , ε_1 - ε_v

B. OED: σ_v - ε_v

2. Reproduction of laboratory curves with a given constitutive model

3. If the level of fit unsatisfactory -> **optimization**

Current works

Parameter identification from laboratory curves

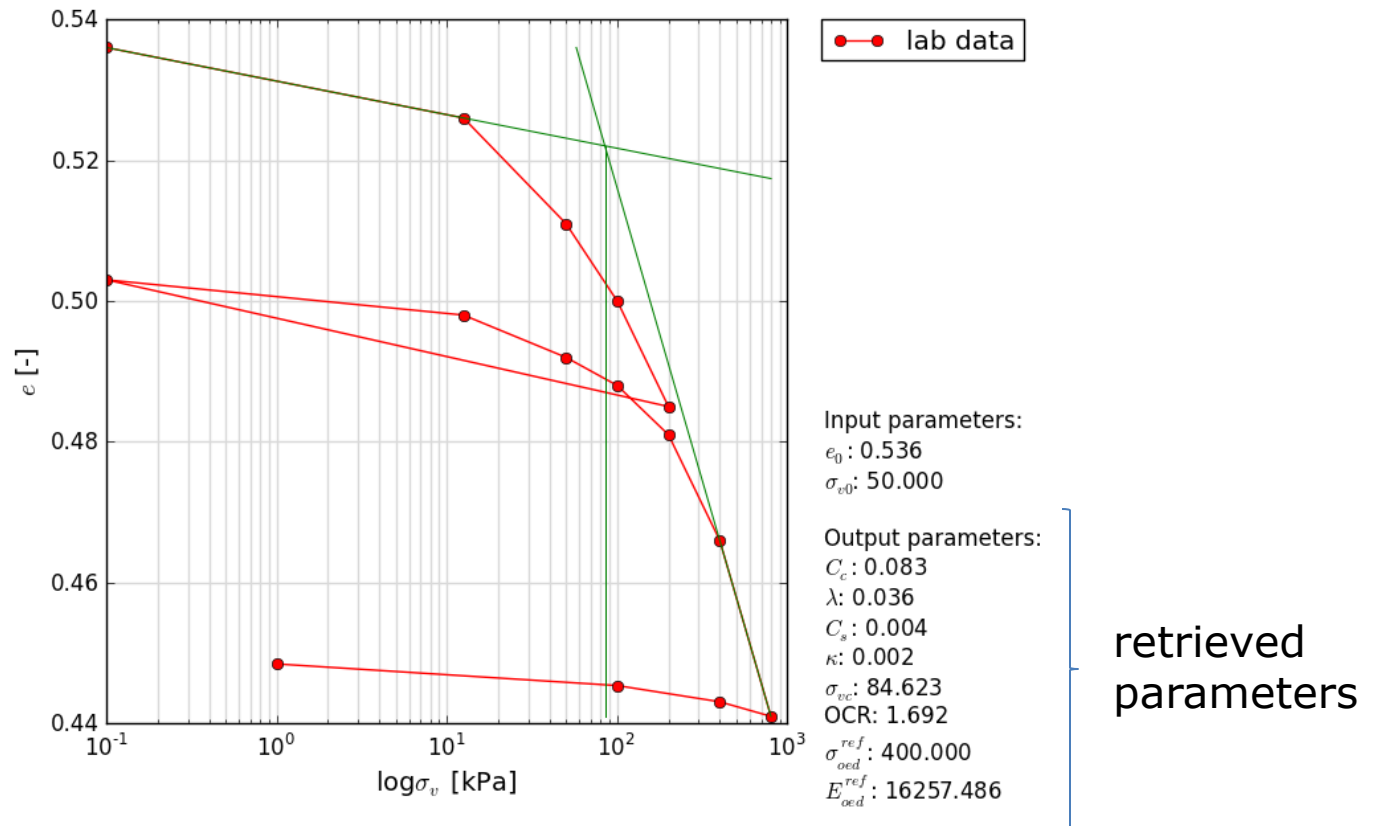
Analysis using a spreadsheet

example of parameter identification from triaxial compression test

	Sample	0.6m			3.0m			Mean
	σ_3 [kPa]	34.5	138	345	34.5	138	345	
	q_f [kPa]	112.3	417.7	994.4	114.9	417.9	994.9	
	σ_1 [kPa]	146.8	555.7	1339.4	149.4	555.9	1339.9	
	$p(q_f)$ [kPa]	71.9	277.2	676.5	72.8	277.3	676.6	
	Identification ϕ and c							
	M [-]	1.488			1.482			
	c^* [kPa]	5.262			7.06			
	ϕ [deg]	36.6			36.5			36.5
	c [kPa]	2.6			3.5			3.1
	Identification R_f and $E_{50}(\sigma_3)$							
	a	8.48E-03	2.12E-03	8.82E-04	8.40E-03	2.11E-03	8.75E-04	
	b	5.01E-05	3.68E-05	1.57E-05	4.90E-05	3.68E-05	1.65E-05	
	R_f [-]	0.95	0.89	0.88	0.97	0.88	0.87	0.92
	E_{50} [kPa]	9988	13605	31786	10210	13576	30358	
	Identification E_{50}^{ref} and m							
	$\ln E_{50}$	9.21	9.52	10.37	9.23	9.52	10.32	
	σ_{ref} [kPa]	100			100			100
	a	0.367	1.367	3.366	0.375	1.363	3.339	
	$\ln(a)$	-1.001	0.313	1.214	-0.981	0.309	1.206	
	m [-]	0.521			0.494			0.51
	E_{50}^{ref} [kPa]	16012.7			15820.5			15917
	$a = (\sigma_3 + c \cot \phi) / (\sigma_{ref} + c \cot \phi)$							
	Identification ψ							
	d	0.049	0.039	0.010	0.048	0.037	0.011	
	ψ [deg]	1.37	1.10		1.35	1.04		1.27

