Creep and aging in nonlinear models for concrete

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Assumptions and hypotheses

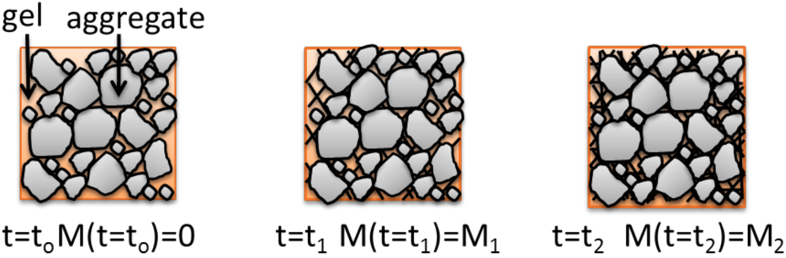
- in meso-scale effect of solidification takes place in the cement paste
- solidification is driven by kinetics of chemical reactions
- stiffness and strength depend on amount of released heat due to hydration

$$
\begin{aligned}
H(M) &= H_\infty \frac{a\,M}{1 + a\,M} \\
M(t) &= \int_{t_d}^{t} e^{\frac{Q}{R}\left[\frac{1}{T_{ref}} - \frac{1}{T(\tau)}\right]} d\tau
\end{aligned}
\qquad (1)
$$

General formulations to handle concrete aging and damage/plasticity at early age

- (1) Layered approach (proposed by Truty, Szarliński and Podleś (*Computers and Concrete 2016*))
- (2) Simplified description of concrete aging following the idea by Bažant, Jirásek and Havlásek (PhD thesis, Prague) (this approach is adopted in ZSoil for modified plastic damage model)

General concept of layered medium



- At any point in the structure, concrete is composed of a set of layers
- Layers are activated in time one by one, basing on amount of released heat
- This way each layer can be modeled as an old one

Notion of effective stress

- Cross section is composed of N_{act} layers at current time instance
- Each layer is carrying a certain part of the applied external forces
- Effective stress

$$\Delta\sigma = \frac{\Delta F}{\Delta A} = \frac{\sum_{K=0}^{N_{\text{act}}-1} \Delta F_k}{\Delta A} = \frac{\sum_{K=0}^{N_{\text{act}}-1} \Delta\sigma_K \; w_K \; \Delta A}{\Delta A} =$$

$$= \sum_{K=0}^{N_{\text{act}}-1} \Delta\sigma_K \; w_K$$

- So far we assume that $\sum_{K=0}^{\text{N-1}} w_K = 1$

Constitutive equations for layers

- Incremental constitutive law for layered model

$$\begin{aligned}\Delta\boldsymbol{\sigma} &= \sum_{K=0}^{N_{\mathrm{act}}-1} \Delta\boldsymbol{\sigma}_K \, w_K \\ \Delta\boldsymbol{\sigma}_K &= \mathbf{D}^{\mathrm{e}} \left(\Delta\varepsilon - \Delta\varepsilon_K^{p} - \Delta\varepsilon_K^{c} - \Delta\varepsilon^{o}\right)\end{aligned}$$

- Conclusion: any implemented constitutive model worked out for the continuum can be used for description of a single layer

Simple elastic-plastic law for layers (1D just for testing)

- Elasto-plastic model with linear softening

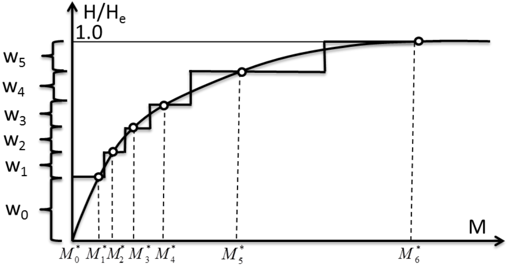
$$
\begin{aligned}
F(\boldsymbol{\sigma}_K) &= \sigma_{1,K} - f_{ct} \leq 0 && (2)\\
\Delta\varepsilon_K^p &= \Delta\lambda \frac{\partial F}{\partial \boldsymbol{\sigma}_K} && (3)\\
\Delta f_{ct} &= H^p \, \Delta\varepsilon_{1,K}^p && (4)
\end{aligned}
$$