Automated parameter retrieving

Example of an automated parameter identification



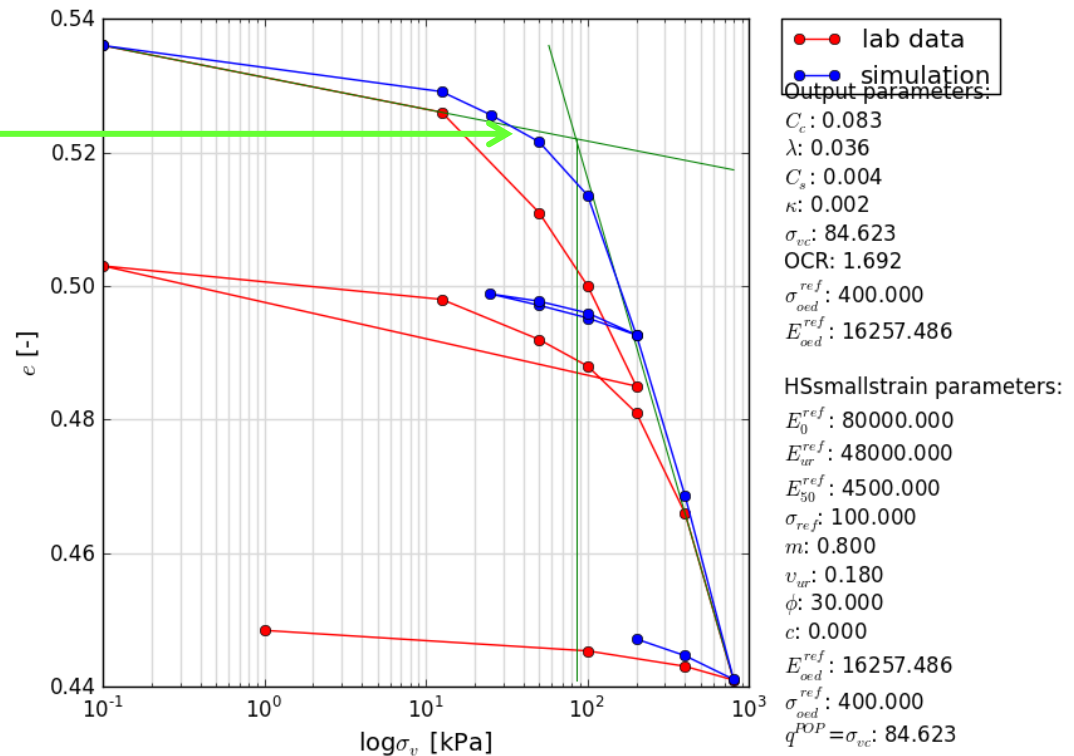
Automated reconstruction of laboratory curves

Automatic step densification

in case of model instability because of:

a) “difficult” vector of parameters

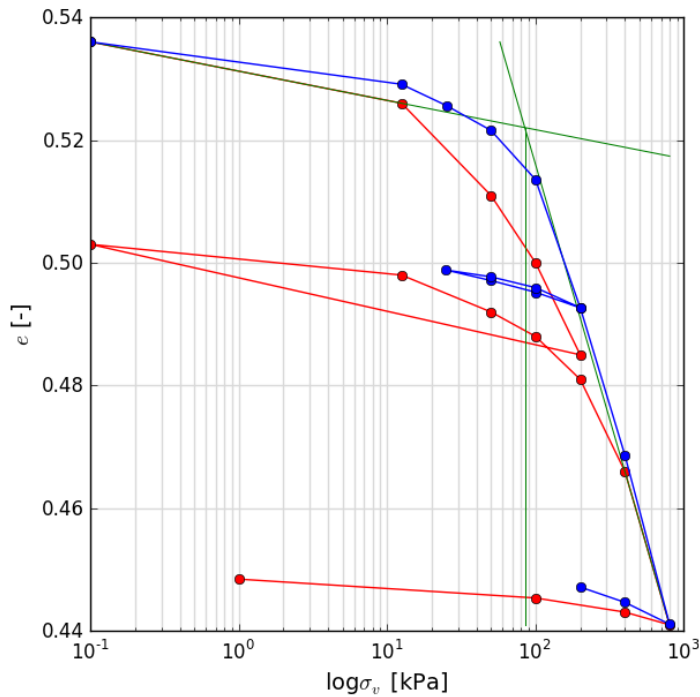
b) excessive load increment



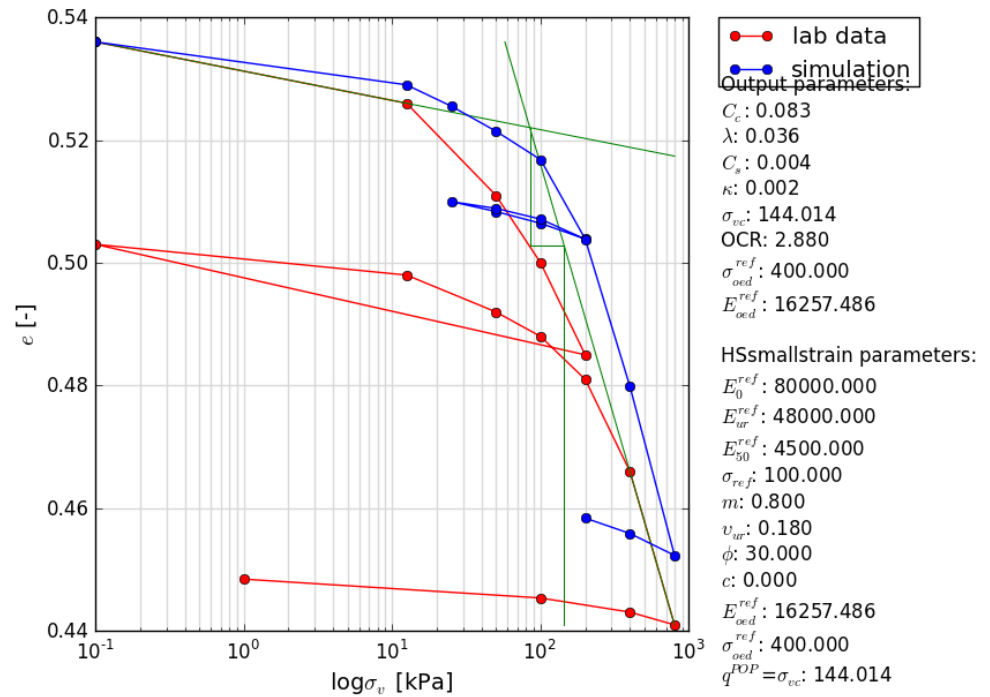
Automated reconstruction of laboratory curves

Simulations of OED test with **HS model** and
prediction of preconsolidation pressure with:

Intersection method



Pacheco-Silva method

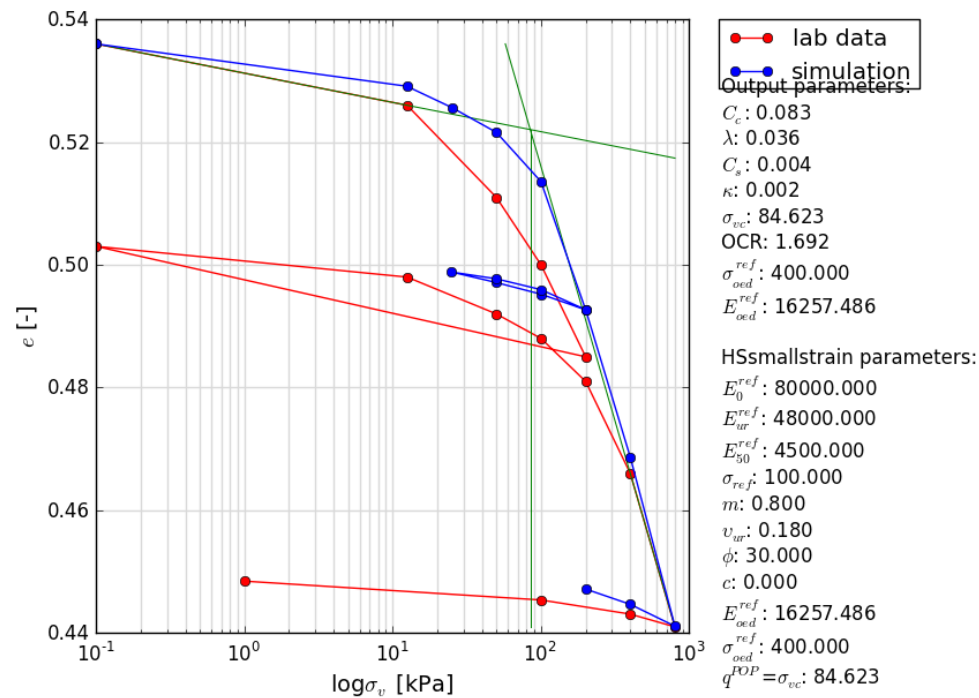


Parameter identification from laboratory curves

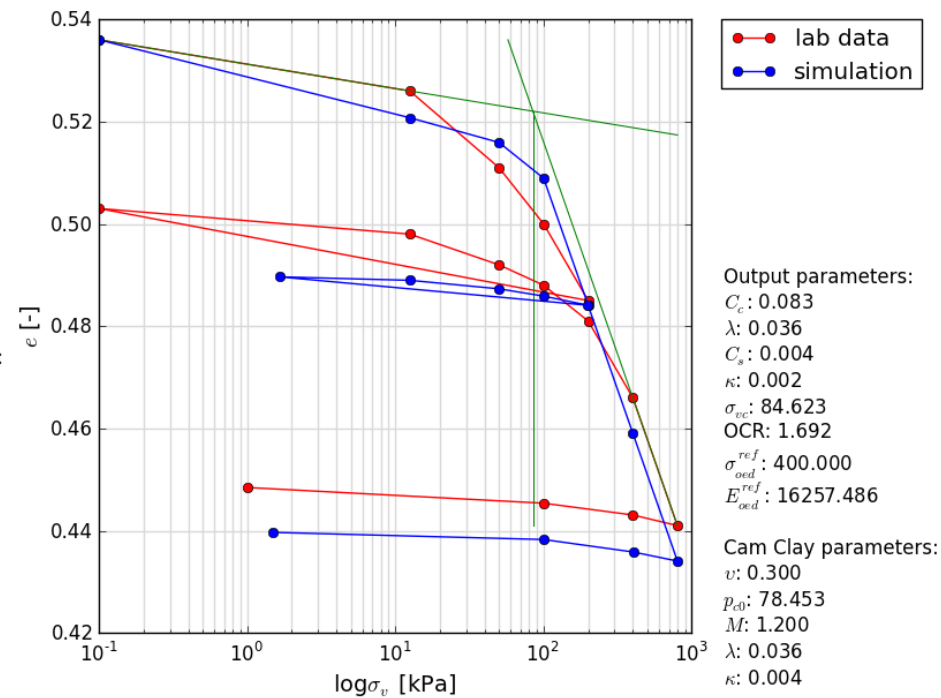
Automated reproducing laboratory curves

Simulations of OED test with **different models**

HS model



Modified Cam-clay



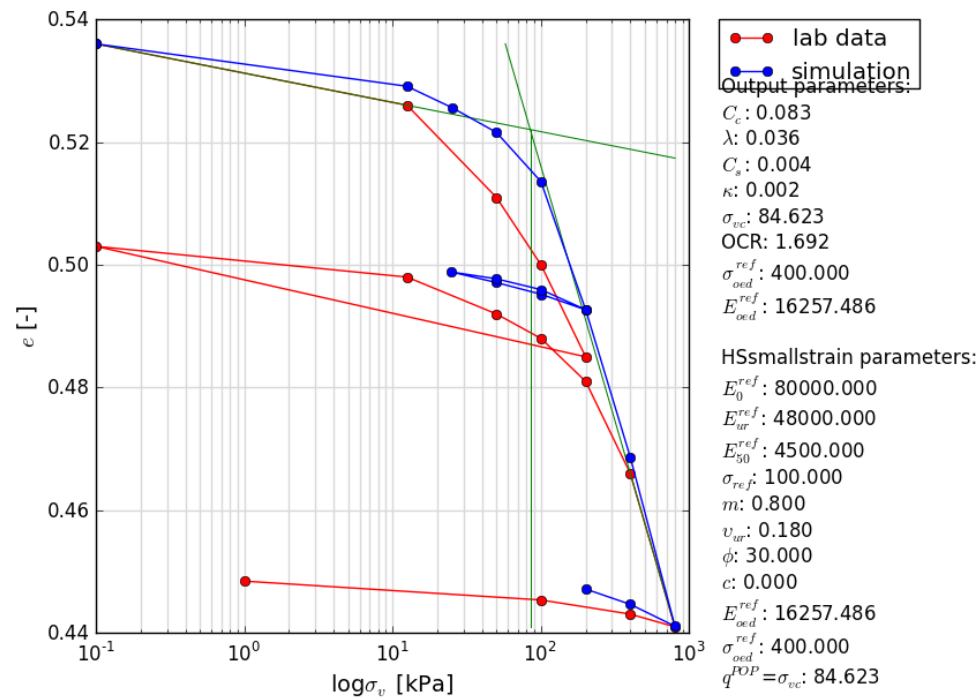
NB. κ correction because identified κ/λ ratio < 0.1