- Softening modulus

$$H^p = -\alpha_{sf} \; E \qquad (5)$$

Procedure for activation of layers in time

- How to set maturities M_K at which $K-$th layer is activated

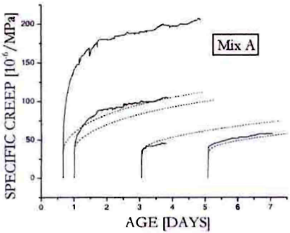


Description of basic creep

Basic creep compliance function $J(t, t')$ by Bažant and Kim

$$J(t, t') = q_1 + F(\boldsymbol{\sigma}) \left(q_2 \; Q(t, t') + q_3 \; \ln(1 + \left(\frac{t - t'}{\lambda_o} \right)^n) + q_4 \; ln \frac{t}{t'} \right)$$

- Parameters λ_o and n are fixed to $\lambda_o = 1$ [days] and $n = 0.1$
- $F(\boldsymbol{\sigma}) = 1$

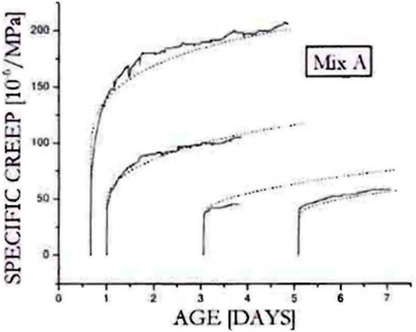


Prediction by classical model

Basic creep in classical solidification theory: theory vs experiment

Experiment by Østegard at al. (2001)

$$\tilde{q}_2 = q_2 \frac{t'}{t' - q_5}$$



Prediction with Østegard's correction

Definition of E modulus and specific creep $C(t, t')$

- Derivation of E modulus from $J(t, t')$

$$E(t') = J^{-1}(t' + \Delta t_o, t')$$

  here Δt_o is usually assumed as 0.001 [days].

- Specific creep

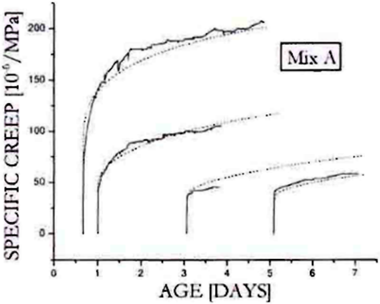
$$C(t, t') = J(t, t') - J(t' + \Delta t_o, t')$$

- Taking into account the fact that each newly created layer is a non-aging one the $C(t, t')$ function can be expressed as follows

$$C(t, t') \approx \underbrace{q_3 \ln \frac{1 + (t - t')^n}{1 + (\Delta t_o)^n}}_{C_{ve}(t,t')} + \underbrace{q_4 \ln \frac{t}{t'}}_{C_{vf}(t,t')}$$

Example 1: Basic creep test

- Constant temperature and humidity



Example 1: Basic creep test

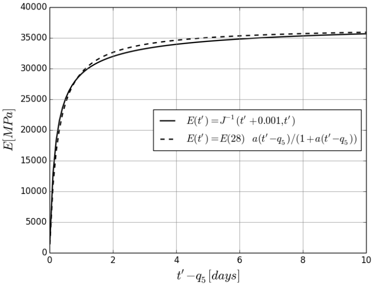
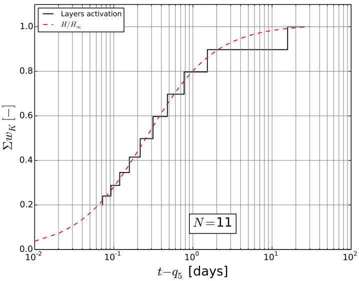
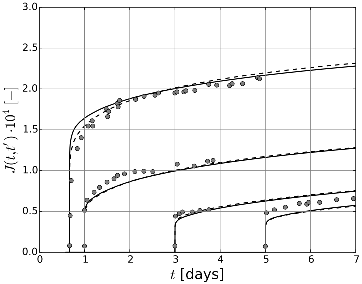
- Material data after Østegard
-
$$E(t') = E(28)\frac{a(t'-q_5)}{1+a(t'-q_5)}$$
- Best fit: $a = 3.876$ [days]$^{-1}$
- 

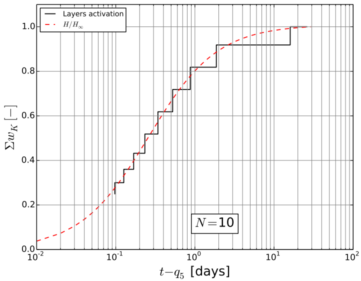
Figure: Evolution of standard Young moduli in time according to experimental data by Østegard