Parameter identification from laboratory curves

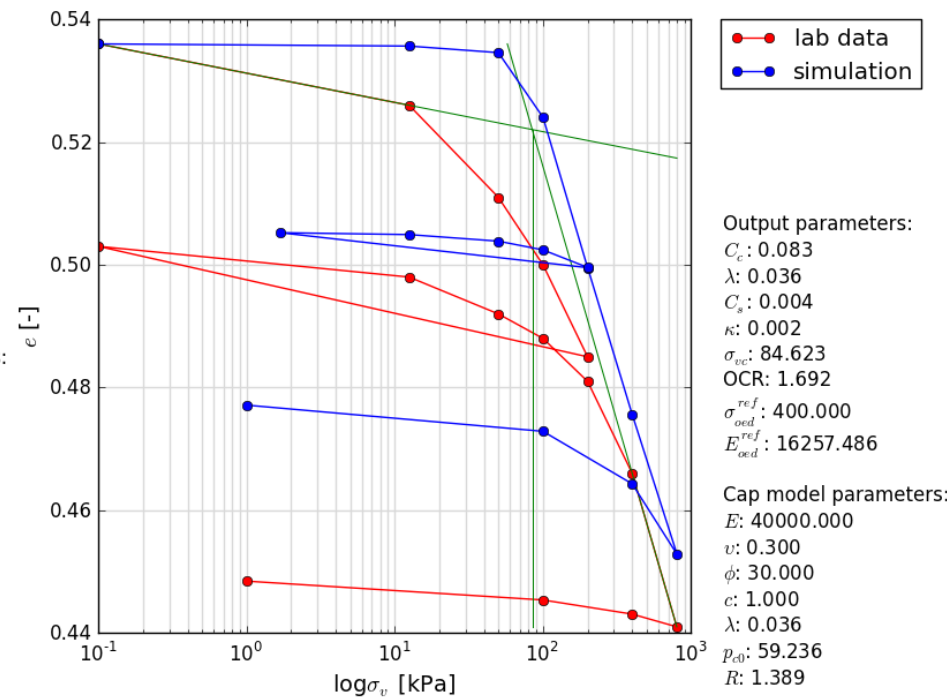
Automated reproducing laboratory curves

Simulations of OED test with **different models**

HS model



CAP model



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- ❑ Introduction

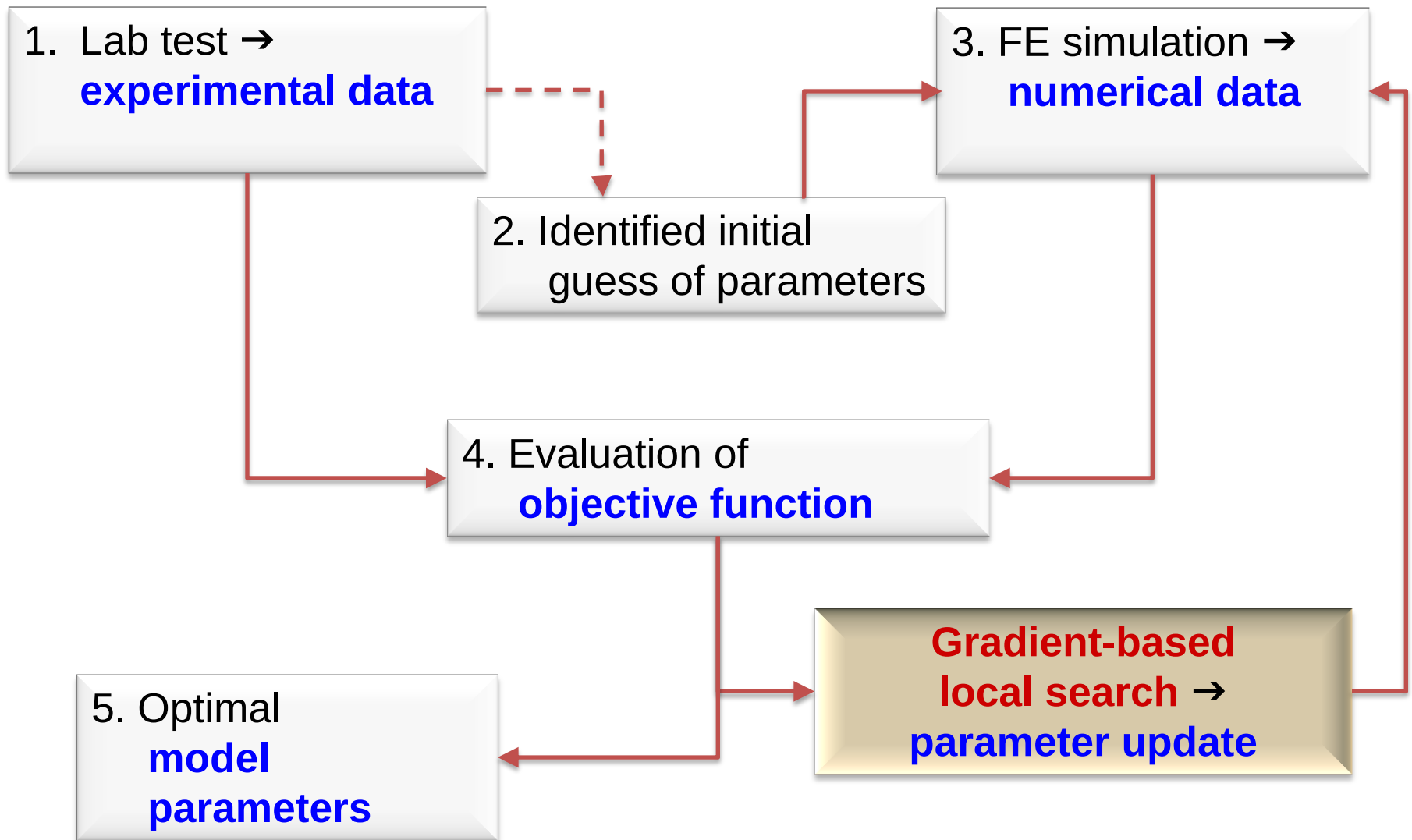
Part 1

- ❑ Parameter estimation toolbox v2012
- ❑ Combined parameter determination based on laboratory and field test results

Part 2

- ❑ Parameter migration from σ'_3 to p' -stress stiffness dependency
- ❑ Parameter identification
- ❑ **Enhanced parameter identification supported by the gradient-based optimization**

Enhanced parameter identification



Objective function and gradient-based algorithm

Objective Function: Weighted Least-Squares

$$S(\mathbf{b}_r) = [\mathbf{y} - \mathbf{y}'(\mathbf{b}_r)]^T \boldsymbol{\omega} \cdot [\mathbf{y} - \mathbf{y}'(\mathbf{b}_r)]$$

vector of
measurement

vector of
numerical results

diagonal matrix of
weights

Gradient-based algorithm: Modified Gauss-Newton

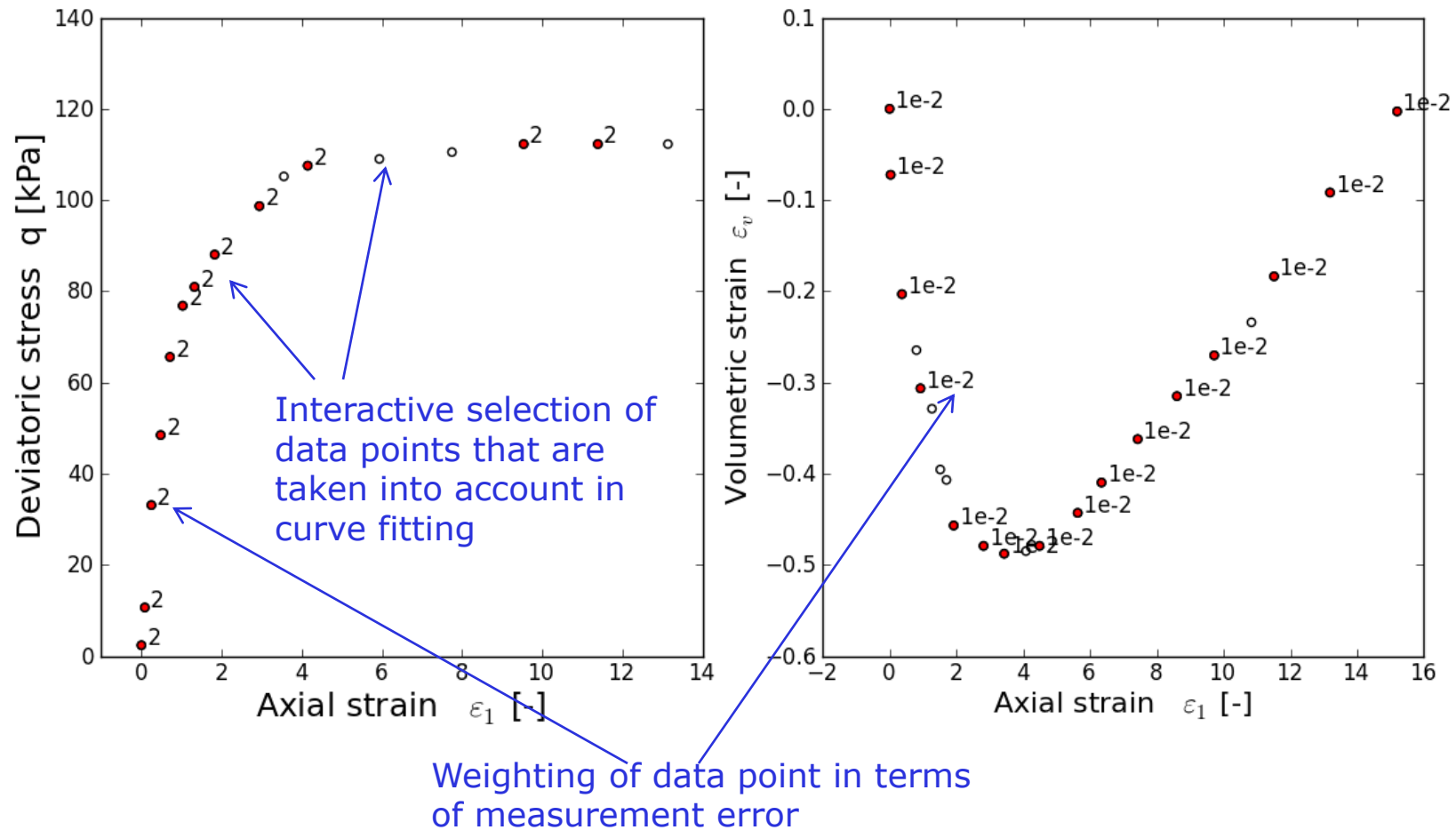
including:

- scaling parameter controlling magnitude of update
- oscillation control

$$(\mathbf{C}^T \mathbf{X}_r^T \boldsymbol{\omega} \cdot \mathbf{X}_r \mathbf{C} + \mathbf{I} \cdot m_r) \mathbf{C}^{-1} \cdot \mathbf{d}_r = \mathbf{C}^T \mathbf{X}_r^T \boldsymbol{\omega} \cdot (\mathbf{y} - \mathbf{y}'(\mathbf{b}_r))$$
$$\mathbf{b}_{r+1} = \rho_r \cdot \mathbf{d}_r + \mathbf{b}_r$$