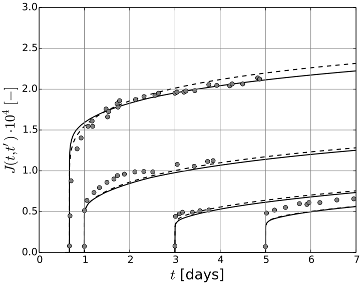
Example 1: Basic creep test ($w_o = 0.2$)





Example 1: Basic creep test ($w_o = 0.25$)





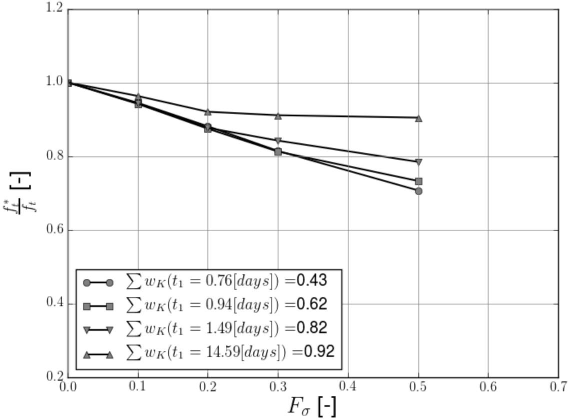
Example 2: Tensile strength of preloaded sample

- Compressive preloading program ($t_2 = 28$ [days])

$$\sigma(t) = 0 \quad \text{for} \quad t \leq t_1 \tag{6}$$
$$\sigma(t) = F_\sigma \, f_c(t_1) \quad \text{for} \quad t = t_1..t_2 \tag{7}$$

- Loading time instances $t_1 = 0.17, 0.35, 0.90, 14.0$ [days]
- Five levels of compressive preloading $F_\sigma =$0.0, 0.1, 0.2, 0.3, 0.5.
- Corresponding levels of solidification: $\sum w_K(t_1) = 0.43, 0.62, 0.82, 0.92.$

Example 2: Tensile strength of preloaded sample (creep ON)



Reduction of peak tensile strength due to initial compressive preloading (creep active)

Example 2: Stresses in layers during preloading

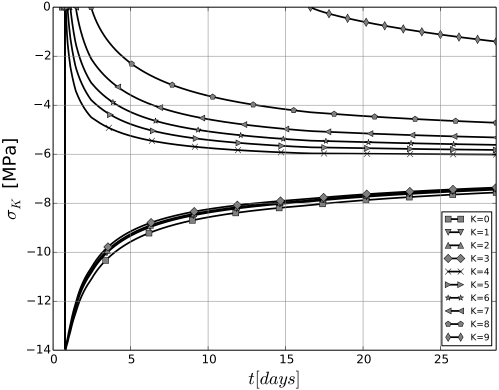


Figure: Effective stress time histories during compressive preloading applied at $t_1 = 0.17$ [days] with loading level $F_\sigma = 0.5$ (creep active)

Example 2: Stresses in layers during tensile loading

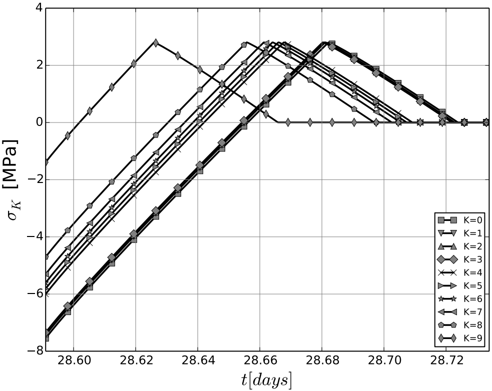


Figure: Effective stress time histories during tensile kinematic loading preceded by the compressive preloading applied at $t_1 = 0.17$ [days] with loading level $F_\sigma = 0.5$ (creep active)

Concluding remarks on layered model

1. Layered framework is an external shell over the standard implemented constitutive model
2. In each Gauss point we need to have at least 10 layers $\rightarrow$ high computational effort
3. Maybe that parallel implementation using CUDA could help to use this kind of a model in commercial codes
4. Activation procedure is designed to keep the approximation error within certain limits
5. Creep of aging concrete can be properly reproduced (much better than in the solidification model)
6. Initial compressive preloading of concrete may significantly reduce its tensile strength (to be verified in the experiment) (hilfe.....)

Extensions of damage model to aging and creep according to EC2 standard

1. Motivation: adapt the model to the EC2 standard concerning creep and aging
2. In the EC2 (EN 1992-1-1:2004+AC:2008) creep is assumed as viscoelastic
3. It is supported in ZSoil for continuum and shell formulations
4. Strength and stiffness do not develop with the same rate during maturing (solid+water are treated in a simplified manner)
5. No distinction is made between creep in compression and tension

Extensions of damage model to aging and creep according to EC2 standard

- Time dependent creep coefficient: $\phi(t,t_o) = \phi_o \beta_c(t,t_o)$
- Basic creep coefficient: $\phi_o = \phi_{RH}\beta(f_{cm})\beta(t_o)$
- $$\phi_{RH} = \begin{cases} 1 + \dfrac{1 - \dfrac{RH}{100}}{0.1\ h_o^{1/3}} & \text{for} \quad f_{cm} \leq 35 \text{ MPa} \\ \left(1 + \dfrac{1 - \dfrac{RH}{100}}{0.1\ h_o^{1/3}}\alpha_1\right)\alpha_2 & \text{for} \quad f_{cm} > 35 \text{ MPa} \end{cases}$$
- $$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$
- $$\beta(t_o) = \frac{1}{0.1 + t_o^{0.2}}$$
- $$\beta_c(t,t_o) = \left[\frac{t - t_o}{\beta_H + t - t_o}\right]^{0.3}$$ → power law

Extensions of damage model to aging and creep according to EC2 standard

- Time parameter t and t_o can be replaced by a corresponding temperature adjusted value t_T defined as follows
$$t_T(t) = \int_{t_1}^{t} \exp\left(-\frac{Q}{R}\left(\frac{1}{273 + T(\tau)} - \frac{1}{273 + T_{ref}}\right)\right) d\tau$$
$(T_{ref} = 20\ [^oC])$

Implementation of the EC2 creep model in ZSoil 2016

- Implementation scheme is partially based on the algorithm given in PhD by Havlásek
- Creep strain increment is computed using the following scheme (Kelvin chain of units)

  $$\Delta\varepsilon^{cr}_{n+1} = \mathbf{D}_o^{-1} \frac{1}{v^{cr}_{n+1/2}} \sum_{\mu=1}^{N} A_\mu \left(1 - \beta_{\mu,n+1}\right) \boldsymbol{\sigma}_{v\mu,n}$$

  where:
  - $\mathbf{D}_o^{-1}$ is an elastic compliance matrix computed for unit Young's modulus
  - $v^{cr}_{n+1/2}$ is an extra scaling factor amplifying creep rate due to aging phenomenon (here it is not equivalent to the fraction of solidified layers)
  - $\boldsymbol{\sigma}_{v\mu,n+1}$ represents viscous effective stresses in $\mu - th$ Kelvin unit
  - A_μ is the ultimate creep strain value in μ-th Kelvin unit

Implementation of the EC2 creep model in ZSoil 2016

- Viscous stress update:
  $\boldsymbol{\sigma}_{v\mu,n+1} = \beta_{\mu,n+1}\boldsymbol{\sigma}_{v\mu,n} + \lambda_{\mu,n+1}\Delta\overline{\boldsymbol{\sigma}}_{n+1}$
- $\lambda_{\mu,n+1} = (1 - \beta_{\mu,n+1})\dfrac{\tau_\mu}{\Delta t_{n+1}}$
- The algorithmic effective Young's modulus is expressed as follows
  $$\overline{E} = \frac{1}{\dfrac{1}{v^E E_{28}} + \dfrac{1}{v^{cr}} \sum_{\mu=1}^{N} \left(1 - \lambda_{\mu,n+1}\right) A_\mu}$$
- $v^E = \sqrt{\beta_{cc}}$
- $\beta_{cc} = \begin{cases} \exp(s(1 - \sqrt{28/t})) & \text{for} \quad t \leq 28 \text{ days} \\ 1 & \text{for} \quad t > 28 \text{ days} \end{cases}$