Parameter optimization – setting weights

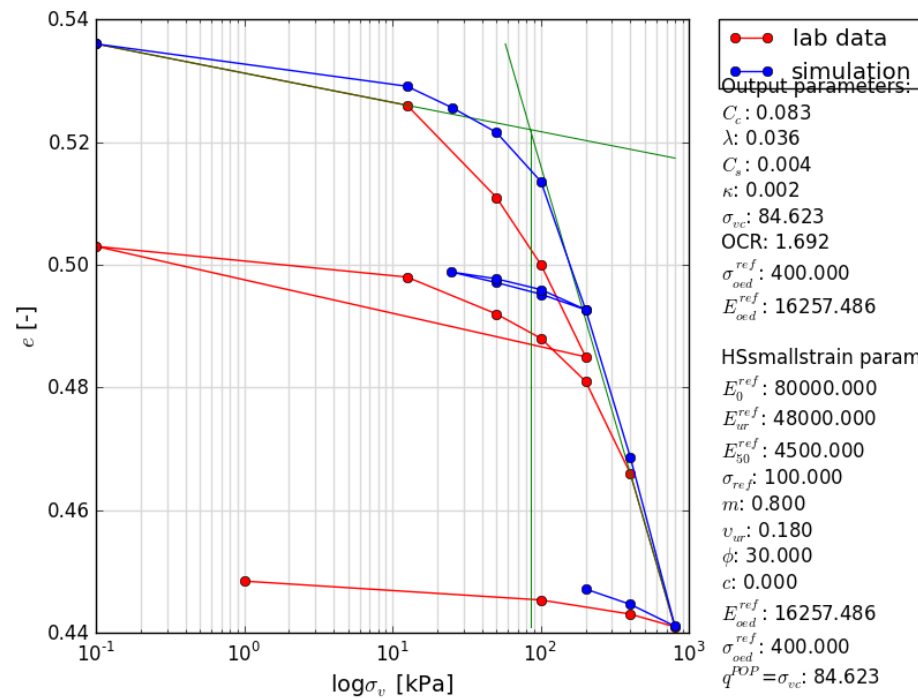
1 specimen – 2 different experimental curves