Derivation of v^{cr} function

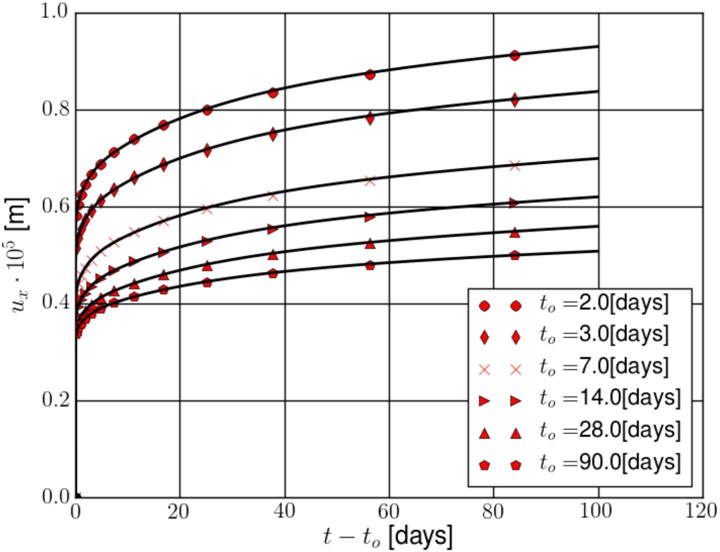
- Evolution of creep strain in time, according to EC2, can be expressed by the following equation
  $$\varepsilon^{cr} = A_1 \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o)$$
  where $A_1 = \phi_{RH} \beta(f_{cm})$
- Evolution of the reference creep strain for concrete loaded at $t_o = 28$ days (matured concrete) can be defined as
  $$\varepsilon^{cr}_{ref} = A_1 \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o = 28)$$
- The reference creep strain curve is taken here as a basis for optimization of A_μ coefficients in chain of nonaging Kelvin units (retardation times τ_μ are predefined by considering duration of carried out analysis time)
- To derive v^{cr} we assume the following creep strain rates compatibility condition
  $$\dot{\varepsilon}^{cr} = \frac{1}{v^{cr}} \dot{\varepsilon}^{cr}_{ref}$$
- This yields the following definition of v^{cr} function
  $$v^{cr} = \frac{\beta(t_o = 28)}{\beta(t_o)}$$
  where t_o is the age of concrete at the beginning of analysis

User interface for creep model

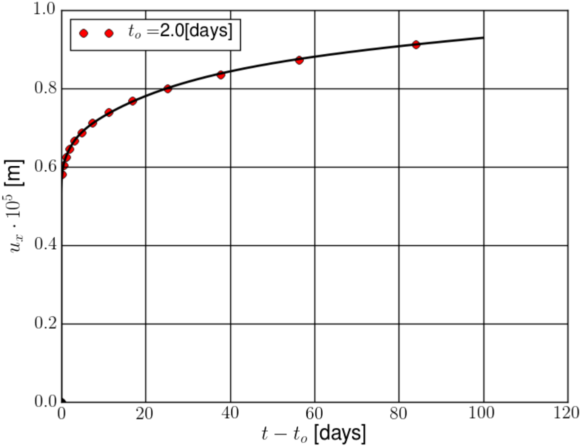
Reference: EC2: EN-1992-1-1:2004+AC:2008

Parameter	Unit	Range	Description
A	[1/MPa]	$9*10^{-5}$ (default)	$A = A_1 = \phi_{RH}\beta(f_{cm})/E$ (see EC2)
B	[day]	500 (default)	$B = \beta_H$ (see EC2)
Initial age	[day]	> 0.5	Age of analyzed concrete
Equivalent time flag		ON/OFF	Flag whether to use temperature adjusted time
$\frac{Q}{R}$	[K]	4000	Ratio between activation energy and universal gas constant
T_{ref}	[C]	20	Reference temperature
s	[-]	0.38	Strength evolution parameter (EC2)
t_{28}	[day]	28.0	Time of 28 days in formula for β_{cc} (EC2)
n	[-]	0.5	Exponent in expression for stiffness modulus (applied to β_{cc}) (EC2)

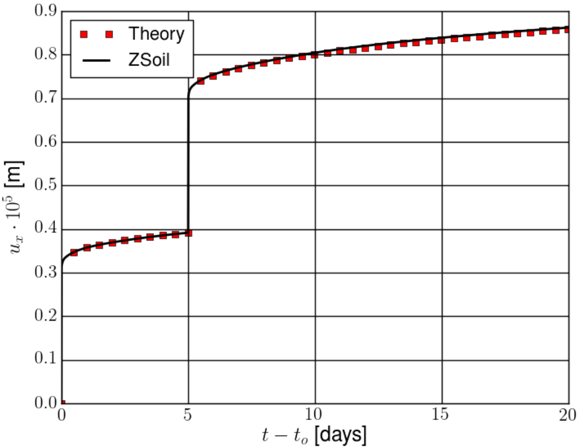
Benchmark: monotonic creep (continuum)



Benchmark: monotonic creep (shell)



Benchmark: creep under variable loading conditions



Conclusions

1. Plastic damage model is the only one that supports EC2 creep/aging
2. It is developed for continuum and shell elements
3. Simplified approach following the idea by Bažant, Jirásek and Havlásek was adopted for the implementation
4. The proposed approach follows the EC2 standard