Automated curve fitting

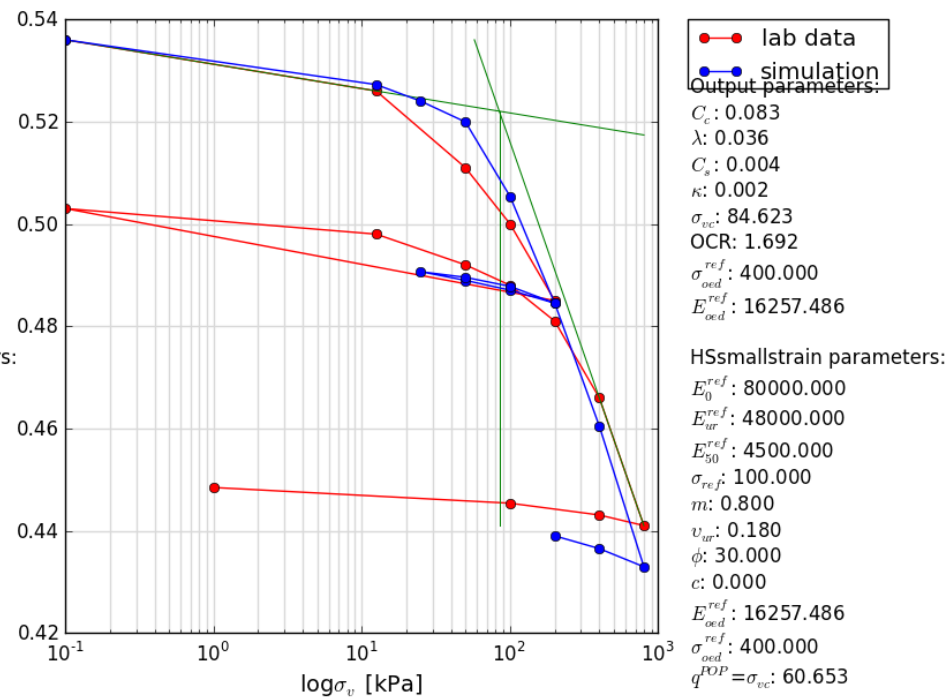
HS model

Initial $\sigma'_{vc} = 84.6$ kPa



HS model

Initial $\sigma'_{vc} = 60.6$ kPa



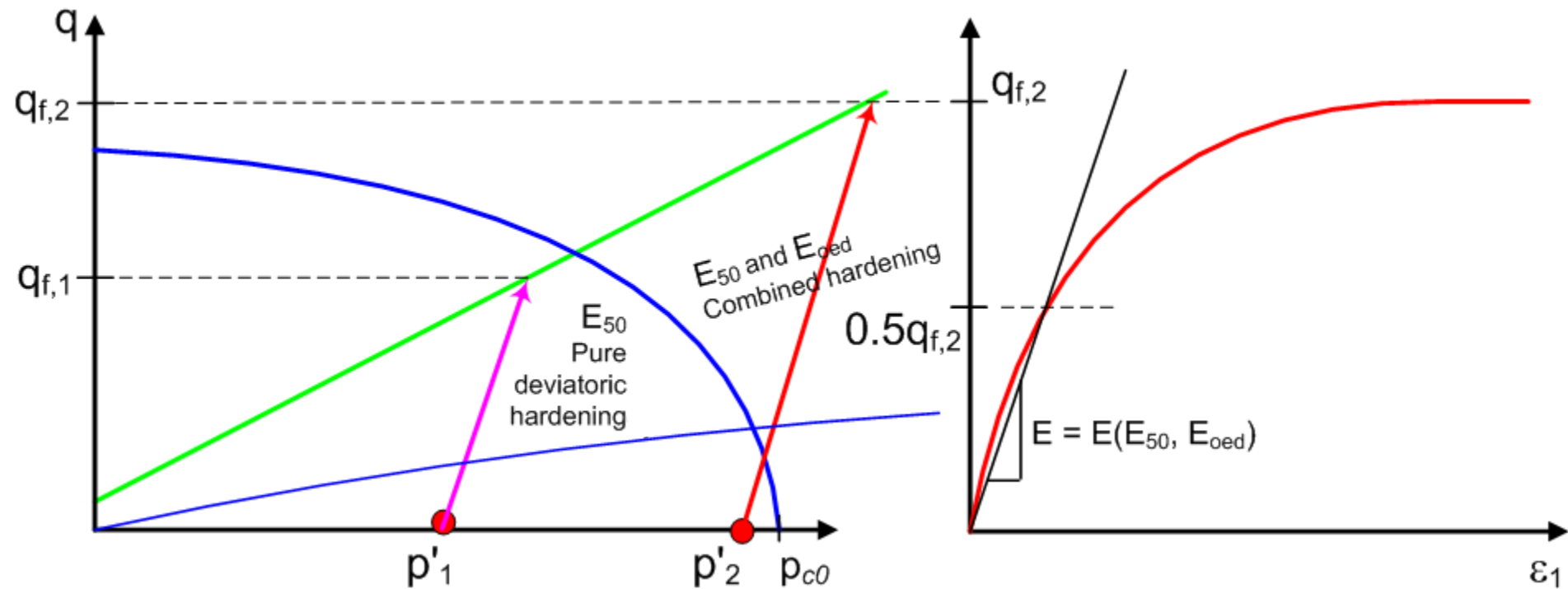
Potential benefits from parameter optimization

Examples:

- ✓ Determination of preconsolidation pressure from an oedometer test
- ✓ Multiple identification from a large number of specimens and different tests
- ✓ Identification of E_{50} and m from normally- or lightly-consolidated specimen
- ✓ ...

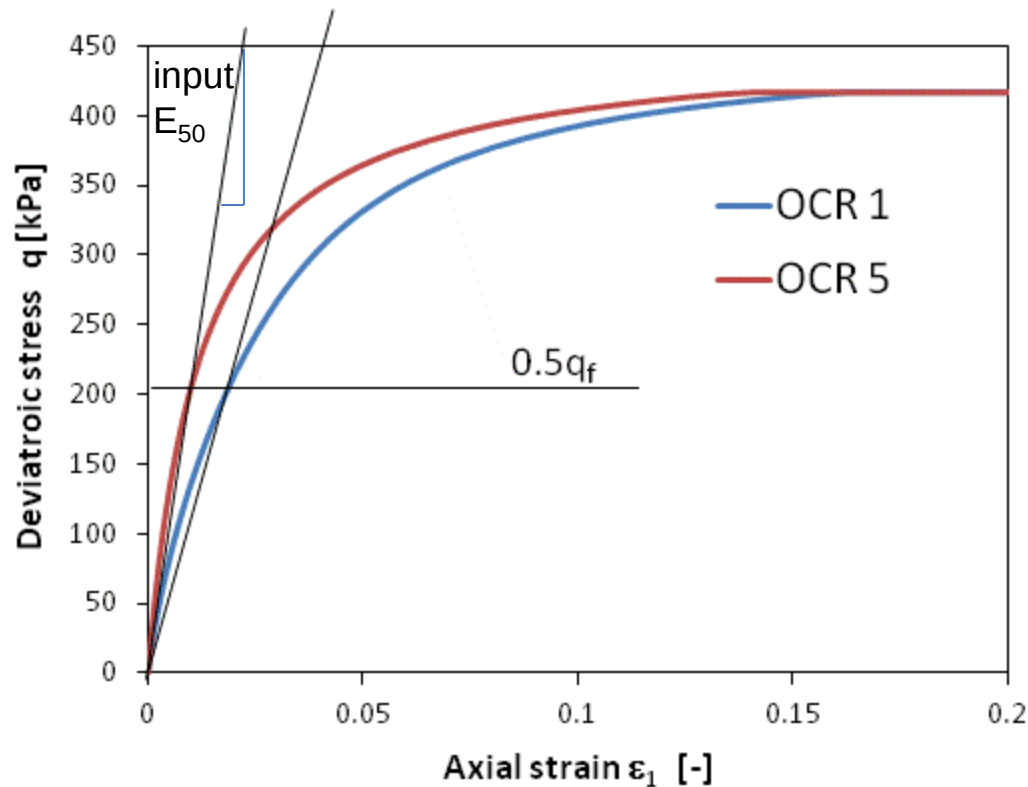
Identification of E_{50} and m

may be affected by volumetric plastic straining in case of **normally- and lightly overconsolidated** soil specimens



Identification of E_{50} and m

Example: 2 simulations for the same E_{50} and confining pressure $\sigma_0 = 138\text{kPa}$ and different $p_{c0} = 138\text{kPa}$ (OCR=1) and $p_{c0} = 690\text{kPa}$ (OCR=5)



Future for parameter selection in ZSoil

I. Basic parameter estimation

II. Advanced parameter estimation

III. Identification from laboratory curves

IV. Reproduction of elementary tests for selected parameters

V. Automated curve-fitting

GOAL:

All-in-one in a user-friendly and intuitive graphical user interface

Available for different models

Thank you for your attention



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